MATHEMATICS

Chapter 10: STRAIGHT LINES

Important Questions

Multiple Choice questions-

Question 1. In a ΔABC, if A is the point (1, 2) and equations of the median through B and C are respectively x + y = 5 and x = 4, then B is

(a) (1, 4)

(b) (7, -2)

(c) none of these

(d) (4, 1)

Question 2. The equation of straight line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0

(a) y – x + 1 = 0

(b) y – x – 1 = 0

(c) y – x + 2 = 0

(d) y – x – 2 = 0

Question 3. The points (-a, -b), (0, 0), (a, b) and (a^2, ab) are

(a) vertices of a square

(b) vertices of a parallelogram

(c) collinear

(d) vertices of a rectangle

Question 4. The equation of the line through the points (1, 5) and (2, 3) is

(a) 2x – y – 7 = 0

(b) 2x + y + 7 = 0

(c) 2x + y – 7 = 0

(d) x + 2y – 7 = 0

Question 5. The slope of a line which passes through points (3, 2) and (-1, 5) is

(a) 3/4

(b) -3/4

(c) 4/3

(d) -4/3

Question 6. The ratio of the 7th to the (n – 1)th mean between 1 and 31, when n arithmetic means are inserted between them, is 5 : 9. The value of n is

(a) 15

(b) 12

(c) 13

(d) 14

Question 7. The ortho centre of the triangle formed by lines xy = 0 and x + y = 1 is :

(a) (0, 0)

(b) none of these

(c) (1/2, 1/2)

(d) (1/3, 1/3)

Question 8. Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel if

(a) $a_1 /a_2 = b_1 /b_2 \neq c_1 /c_2$

(b) $a_1 /a_2 \neq b_1 /b_2 = c_1 /c_2$

(c) $a_1 /a_2 \neq b_1 /b_2 \neq c_1 /c_2$

(d) $a_1 /a_2 = b_1 /b_2 = c_1 /c_2$

Question 9. If the line x/a + y/b = 1 passes through the points (2, -3) and (4, -5), then (a, b) is

(a) a = 1 and b = 1

(b) a = 1 and b = −1

(c) a = −1 and b = 1

(d) a = −1 and b = -1

Question 10. The angle between the lines x – 2y = y and y – 2x = 5 is

(a) $\tan^{-1}$ (1/4)

(b) $\tan^{-1}$ (3/5)

(c) $\tan^{-1}$ (5/4)

(d) $\tan^{-1}$ (2/3)

Very Short Questions:

1. Find the slope of the lines passing through the point (3,-2) and (-1,4)

2. Three points $P(h,k)$, $Q(x_1,y_1)$ and $R(x_2,y_2)$ lie on a line. Show that $(h-x_1)(y_2-y_1)=(k-y_1)(x_2-x_1)$

3. Write the equation of the line through the points (1, -1) and (3, 5)
4. Find the measure of the angle between the lines x + y + 7 = 0 and x – y + 1 = 0.
5. Find the equation of the line that has y-intercept 4 and is ⊥ to the line y = 3x – 2.
6. Find the equation of the line, which makes intercepts -3 and 2 on the x and y-axis respectively.
7. Equation of a line is 3x – 4y + 10 = 0 find its slope.
8. Find the distance between the parallel lines 3x – 4y + 7 = 0 and 3x – 4y + 5 = 0.
9. Find the equation of a straight line parallel to y-axis and passing through the point (4,-2)
10. If 3x – by + 2 = 0 and 9x + 3y + a = 0 represent the same straight line, find the values of a and b.

Short Questions:

1. If p is the length of the from the ⊥ origin on the line whose intercepts on the axes are a and b. show that

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

2. Find the value of p so that the three lines 3x + y – 2 = 0, px + 2y – 3 = 0 and may intersect at one point.
3. Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign.
4. By using area of Δ . Show that the points (a, b + c), and (c, a + b) are collinear.
5. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point p(0, 4) and Q (8, 0)

Long Questions:

1. Find the values of for the line $(k-3)x – (4 - k^2) y + k^2 – 7k + 6 = 0$

   (a). Parallel to the x-axis

   (b). Parallel to y-axis

   (c). Passing through the origin.

2. If p and q are the lengths of ⊥from the origin to the lines.

   $x\cos\theta - y\sin\theta = k\cos 2\theta$, and $x\sec\theta + y\operatorname{cosec}\theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$

3. Prove that the product of the ⊥ drawn from the points $(\sqrt{a^2-b^2}, 0)$ and $(-\sqrt{a^2-b^2}, 0)$ to the line.
4. Find equation of the line mid way between the parallel lines 9x + 6y -7 = 0 and 3x + 2y + 6 = 0.

5. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line x – 3y + 4 = 0.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The point (3, 0) is at 3 units distance from the Y -axis measured along the positive X -axis and has zero distance from the X -axis.

   Reason (R) : The point (3, 0) is at 3 units distance from the X -axis measured along the positive Y -axis and has zero distance from the Y -axis.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : Slope of X -axis is zero and slope of Y -axis is not defined.

   Reason (R) : Slope of X -axis is not defined and slope of Y -axis is zero.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (b) (7, -2)

2. (b) y – x – 1 = 0

3. (c) collinear

4. (c) 2x + y – 7 = 0

5. (b) -3/4

6. (d) 14

7. (a) (0, 0)

8. (a) $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

9. (d) a = −1 and b = -1

10.(c) $\tan^{-1}(5/4)$

Very Short Answer:

1. Slope of line through (3,-2) and (-1, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-(-2)}{-1-3}$$

$$= \frac{6}{-4} = \frac{-3}{2}$$

2. Since P, Q, R are collinear

Slope of PQ = slope of QR

$$\frac{y_1 - k}{x_1 - h} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{(k - y_1)}{(h - x_1)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

3.

Req. eq. $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y + 1 = \frac{5+1}{2}(x - 1)$$

$$-3x + y + 4 = 0$$

4.

$$x + y + 7 = 0$$

$$m_1 = \frac{-1}{1}$$

$$x - y + 1 = 0$$

$$m_2 = \frac{-1}{-1} = 1$$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

5.

$y = 3x - 2$

Slope $(m) = \frac{-3}{-1} = 3,$ slope of any line $\perp$ it is $-\frac{1}{3}$

$C = 4$

Req. eq. is $y = mx + c$

$y = \frac{-1}{3}x + 4$

6.

Req. eq. $\frac{x}{a} + \frac{y}{b} = 1$

$a = -3, b = 2$

$\therefore \frac{x}{-3} + \frac{y}{2} = 1$

2x - 3y + 6 = 0

7.

$m = \frac{-\text{coff. of } x}{\text{coff. of } y}$

$= \frac{-3}{-4} = \frac{3}{4}$

8.

$A = 3, B = -4, C_1 = 7$ and $C_2 = 5$

$d = \frac{|C_1 - C_2|}{\sqrt{a^2 + b^2}}$

$= \frac{|7 - 5|}{\sqrt{(3)^2 + (-4)^2}}$

$= \frac{2}{5}$

9.

Equation of line parallel to y-axis is $x = a.....(i)$

Eq. (i) passing through (-4,2)

$a=-4$

So $x=-4$

$x+4=0$

10.

ATQ

$\frac{3}{9}=\frac{-b}{3}=\frac{2}{a}$

$b=-1$

$\Rightarrow a=6$

Short Answer:

1. Equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{x}{a}+\frac{y}{b}-1=0$

The distance of this line from the origin is P

$\therefore P=\frac{\left|\frac{0}{a}+\frac{0}{b}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$ $\left[d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}\right]$

$\frac{P}{1}=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$

$\frac{1}{P}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$

Sq. both side

$\frac{1}{P^2}=\frac{1}{a^2}+\frac{1}{b^2}$

2.

$3x+y-2=0.....(i)$

$px+2y-3=0.....(ii)$

$2x-y+3=0.....(iii)$

On solving eq. (i) and (iii)

$x=1$, And $y=-1$

Put x, y in eq. (ii)

$P(1)+2(-1)-3=0$

$p-2-3=0$

$p=5$

3.

Let intercept be a and –a the equation of the line is

$\frac{x}{a}+\frac{y}{-a}=1$

$\Rightarrow x-y=a.....(i)$

Since it passes through the point (3, 4)

$3-4=a$

$a=-1$

Put the value of a in eq. (i)

$x-y=-1$

$x-y+1=0$

4.

Area of $\Delta=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$

$=\frac{1}{2}|a(c+a)-b(b+c)+b(a+b)-c(c+a)+c(b+c)-a(a+b)|$

$=\frac{1}{2}.0=0$

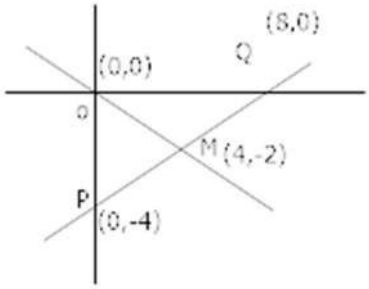
5.

Let m be the midpoint of segment PQ then $M=\left(\frac{0+8}{2},\frac{-4+0}{2}\right)$

$=(4,-2)$

Slope of $OM=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-2-0}{4-0}=\frac{-1}{2}$



Long Answer:

1. (a) The line parallel to x-axis if coeff. Of x =0

$k-3=0$

$k=3$

(b) The line parallel to y-axis if coeff. Of y =0

$4-k^2=0$

$k=\pm 2$

(c)Given line passes through the origin if (0, 0) lies on given eq.

$(k-3).(0)-(4-k^2)(0)+k^2-7k+6=0$

$(k-6)(k-1)=0$

$k=6,1$

2.

$$P=\frac{|0.\cos\theta-0\sin\theta-k\cos 2\theta|}{\sqrt{(\cos\theta)^2+(-\sin\theta)^2}} \quad \left[\begin{array}{l}\perp \text{ from origin}\\ \because (0,0)\end{array}\right.$$

$P=K\cos 2\theta.....(i)$

$$q=\frac{|0.\sec\theta+0\,\mathrm{cosec}\theta-k|}{\sqrt{\sec^2\theta+\mathrm{cosec}^2\theta}}$$

$$=\frac{K}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}$$

$$=\frac{k\cos\theta.\sin\theta}{\sqrt{\sin^2\theta+\cos^2\theta}}=\frac{1}{2}k.\sin\theta.\cos\theta$$

$2q=k.\sin 2\theta.....(ii)$

Squaring (i) and (ii) and adding

$P^2+(2q)^2=K^2\cos^2 2\theta+K^2\sin^2 2\theta$

$$P^2+4q^2=K^2\left(\cos^2 2\theta+\sin^2 2\theta\right)$$

$$p^2+4q^2=k^2$$

3. Let

$$p_1=\frac{\left|\frac{\sqrt{a^2-b^2}}{a}.\cos\theta-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}\left[\because\perp \text{ from the points}\sqrt{a^2-b^2},0\right]$$

Similarly p_2 be the distance $\left(-\sqrt{a^2-b^2},0\right)$ from to given line

$$p_2=\frac{\left|-\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$$

$$p_1p_2=\frac{\left|\left(\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\left(-\frac{\sqrt{a^2-b^2}}{a}\cos\theta-1\right)\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$=\frac{\left|\left(\frac{a^2-b^2}{a^2}\right).\cos^2\theta-1\right|}{\frac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2b^2}}$$

$$=\frac{\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\right|a^2b^2}{a^2(a^2\sin^2\theta+b^2\cos^2\theta)}$$

$$=\frac{\left|-(a^2\sin^2\theta+b^2\cos^2\theta)\right|b^2}{a^2\sin^2\theta+b^2\cos^2\theta}\qquad\left[\because a^2\cos^2\theta-a^2=a^2(\cos^2\theta-1)\right.$$

$$=\frac{(a^2\sin^2\theta+b^2\cos^2\theta)\,b^2}{a^2\sin^2\theta+b^2\cos^2\theta}$$

$$=b^2$$

4.

The equations are

$$9x+6y-7=0$$

$$3\left(3x+2y-\frac{7}{3}\right)=0$$

$3x+2y-\frac{7}{3}=0.....(i)$

$3x+2y+6=0.....(ii)$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$3x+2y+k=0.....(iii)$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left|\frac{K+\frac{7}{3}}{\sqrt{9+4}}\right|=\left|\frac{K-6}{\sqrt{9+4}}\right| \quad \left[\because d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}\right]$$

$K+\frac{7}{3}=K-6$

$K=\frac{11}{6}$

Req. eq. is

$3x+2y+\frac{11}{6}=0$

5.

Let Q(h, k) is the image of the point p(1, 2) in the line.

$x-3y+4=0.....(i)$

Coordinate of midpoint of $PQ=\left(\frac{h+1}{2},\frac{k+2}{2}\right)$

This point will satisfy the eq. ……(i)

$\left(\frac{h+1}{2}\right)-3\left(\frac{k+2}{2}\right)+4=0$

$h-3k=-3.....(i)$

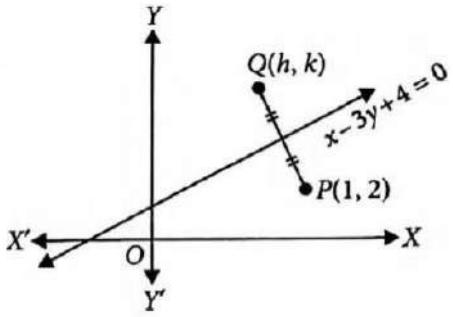
(Slope of line PQ) × (slope of line x – 3y + 4 = 0) = -1

$\left(\frac{k-2}{h-1}\right)\left(\frac{\cancel{-1}}{\cancel{-3}}\right)=-1$

$3h+k=5.....(ii)$

On solving (i) and (ii)

$h=\frac{6}{5}$ and $k=\frac{7}{5}$



Assertion Reason Answer:

1. (iii) Assertion is true but reason is false.
2. (iii) Assertion is true but reason is false.