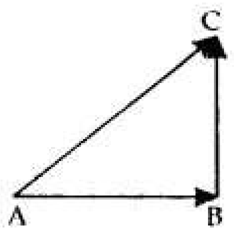
MATHEMATICS

Swotters

Important Questions

Multiple Choice questions-

1. In ΔABC, which of the following is not true?



(a) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(b) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

(d) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

2. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(a) $\vec{b} = \lambda\vec{a}$ tor some scalar λ.

(b) $\vec{a} = \pm\vec{b}$

(c) the respective components of $\vec{a}$ and $\vec{b}$ are proportional

(d) both the vectors $\vec{a}$ and $\vec{b}$ have the same direction, but different magnitudes.

3. If a is a non-zero vector of magnitude 'a' and λa non-zero scalar, then $\lambda\vec{a}$ is unit vector if:

(a) λ = 1

(b) λ = -1

(c) a = |λ|

(d) a = $\frac{1}{|\lambda|}$

4. Let λ be any non-zero scalar. Then for what possible values of x, y and z given below, the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $x\hat{\imath} - y\hat{\jmath} + z\hat{k}$ are perpendicular:

(a) x = 2λ. y = λ, z = λ

(b) x = λ, y = 2λ, z = -λ

(c) x = -λ, y = 2λ, z = λ

(d) x = -λ, y = -2λ, z = λ.

5. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|$ = 3 and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

6. Area of a rectangle having vertices

A $(-\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k})$,

B $(\hat{\imath} + \frac{1}{2}\hat{\jmath} + 4\hat{k})$,

C $(\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k})$,

D $(-\hat{\imath} - \frac{1}{2}\hat{\jmath} + 4\hat{k})$ is

(a) $\frac{1}{2}$ square unit

(b) 1 square unit

(c) 2 square units

(d) 4 square units.

7. If θ is the angle between two vectors $\vec{a}, \vec{b}$, then $\vec{a}.\vec{b} \geq 0$ only when

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

8. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and 6 is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if:

(a) $\theta = \frac{\pi}{4}$

(b) $\theta = \frac{\pi}{3}$

(c) $\theta = \frac{\pi}{2}$

(d) $\theta = \frac{2\pi}{3}$

9. If $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ are the usual three perpendicular unit vectors, then the value of:

$\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$ is

(a) 0

(b) -1

(c) 1

(d) 3

10. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to:

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π

Very Short Questions:

1. Classify the following measures as scalar and vector quantities:

   (i) 40°

   (ii) 50 watt

   (iii) $10gm/cm^3$

(iv) 20 m/sec towards north

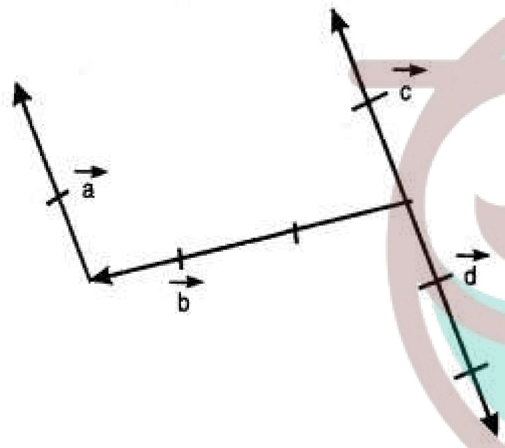
(v) 5 seconds. (N.C.E.R.T.)

2. In the figure, which of the vectors are:

   (i) Collinear

   (ii) Equal

   (iii) Co-initial. (N.C.E.R.T.)



3. Find the sum of the vectors:

   $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{a} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. (C.B.S.E. 2012)

4. Find the vector joining the points P (2,3,0) and Q (-1, – 2, – 4) directed from P to Q. (N.C.E.R.T.)

5. If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors, then write the value of x + y + z. (C.B.S.E. 2013)

6. Find the unit vector in the direction of the sum of the vectors:

   $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$ (N.C.E.R.T.)

7. Find the value of 'p' for which the vectors: $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$ are parallel. (A.I.C.B.S.E. 2014)

8. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5$, find the value of $|\vec{b}|$ (A.I.C.B.S.E. 2014)

9. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such

that the angle between them is 60° and their scalar product is $\frac{9}{2}$ (C.B.S.E. 2018)

10. Find the area of the parallelogram whose diagonals are represented by the vectors: $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ (C.B.S.E. Sample Paper 2018-19)

Short Questions:

1. If θ is the angle between two vectors:

   $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find sinθ. (C.B.S.E. 2018)

2. X and Y are two points with position vectors $\overrightarrow{3a} + \vec{b}$ and $\vec{a} - \overrightarrow{3b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2:1 externally. (C.B.S.E. Outside Delhi 2019)

3. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where:

   $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$ and $\vec{b} = -\hat{j} + \hat{k}$

4. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ. (C.B.S.E. 2019 C)

5. Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and

   $(\vec{a} - \vec{b})$ are perpendicular to each other. (C.B.S.E. Outside Delhi 2019)

6. If the sum of two-unit vectors is a unit vector, prove that the magnitude of their difference

   is √3. (C.B.S.E. 2019)

7. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{a}| = 7$, then find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$

   (C.B.S.E. Sample Paper 2019-20)

8. Find $|\vec{a} - \vec{b}|$, if two vectors a and b are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$. (N.C.E.R.T.)

Long Questions:

1. Let $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$. Find a vector $\vec{a}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d}\cdot\vec{a} = 21$. (C.B.S.E. 2018)

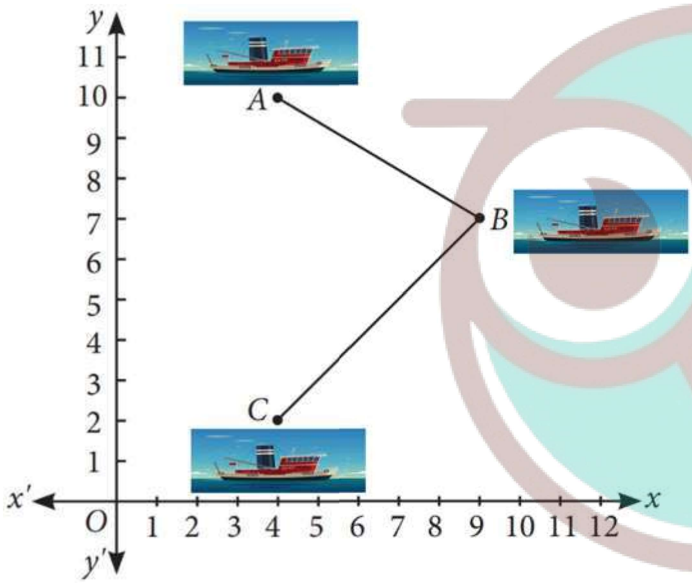
2. If $\vec{p} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{q} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude 5√3 units perpendicular to the vector $\vec{q}$. and coplanar with vector $\vec{p}$ and $\vec{q}$. (C.B.S.E. 2018)

3. If $\hat{\imath} + \hat{\jmath} + \hat{k}$, $2\hat{\imath} + 5\hat{\jmath}$, $3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find

4. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, and $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$. (C.B.S.E. 2014)

Case Study Questions:

1. A barge is pulled into harbour by two tug boats as shown in the figure.



Based on the above information, answer the following questions.

i. Position vector of A is:

a. $4\hat{i} + 2\hat{j}$

b. $4\hat{i} + 10\hat{j}$

c. $4\hat{i} - 10\hat{j}$

d. $4\hat{i} - 2\hat{j}$

ii. Position vector of B is:

a. $4\hat{i} + 4\hat{j}$

b. $6\hat{i} + 6\hat{j}$

c. $9\hat{i} + 7\hat{j}$

d. $3\hat{i} + 3\hat{j}$

iii. Find the vector $\overline{AC}$ in terms of $\hat{i}, \hat{j}$.

a. $8\hat{j}$

b. $-8\hat{j}$

c. $8\hat{i}$

d. None of these

iv. If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, then its unit vector is:

a. $\frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$

b. $\frac{3\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$

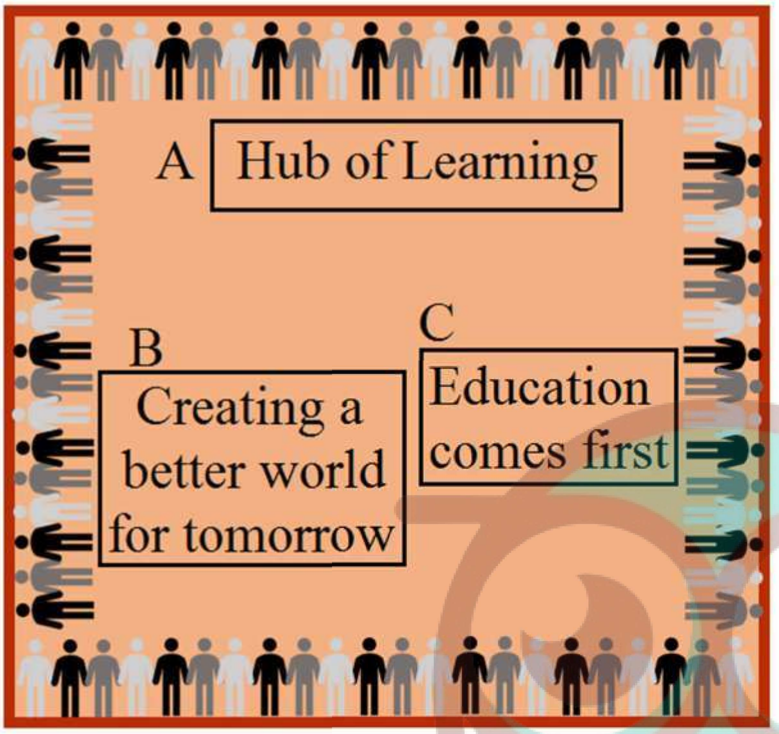
c. $\frac{2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$

d. None of these

v. If $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$, then $|\vec{A}| + |\vec{B}| =$

a. 12
b. 13
c. 14
d. 10

2. Three slogans on chart papers are to be placed on a school bulletin board at the points A, Band C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.



Based on the above information, answer the following questions.

i. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then $\vec{a}+\vec{b}+\vec{c}$ is equal to:

a. $2\hat{i}+3\hat{j}+6\hat{k}$

b. $2\hat{i}-3\hat{j}-6\hat{k}$

c. $2\hat{i}+8\hat{j}+3\hat{k}$

d. $2(7\hat{i}+8\hat{j}+3\hat{k})$

ii. Which of the following is not true?

a. $\overline{AB}+\overline{BC}+\overline{CA}=\vec{0}$

b. $\overline{AB}+\overline{BC}-\overline{AC}=\vec{0}$

c. $\overline{AB}+\overline{BC}-\overline{CA}=\vec{0}$

d. $\overline{AB}-\overline{CB}+\overline{CA}=\vec{0}$

iii. Area of $\triangle ABC$ is:

a. 19 sq. units

b. $\sqrt{1937}$sq. units

c. $\frac{1}{2}\sqrt{1937}$sq. units

d. $\sqrt{1837}$sq. units

iv. Suppose, if the given slogans are to be placed on a straight line,

then the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ will be equal to:

a. -1
b. -2
c. 2
d. 0

v. If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then unit vector in the direction of vector $\vec{a}$ is:

a. $\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

b. $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

c. $\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

d. None of these

Answer Key-

Multiple Choice questions-

1. Answer: (c) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$
2. Answer: (d) both the vectors $\vec{a}$ and $\vec{b}$ have the same direction, but different magnitudes.
3. Answer: (d) a = $\frac{1}{|\lambda|}$
4. Answer: (c) x = -λ, y = 2λ, z = λ
5. Answer: (b) $\frac{\pi}{4}$
6. Answer: (c) 2 square units

7. Answer: (b) $0 \leq \theta \leq \frac{\pi}{2}$

8. Answer: (d) $\theta = \frac{2\pi}{3}$

9. Answer: (d) 3

10. Answer: (b) $\frac{\pi}{4}$

Very Short Answer:

1. Solution:

   (i) Angle-scalar

   (ii) Power-scalar

   (iii) Density-scalar

   (iv) Velocity-vector

   (v) Time-scalar.

2. Solution:

   (i) $\vec{a}$, $\vec{c}$ and $\vec{a}$ are collinear vectors.

   (ii) $\vec{a}$ and $\vec{c}$ are equal vectors.

   (iii) $\vec{b}$, $\vec{c}$ and $\vec{d}$ are co-initial vectors.

3. Solution:

   Sum of the vectors = $\hat{a} + \hat{b} + \hat{c}$

   $= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$

   $= (\hat{i} - 2\hat{i} + \hat{i}) + (-2\hat{j} + 4\hat{j} - 6\hat{j}) + (\hat{k} + 5\hat{k} - 7\hat{k})$

   $= -4\hat{j} - \hat{k}.$

4. Solution:

   Since the vector is directed from P to Q,

   ∴ P is the initial point and Q is the terminal point.

∴ Reqd. vector = $\overrightarrow{PQ}$

$$= (-\hat{i} - 2\hat{j} - 4\hat{k}) - (2\hat{i} + 3\hat{j} + 0\hat{k})$$
$$= (-1-2)\hat{i} + (-2-3)\hat{j} + (-4-0)\hat{k}$$
$$= -3\hat{i} - 5\hat{j} - 4\hat{k}.$$

5. Solution:

Here

$$\vec{a} = \vec{b} \Rightarrow x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$$

Comparing, A: = 3,2 = -y i.e. y = -2, ~ z= 1 i.e. z = -1.

Hence, x + y + z = 3 – 2 – 1 = 0.

6. Solution:

We have : $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$

and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

∴ $\vec{c} = \vec{a} + \vec{b}$

$$= (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k})$$
$$= \hat{i} + 0.\hat{j} + 5\hat{k}.$$

$$\therefore |\vec{c}| = \sqrt{1^2 + 0^2 + 5^2}$$
$$= \sqrt{1+0+25} = \sqrt{26}.$$

∴ Reqd. unit vector $= \hat{c} = \dfrac{\vec{c}}{|\vec{c}|}$

$$= \frac{\hat{i} + 0\hat{j} + 5\hat{k}}{\sqrt{26}} = \frac{\hat{i} + 5\hat{k}}{\sqrt{26}}.$$

7. Solution:

The given vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$ are parallel

If $\frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$ if 3 = $\frac{1}{-p}$ = 3

if p = $-\frac{1}{3}$

8. Solution:

We have : $|\vec{a}+\vec{b}| = 13.$

Squaring, $(\vec{a}+\vec{b})^2 = 169$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 169$

$\Rightarrow (5)^2 + |\vec{b}|^2 + 2(0) = 169$

[$\because \vec{a}$ and $\vec{b}$ are perpendicular $\Rightarrow \vec{a}\,\vec{b} = 0$]

$\Rightarrow |\vec{b}|^2 = 169 - 25 = 144.$

Hence, $|\vec{b}| = 12.$

9. Solution:

By the question, $|\vec{a}| = |\vec{b}|$ ...(1)

Now $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$

$\Rightarrow \frac{9}{2} = |\vec{a}||\vec{a}| \cos 60°$ [Using (1)]

$\Rightarrow \frac{9}{2} = |\vec{a}|^2 \left(\frac{1}{2}\right)$

$\Rightarrow |\vec{a}|^2 = 9.$

Hence, $|\vec{a}| = |\vec{b}| = 3.$

10. Solution:

We have: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

$$\therefore \ \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & -1 & 2 \end{vmatrix}$$

$= \hat{i}(-6+4) - \hat{j}(4-8) + \hat{k}(-2+6)$

$= -2\hat{i} + 4\hat{j} + 4\hat{k}.$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{4+16+16} = \sqrt{36} = 6.$

$\therefore$ Area of the parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$= \frac{1}{2}(6) = 3$ sq. units.

Short Answer:

1. Solution:

We know that $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

$\Rightarrow \sin\theta = \frac{(\hat{i}-2\hat{j}+3\hat{k})\times(3\hat{i}-2\hat{j}+\hat{k})}{|\hat{i}-2\hat{j}+3\hat{k}||3\hat{i}-2\hat{j}+\hat{k}|}$ ...(1)

Now $(\hat{i}-2\hat{j}+3\hat{k})\times(3\hat{i}-2\hat{j}+\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(-2+6) - \hat{j}(1-9) + \hat{k}(-2+6)$

$= 4\hat{i} + 8\hat{j} + 4\hat{k}$.

$\therefore \left|4\hat{i}+8\hat{j}+4\hat{k}\right| = \sqrt{16+64+16}$

$= \sqrt{96} = 4\sqrt{6}$

and $\left|i-2\hat{j}+3\hat{k}\right| = \sqrt{1+4+9} = \sqrt{14}$;

$|3\hat{i}-2\hat{j}+\hat{k}| = \sqrt{9+4+1} = \sqrt{14}$.

$\therefore$ From (1), $\sin\theta = \frac{4\sqrt{6}}{\sqrt{14}\sqrt{14}} = \frac{4\sqrt{6}}{14}$.

Hence, $\sin\theta = \frac{2\sqrt{6}}{7}$.

2. Solution:

Position vector of

$A = \frac{2(\vec{a}-3\vec{b})-(3\vec{a}+\vec{b})}{2-1} = -\vec{a} - 7\vec{b}$

3. Solution:

We have : $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$, $\vec{b} = -\hat{j} + \hat{k}$.

$$\therefore \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+8) - \hat{j}(4-0) + \hat{k}(-4+0)$$

$$= 7\hat{i} - 4\hat{j} - 4\hat{k}.$$

$$\therefore \quad |\vec{a} \times \vec{b}| = \sqrt{(7)^2 + (-4)^2 + (-4)^2}$$

$$= \sqrt{49+16+16} = \sqrt{81} = 9.$$

Hence, the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{7\hat{i} - 4\hat{j} - 4\hat{k}}{9} = \frac{7}{9}\hat{i} - \frac{4}{9}\hat{j} - \frac{4}{9}\hat{k}.$$

4. Solution:

We have:

a = $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$$\therefore \vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$$

$= (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3+\lambda)\hat{k}.$

Now, $(\vec{a} + \lambda\vec{b})$ is perpendicular to c ,

$$\therefore (\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$\Rightarrow ((2 - \lambda)\ \hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda\hat{k}).\ (3\hat{i} + \hat{j}) = 0$

$\Rightarrow (2 - \lambda)\ (3) + (2 + 2\lambda)\ (1) + (3 + \lambda)(0) = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

⇒ -λ, + 8 = 0.

Hence, λ, = 8.

5. Solution:

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k})$

$= 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - j + 2k)$

$= -2\hat{i} + 3\hat{j} - 5\hat{k}$.

Now,

$\vec{a} + \vec{b} . \vec{a} - \vec{b} = (4\hat{i} + \hat{j} - \hat{k}) \cdot (-2\hat{i} + 3\hat{j} - 5\hat{k})$

= (4) (- 2) + (1) (3) + (-1) (- 5)

= – 8 + 3 + 5 = 0.

Hence $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} - \vec{b}$.

6. Solution:

We have : $|\vec{a}| = |\vec{b}| = 1, \ |\vec{a} + \vec{b}| = 1.$

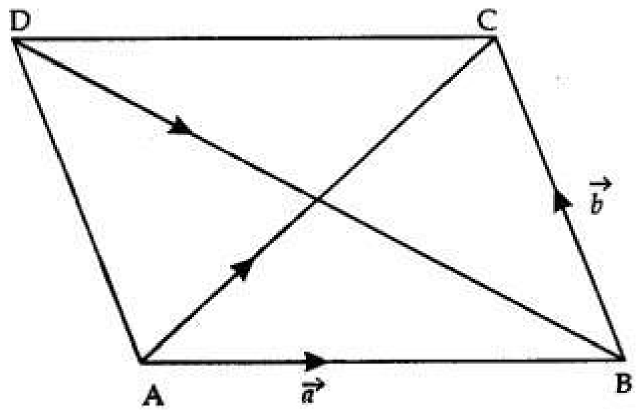
Let $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}.$

Then, $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$

and $\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$

$= -\overrightarrow{AD} + \overrightarrow{AB} = \overrightarrow{AB} - \overrightarrow{AD}$

$= \vec{a} - \vec{b}.$



By the question,

$$|\overrightarrow{AB}| = |\overrightarrow{\mathbf{BC}}| = |\overrightarrow{AC}| = 1$$

⇒ ΔABC is equilateral, each of its angles being 60°

⇒ ∠DAB = 2 x 60° = 120° and ∠ADB = 30°.

By Sine Formula,

$$\frac{DB}{\sin \angle DAB} = \frac{AB}{\sin \angle ADB}$$

$$\Rightarrow \quad \frac{|\overrightarrow{DB}|}{\sin 120°} = \frac{|\overrightarrow{AB}|}{\sin 30°}$$

$$\Rightarrow \quad |\overrightarrow{DB}| = \frac{\sin 120°}{\sin 30°} |\overrightarrow{AB}|$$

$$= \frac{\sqrt{3}/2}{1/2} \times 1 = \sqrt{3}.$$

Hence, $|\vec{a} - \vec{b}| = \sqrt{3}$.

7. Solution:

Here, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}$
$+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 3^2+5^2+7^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-(9+25+49).$

Hence, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{83}{2}.$

8. Solution:

Here, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}$
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$\Rightarrow 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-(9+25+49).$

Hence, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{83}{2}.$

Long Answer:

1. Solution:

Wehave: $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$

$\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and

$\vec{c}=3\hat{i}+\hat{j}-\hat{k}$

Let $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}$

since $\vec{d}$ is perpendicular to both $\vec{c}$ and $\vec{b}$

$\vec{d} \cdot \vec{c} = 0$ and $\vec{d} \cdot \vec{b} = 0$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) . (3\hat{i} + \hat{j} - \hat{k}) = 0$

and $(x\hat{i} + y\hat{j} + z\hat{k}) . (\hat{i} - 4\hat{j} + 5\hat{j}) = 0$

⇒ 3x + y - z = 0 ...(1)

and x - 4y + 5z = 0 ...(2)

Also, $\vec{d} \cdot \vec{a} = 21$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}). (4\hat{i} + 5\hat{j} - \hat{k}) = 21$

⇒ 4x + 5y-z = 21 ...(3)

Multiplying (1) by 5,

1 5x + 5y – 5z = 0 ...(4)

Adding (2) and (4),

16x + y = 0 ...(5)

Subtracting (1) from (3),

x + 4y = 21 ...(6)

From (5),

y = -16x ...(7)

Putting in (6),

x – 64x = 21

-63x: = 21

Putting in (7), $y = -16\left(-\frac{1}{3}\right) = \frac{16}{3}$

Putting in (1), $3\left(-\frac{1}{3}\right) + \frac{16}{3} - z = 0$

z = 13/3

Hence $\vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$

2. Solution:

Let $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ be the vector.

Since $\vec{r} \perp \vec{q}$

(1) (a) + (-2) (b) + 1 (c) = 0

⇒ a – 2b + c = 0

Again, $\vec{p}$, $\vec{q}$ and $\vec{r}$ and coplanar,

$\therefore\ [\vec{p}\ \vec{q}\ \vec{r}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ a & b & c \end{vmatrix} = 0$$

⇒ (1) (-2c – b) – (1) (c – a) + (1) (b + 2a) = 0

⇒ -2c-b-c + a + b + 2a = 0

⇒ 3a – 3c = 0

⇒ a – c – 0

Solving (1) and (2),

$$\frac{a}{2-0} = \frac{b}{1+1} = \frac{c}{0+2}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{2} = \frac{c}{2}$$

$$\Rightarrow \quad \frac{a}{1} = \frac{p}{1} = \frac{c}{1}.$$

$$\therefore \quad \vec{r} = 1\hat{i} + 1\hat{j} + 1\hat{k}.$$

$$\therefore |\vec{r}| = \sqrt{3}.$$

$$\therefore \text{Unit vector } \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}.$$

Hence, the required vector $= 5\sqrt{3}\,\hat{r}$

$$= 5\sqrt{3}\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) = 5\,(\hat{i}+\hat{j}+\hat{k}).$$

3. Solution:

Note: If 'θ' is the angle between AB and CD,

then θ is also the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Now $\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= (2\hat{i} + 5\hat{j}) - (\hat{i} + \hat{j} + \hat{k}) = \hat{i} + 4\hat{j} - \hat{k}.$$

$$\therefore |\overline{AB}| = \sqrt{(1)^2 + (4)^2 + (-1)^2} = 3\sqrt{2}.$$

Similarly, $\overline{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$

and $|\overline{CD}| = 6\sqrt{2}$.

$$\text{Thus} \quad \cos\theta = \frac{\overline{AB}\cdot\overline{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}$$

$$= \frac{1(-2) + 4(-8) + (-1)(2)}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{36} = -1.$$

Since 0 ≤ θ ≤ π, it follows that θ = π. This shows that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

Alternatively, $\overrightarrow{AB} = -\frac{1}{2}\overrightarrow{CD}$ which implies that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear vectors.

4. Solution:

Since $\vec{a}+\vec{b}+\vec{c}=\vec{0}$,

$\therefore \quad \vec{a}+\vec{b}=-\vec{c}$.

Squaring, $\left(\vec{a}+\vec{b}\right)^2=\vec{c}^2$

$\Rightarrow \vec{a}^2+\vec{b}^2+2\vec{a}.\vec{b}=\vec{c}^2$

$\Rightarrow \left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\left|\vec{c}\right|^2$,

where 'θ' is the angle between a and b

$\Rightarrow (3)2 + (5)2 + 2 (3) (5) \cos\theta = (7)2$

$\Rightarrow 9 + 25 + 30 \cos\theta = 49$

$\Rightarrow 30 \cos\theta = 49 - 34 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60°$.

Hence, the angle between $\vec{a}$ and $\vec{b}$ is 60°.

Case Study Answers:

1. Answer :

i. (b) $4\hat{i}+10\hat{j}$

Solution:

Here, (4, 10) are the coordinates of A.

$\therefore$ P.V of A $=4\hat{i}+10\hat{j}$

ii. (c) $9\hat{i} + 7\hat{j}$

Solution:

Here, (9, 7) are the coordinates of B.

$\therefore \text{P.V of B} = 9\hat{i} + 7\hat{j}$

iii. (b) $-8\hat{j}$

Solution:

Here, P.V. of $A = 4\hat{i} + 10\hat{j}$ and P.V. of

$C = 4\hat{i} + 2\hat{j}$

$\therefore \overline{AC} = (4-4)\hat{i} + (2-10)\hat{j} = -8\hat{j}$

iv. (a) $\frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$

Solution:

Here, $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore |\vec{A}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

$\therefore \vec{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$

$= \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

v. (d) 10

Solution:

We have, $\vec{A} = 4\hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

$\therefore |\vec{A}| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

And $|\vec{B}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

Thus, $|\vec{A}| + |\vec{B}| = 5 + 5 = 10.$

2. Answer :

i. (a) $2\hat{i}+3\hat{j}+6\hat{k}$

Solution:

$\vec{a}=\hat{i}+4\hat{j}+2\hat{k}, \overrightarrow{b}=3\hat{i}-3\hat{j}-2\hat{k}$

And $\vec{c}=2\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \vec{a}+\vec{b}+\vec{c}=2\hat{i}+3\hat{j}+6\hat{k}$

ii. (c) $\overline{AB}+\overline{BC}-\overline{CA}=\vec{0}$

Solution:

Using triangle law of addition in $\triangle ABC,$

we get $\overline{AB}+\overline{BC}-\overline{CA}=\vec{0}$ which can be rewritten as,

$\overline{AB}+\overline{BC}-\overline{CA}=\vec{0}$ or $\overline{AB}-\overline{CB}+\overline{CA}=\vec{0}$

iii. (c) $\frac{1}{2}\sqrt{1937}\text{sq. units}$

Solution:

We have, A(1 ,4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overline{AB}=\vec{b}-\vec{a}=2\hat{i}-7\hat{j}-4\hat{k}$

And $\overline{AC}=\vec{c}-\vec{a}=-3\hat{i}-2\hat{j}+4\hat{k}$

$$\therefore \overline{AB}\times\overline{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & -4\\ -3 & -2 & 4\end{vmatrix}$$

$$=\hat{i}(-28-8)-\hat{j}(8-12)+\hat{k}(-4-21)$$

$= -36\hat{i} + 4\hat{j} - 25\hat{k}$

Now, $|\overline{AB} \times \overline{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

$= \sqrt{1296 + 16 + 625} = \sqrt{1937}$

$\therefore$ Area of $\triangle ABC = \frac{1}{2}|\overline{AB} \times \overline{AC}|$

$= \frac{1}{2}\sqrt{1937} \text{sq. units.}$

iv. (d) 0

Solution:

If the given points lie on the straight line,

then the points will be collinear and so area of $\triangle ABC = 0.$

$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$

[$\because$ If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the three vertices

A, B and C of $\triangle ABC,$ then area of triangle

$= \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$]

v. (b) $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

Solution:

Here, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 6 + 36}$

$= \sqrt{49} = 7$

$\therefore$ Unit vector in the direction of vector $\vec{a}$ is

$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7}$

$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$