MATHEMATICS

Chapter 11: CONIC SECTIONS

Important Questions

Multiple Choice questions-

Question 1. The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if

(a) $\sqrt{a^2 \times (1 + m^2)} < c$

(b) $\sqrt{a^2 \times (1 - m^2)} < c$

(c) $\sqrt{a^2 \times (1 + m^2)} > c$

(d) $\sqrt{a^2 \times (1 - m^2)} > c$

Question 2. Equation of the directrix of the parabola $x^2 = 4ay$ is

(a) $x = -a$

(b) $x = a$

(c) $y = -a$

(d) $y = a$

Question 3. The equation of parabola with vertex at origin and directrix $x - 2 = 0$ is

(a) $y^2 = -4x$

(b) $y^2 = 4x$

(c) $y^2 = -8x$

(d) $y^2 = 8x$

Question 4. The perpendicular distance from the point (3, -4) to the line $3x - 4y + 10 = 0$

(a) 7

(b) 8

(c) 9

(d) 10

Question 5. The equation of a hyperbola with foci on the x-axis is

(a) $x^2/a^2 + y^2/b^2 = 1$

(b) $x^2/a^2 - y^2/b^2 = 1$

(c) $x^2 + y^2 = (a^2 + b^2)$

(d) $x^2 - y^2 = (a^2 + b^2)$

Question 6. If the line $2x - y + \lambda = 0$ is a diameter of the circle $x^2 + y^2 + 6x - 6y + 5 = 0$ then λ =

(a) 5

(b) 7

(c) 9

(d) 11

Question 7. The number of tangents that can be drawn from (1, 2) to $x^2 + y^2 = 5$ is

(a) 0

(b) 1

(c) 2

(d) More than 2

Question 8. The equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent a real circle if

(a) $g^2 + f^2 - c < 0$

(b) $g^2 + f^2 - c \geq 0$

(c) always

(d) None of these

Question 9. The equation of parabola whose focus is (3, 0) and directrix is $3x + 4y = 1$ is

(a) $16x^2 - 9y^2 - 24xy - 144x + 8y + 224 = 0$

(b) $16x^2 + 9y^2 - 24xy - 144x + 8y - 224 = 0$

(c) $16x^2 + 9y^2 - 24xy - 144x - 8y + 224 = 0$

(d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

Question 10. If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is

(a) 2/3

(b) 4/3

(c) 1/3

(d) 4

Short Questions:

1. Show that the equation $x^2 + y^2 - 6x + 4y - 36 = 0$ represent a circle, also find its centre & radius?
2. Find the equation of an ellipse whose foci are $(\pm 8, 0)$ & the eccentricity is $\frac{1}{4}$?
3. Find the equation of an ellipse whose vertices are $(0, \pm 10)$ & e = $\frac{4}{5}$

4. Find the equation of hyperbola whose length of latus rectum is 36 & foci are $(0, \pm 12)$
5. Find the equation of a circle drawn on the diagonal of the rectangle as its diameter, whose sides are
   $x = 6, x = -3, y = 3$ & $y = -1$
6. Find the coordinates of the focus & vertex, the equations of the diretrix & the axis & length of latus rectum of the parabola x = -8y.
7. Show that the equation $6x^2 + 6y^2 + 24x - 36y - 18 = 0$ represents a circle. Also find its centre & radius.
8. Find the equation of the parabola with focus at F(5, 0) & directrix is x = -5. x = -5.
9. Find the equation of the hyperbola with centre at the origin, length of the transverse axis 18 & one focus at (0,4)
10. Find the equation of an ellipse whose vertices are $(0, \pm 13)$ & the foci are $(0, \pm 5)$

Long Questions:

1. Find the length of major & minor axis- coordinate's of vertices & the foci, the eccentricity & length of latus rectum of the ellipse $16x^2 + y^2 = 16$
2. Find the lengths of the axis , the coordinates of the vertices & the foci the eccentricity & length of the lat us rectum of the hyperbola $25x^2 - 9y^2 = 225$.
3. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.
4. A man running in a race course notes that the sum of the distances of the two flag posts from him is always 12 m & the distance between the flag posts is 10 m. find the equation of the path traced by the man.
5. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ so that one angular point of the triangle is at the vertex of the parabola. Find the length of each side of the triangle.

Answer Key:

MCQ:

1. (c) $\sqrt{a^2 \times (1 + m^2)} > c$
2. (c) $y = -a$
3. (c) $y^2 = -8x$
4. (a) 7
5. (b) $x^2/a^2 - y^2/b^2 = 1$
6. (c) 9
7. (b) 1
8. (b) $g^2 + f^2 - c \geq 0$
9. (d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
10. (b) 4/3

Short Answer:

1. This is of the form $x^2 + y^2 + 2gx + 2Fy + c = 0,$

where $2g = -6, 2f = 4$ &

$\therefore q = -3, f = 2 \,\&\, c = -36$

So, centre of the circle $= (-g, -f) = (3, -2)$

&

Radius of the circle $= \sqrt{q^2 + f^2 - c} = \sqrt{9 + 4 + 36}$

= 7 units

2. Let the required equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where $a^2 > b^2$

let the foci be $(\pm c, 0), c = 8$

&

$e = \frac{c}{a} \Leftrightarrow a = \frac{c}{e} = \frac{8}{\frac{1}{4}} = 32$

Now $c^2 = a^2 - b^2 \Leftrightarrow b^2 = a^2 - c^2 = 1024 - 64 = 960$

$\therefore a^2 = 1024$ & $b^2 = 960$

Hence equation is $\frac{x^2}{1024} + \frac{y^2}{960} = 1$

3. Let equation be

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& its vertices are $(0, \pm a)$ & $a = 10$

Let

$c^2 = a^2 - b^2$

Then $e = \frac{c}{a} \Rightarrow c = ae = 10 \times \frac{4}{5} = 8$

Now

$c^2 = a^2 - b^2 \Leftrightarrow b^2 = (a^2 - c^2) = 100 - 64 = 36$

$\therefore a^2 = (10)^2 = 100$ & $b^2 = 36$

Hence the equation is

$\frac{x^2}{36} + \frac{y^2}{100} = 1$

4. Clearly C = 12

Length of cat us rectum $= 36 \Leftrightarrow \frac{2b^2}{a} = 36$

$\Rightarrow b^2 = 18a$

Now $c^2 = a^2 + b^2 \Leftrightarrow a^2 = c^2 - b^2 = 144 - 18a$

$a^2 + 18a - 144 = 0$

$(a+24)(a-6) = 0 \Leftrightarrow a = 6 \quad [\because a \text{ is non negative}]$

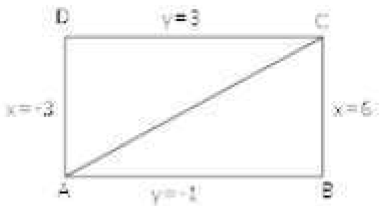
This $a^2 = 6^2 = 36$ & $b^2 = 108$

Hence, $\frac{x^2}{36} + \frac{y^2}{108} = 1$

5.

Let ABCD be the given rectangle & $AD = x = -3$, $BC = x = 6$, $AB = y = -1$ & $CD = y = -3$

Then $A(-3, -1)$ & $c(6, 3)$



So the equation of the circle with AC as diameter is given as

$(x+3)(x-6) + (y+1)(y-3) = 0$

$\Rightarrow x^2 + y^2 - 3x - 2y - 21 = 0$

6.

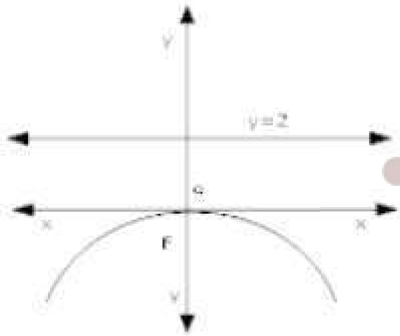
$x^2 = -8y$

& $x^2 = -4ay$

So, $4a = 8 \Leftrightarrow a = 2$

So it is case of downward parabola

o, foci is $F(0,-a)$ *ie* $F(0,-2)$



Its vertex is $0(0,0)$

So, $y = a = 2$

Its axis is y – axis, whose equation is $x = 0$ length of lotus centum

$= 4a = 4 \times 2 = 8$ units.

7.

$6x^2 + 6y^2 + 24x - 36y + 18 = 0$

So $x^2 + y^2 + 4x - 6y + 3 = 0$

Where, $2g = 4, 2f = -6 \& C = 3$

$\therefore g = 2, f = -3 \& C = 3$

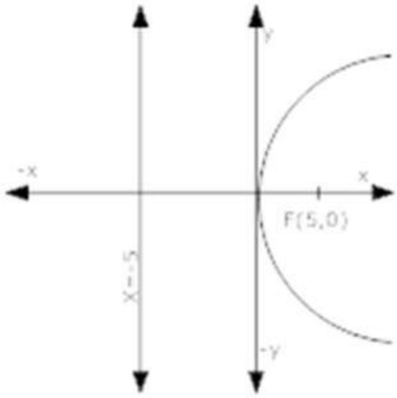
Hence, centre of circle $= (-g, -f) = (-2, 3)$

&

Radius of circle $= \sqrt{4+9+9} = \sqrt{20}$

$= 2\sqrt{5}$ units

8. Focus F (5, 0) lies to the right hand side of the origin



So, it is right hand parabola.

Let the required equation be

$y^2 = 4ax \quad \& \quad a = 5$

So, $y^2 = 20x$

9. Let its equation be $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Clearly, C = 4.

length of the transverse axis $= 8 \Leftrightarrow 2a = 18$

$a = 9$

Also,

$b^2 = c^2 \quad C^2 = (a^2 + b^2)$

So, $a^2 = 81 \quad \& \quad b^2 = -65$

So, equation is

$\frac{y^2}{81} + \frac{x^2}{65} = 1$

10. Let the equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

& a = 13

Let its foci be $(0, \pm c)$ then c = 5

$\therefore b^2 = a^2 - c^2 = 169 - 25 = 144$

So, $a^2 = 169 \quad \& \quad b^2 = 144$

So, equation be $\frac{x^2}{144} + \frac{y^2}{169} = 1$

Long Answer:

1. $16x^2 + y^2 = 16$

Dividing by 16,

$x^2 + \frac{y^2}{16} = 1$

So $b^2 = 1$ & $a^2 = 16$ & $b = 1$ & $a = 4$

&

$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1}$

$= \sqrt{15}$

Thus $a = 4,\ b = 1$ & $c = \sqrt{15}$

(i) Length of major axis $= 2a = 2 \times 4 = 8$ units

Length of minor axis $= 2b = 2 \times 1 = 2$ units

(ii) Coordinates of the vertices are $A(-a, 0)$ & $B(a, 0)$ ie $A(-4, 0)$ & $B(4, 0)$

(iii) Coordinates of foci are $F_1(-c, 0)$ & $F_2(c, 0)$ ie $F_1(-\sqrt{15}, 0)$ & $F_2(\sqrt{15}, 0)$

(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

(v) Length of latus rectum $= \frac{2b^2}{a} = \frac{2}{4} = \frac{1}{2}$ units

2.

$25x^2 - 9y^2 = 225 \Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$

So, $a^2 = 9$ & $b^2 = 25$

& $c = \sqrt{a^2 + b^2} = \sqrt{9 + 25} = \sqrt{34}$

(i) Length of transverse axis $= 2a = 2 \times 3 = 6$ units

Length of conjugate axis $= 2b = 2 \times 5 = 10$ units

(ii) The coordinates of vertices are $A(-a, 0)$ & $B(a, 0)$ ie $A(-3, 0)$ & $B(3, 0)$

(iii) The coordinates of foci are $F_1(-c, 0)$ & $F_2(c, 0)$ ie $F_1(-\sqrt{34}, 0)$ & $F_2(\sqrt{34}, 0)$

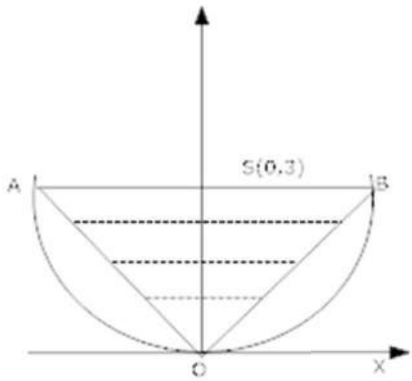
(iv) Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$

(v) Length of the lat us rectum $\frac{2b^2}{a} = \frac{50}{3}$ units

3. The vertex of the parabola $x^2 = 12y$ ie o(0, 0)

0	0	1
6	3	1
-6	3	1

Comparing $x^2 = 12y$ with $x^2 = 4ay$ we get a = 3 the coordinates of its focus S are (0, 3).



Clearly, the ends of its latus rectum are: A(-2a, a) & B (2a, a)

Let A(-6, 3) & B(6, 3)

$\therefore$ are of $\Delta OBA = \frac{1}{2}$

$= \frac{1}{2}[1 \times (18 + 18)]$

= 18 units.

4. We know that on ellipse is the locus of a point that moves in such a way that the sum of its distances from two fixed points (caked foci) is constant.

So, the path is ellipse.

Let the equation of the ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$

where, $b^2 = a^2 (1 - c^2)$

Clearly, $2a = 12$ & $2ae = 10$

$\Rightarrow a = b$ & $e = \frac{5}{6}$

$\Rightarrow b^2 = a^2(1 - e^2) = 36\left(1 - \frac{25}{36}\right)$

$\Rightarrow b^2 = 11$

Hence, the required equation is $\frac{x^2}{36} + \frac{y^2}{11} = 1$

5. Let ΔPOR be an equilateral triangle inscribed in the parabola $y^2 = 4ax$

Let QP = QP = QR = PR = C

Let ABC at the x-axis at M.

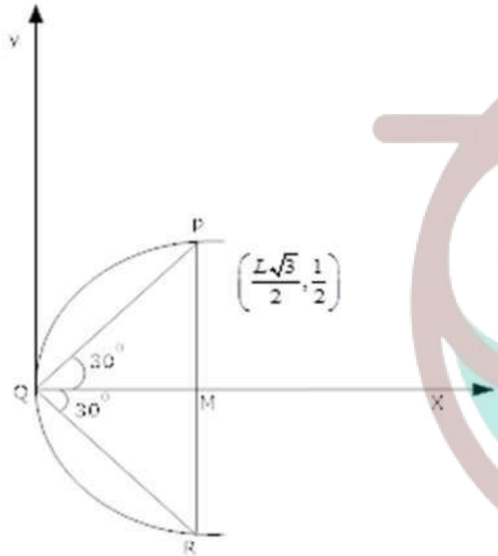
Then , $\angle PQM = \angle RQWM = 30^\circ$

$$\therefore \frac{QM}{QP} = \cos 30^\circ \Rightarrow QM = \angle \cos 30^\circ$$

$$\Rightarrow \frac{L\sqrt{3}}{2}$$

$$\Rightarrow \frac{PM}{QP} = \sin 30^\circ \Rightarrow PM = \angle \sin 30^\circ$$

$$\Rightarrow \frac{L}{2}$$



the coordinates of are $[\frac{L\sqrt{3}}{2}, \frac{L}{2}]$

Since P lies on the parabola $y^2 = 4ax$, we have

$$l^2 = 4a \times \frac{L\sqrt{3}}{2} \Rightarrow l = 8a\sqrt{3}$$

Hence length of each side of the triangle is $8a\sqrt{3}$ units.