MATHEMATICS

Important Questions

Multiple Choice questions-

1. Distance between two planes:

2x + 3y + 4z = 5 and 4x + 6y + 8z = 12 is

(a) 2 units

(b) 4 units

(c) 8 units

(d) $\frac{1}{\sqrt{29}}$ units.

2. The planes 2x – y + 4z = 3 and 5x – 2.5y +10 z = 6 are

(a) perpendicular

(b) parallel

(c) intersect along y-axis

(d) passes through $\left(0, 0, \frac{5}{4}\right)$

3. The co-ordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by:

(a) (2, 0, 0)

(b) (0, 5, 0)

(c) (0, 0, 7)

(d) (0, 5, 7).

4. If α, ß, γ are the angles that a line makes with the positive direction of x, y, z axis, respectively, then the direction-cosines of the line are:

(a) < sin α, sin ß, sin γ >

(b) < cos α, cos ß, cos γ >

(c) < tan α, tan ß, tan γ >

(d) $< \cos^2 \alpha, \cos^2 \beta, \cos^2 \gamma >$.

5. The distance of a point P (a, b, c) from x-axis is

(a) $\sqrt{a^2 + c^2}$

(b) $\sqrt{a^2 + b^2}$

(c) $\sqrt{b^2 + c^2}$

(d) $b^2 + c^2$.

6. If the direction-cosines of a line are $< k, k, k >$, then

(a) $k > 0$

(b) $0 < k < 1$

(c) $k = 1$

(d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

7. The reflection of the point (α, β, γ) in the xy-plane is:

(a) (α, β, 0)

(b) (0, 0, γ)

(c) (-α, -β, γ)

(d) (α, β, -γ).

8. What is the distance (in units) between two planes:

$3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

(a) 0

(b) 3

(c) $\frac{6}{\sqrt{83}}$

(d) 6.

9. The equation of the line in vector form passing through the point (-1, 3, 5) and parallel to line $\frac{x-3}{2} = \frac{y-4}{3}$, $z = 2$ is

(a) $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda (2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

(b) $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda (2\hat{\imath} + 3\hat{\jmath})$

(c) $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + \lambda (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$

(d) $\vec{r} = (2\hat{\imath} + 3\hat{\jmath}) + \lambda (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$.

10. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-2}{2}$ lie in the plane x + 3y – αz + ß = 0. Then (α, ß) equals:

(a) (-6, -17)

(b) (5, -15)ss

(c) (-5, 5)

(d) (6, -17).

Very Short Questions:

1. Find the acute angle which the line with direction-cosines $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}, n >$ makes with positive direction of z-axis. (C.B.S.E. Sample Paper 2018-19)
2. Find the direction-cosines of the line.

   $\frac{x-1}{2} = -y = \frac{z+1}{2}$ (C.B.S.E. Sample Paper 2018-19)
3. If α, β, γ are direction-angles of a line, prove that cos 2a + cos 2P + cos 2y +1 = 0. (N.C.E.R.T.)
4. Find the length of the intercept, cut off by the plane 2x + y – z = 5 on the x-axis. (C.B.S.E. Outside Delhi 2019)
5. Find the length of the perpendicular drawn from the point P(3, -4,5) on the z-axis.
6. Find the vector equation of a plane, which is at a distance of 5 units from the origin and whose normal vector is $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$
7. If a line makes angles 90°, 135°, 45° with the x,y and z-axes respectively, find its direction cosines.
8. Find the co-ordinates of the point where the line through the points A (3,4,1) and B (5,1, 6) crosses the xy-plane.
9. find the vector equation of the line which passes through the point (3,4,5) and is parallel to

the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$

Short Questions:

1. Find the acute angle between the lines whose direction-ratios are:

   < 1,1,2 > and <-3, -4,1 >.

2. Find the angle between the following pair of lines:

   and

   $$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$

   and check whether the lines are parallel or perpendicular. (C.B.S.E. 2011)

3. Find the vector equation of the line joining (1.2.3) and (-3,4,3) and show that it is perpendicular to the z-axis. (C.B.S.E. Sample Paper 2018-19)

4. Find the vector equation of the plane, which is $\frac{6}{\sqrt{29}}$ at a distance of

   units from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also, find its cartesian form. (N.C.E.R.T.)

5. Find the direction-cosines of the unit vector perpendicular to the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$ through the origin. (N.C.E.R.T.)

6. Find the acute angle between the lines

   $$\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5} \text{ and } \frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$$

7. Find the angle between the line:

   $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 4$ Also, find whether the line is parallel to the plane or not .

8. Find the value of 'λ', so that the lines:

   $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not

Long Questions:

1. Find the shortest distance between the lines: $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$ (C.B.S.E. 2018)

2. A line makes angles α, β, γ, δ with the four diagonals of a cube, prove that:

   $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$. (N.C.E.R.T.)

3. Find the equation of the plane through the line $\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ and parallel to the line:

   $\frac{x+1}{2} = \frac{1-y}{4} = \frac{z+2}{1}$

   Hence, find the shortest distance between the lines. (C.B.S.E. Sample Paper 2018-19)

4. Find the Vector and Cartesian equations of the plane passing through the points (2, 2, -1), (3,4,2) and (7,0,6). Also, find the vector equation of a plane passing through (4,3,1) and parallel to the plane obtained above. (C.B.S.E. 2019)

Case Study Questions:

1. Suppose the floor of a hotel is made up of mirror polished Kota stone. Also, there is a large crystal chandelier attached at the ceiling of the hotel. Consider the floor of the hotel as a plane having equation x - 2y + 2z = 3 and crystal chandelier at the point (3, -2, 1).

Based on the above information, answer the following questions.

(i) The d.r'.s of the perpendicular from the point (3, -2, 1) to the plane x - 2y + 2z = 3, is:

a. < 1, 2, 2 >
b. < 1, -2, 2 >
c. < 2, 1, 2 >
d. < 2, -1, 2 >

(ii) The length of the perpendicular from the point (3, -2, 1) to the plane x - 2y + 2z = 3, is:

a. $\frac{2}{3}$ units
b. 3 units
c. 2 units
d. None of these

(iii) The equation of the perpendicular from the point (3, -2, 1) to the plane x - 2y + 2z = 3, is:

a. $\frac{x-3}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$

b. $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z-1}{2}$

c. $\frac{x+3}{1} = \frac{y+2}{-2} = \frac{z-1}{2}$

d. None of these

(iv) The equation of plane parallel to the plane x - 2y + 2z = 3, which is at a unit distance from the point (3, -2, 1) is:

a. x - 2y + 2z = 0
b. x - 2y + 2z = 6
c. x - 2y + 2z = 12
d. Both (b) and (c)

(v) The image of the point (3, -2, 1) in the given plane is:

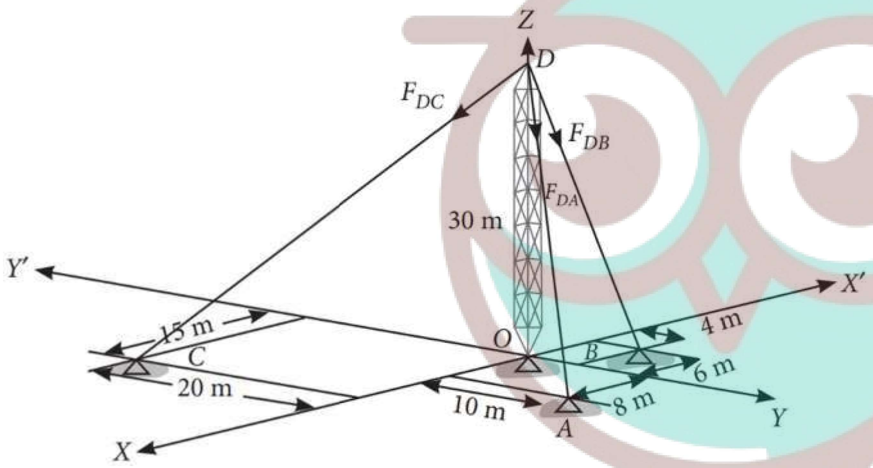
a. $\left(\frac{5}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

b. $\left(\frac{-5}{3}, \frac{-2}{3}, \frac{5}{3}\right)$

c. $\left(\frac{-5}{3}, \frac{2}{3}, \frac{5}{3}\right)$

d. None of these

2. Consider the following diagram, where the forces in the cable are given.



Based on the above information, answer the following questions.

i. The equation of line along the cable AD is:

a. $\frac{x}{5} = \frac{y}{4} = \frac{z-30}{15}$

b. $\frac{x}{4} = \frac{y}{5} = \frac{z-30}{15}$

c. $\frac{x}{5} = \frac{y}{4} = \frac{30-z}{15}$

d. $\frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$

ii. The length of cable DC is:

a. $4\sqrt{61}\mathrm{m}$

b. $5\sqrt{61}\mathrm{m}$

c. $6\sqrt{61}\mathrm{m}$

d. $7\sqrt{61}\mathrm{m}$

iii. The vector DB is:

a. $-6\hat{i}+4\hat{j}-30\hat{k}$

b. $6\hat{i}-4\hat{j}+30\hat{k}$

c. $6\hat{i}+4\hat{j}+30\hat{k}$

d. None of these

iv. The sum of vectors along the cables is:

a. $17\hat{i}+6\hat{j}+90\hat{k}$

b. $17\hat{i}-6\hat{j}-90\hat{k}$

c. $17\hat{i}+6\hat{j}-90\hat{k}$

d. None of these

v. The sum of distances of points A, B and C from the origin, i.e., OA + OB + OC is:

a. $\sqrt{164}+\sqrt{52}+\sqrt{625}$

b. $\sqrt{52}+\sqrt{625}+\sqrt{48}$

c. $\sqrt{164}+\sqrt{625}+\sqrt{49}$

d. None of these

Answer Key-

Multiple Choice questions-

1. Answer: (d) $\frac{1}{\sqrt{29}}$ units.

2. Answer: (b) parallel
3. Answer: (a) (2, 0, 0)
4. Answer: (b) < cos α, cos ß, cos γ >
5. Answer: (c) $\sqrt{b^2 + c^2}$
6. Answer: (c) k = 1
7. Answer: (d) (α, ß, -γ).
8. Answer: (a) 0
9. Answer: (b) $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda\,(2\hat{\imath} + 3\hat{\jmath})$
10. Answer: (a) (-6, -17)

Very Short Answer:

1. Solution:

$l^2 + m^2 + n^2 = 1$

$\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + n^2 = 1$

$\Rightarrow \frac{1}{3} + \frac{1}{6} + n^2 = 1$

$n^2 = 1 - \frac{1}{2}$

$n^2 = \frac{1}{2}$

$n = \frac{1}{\sqrt{2}}$

Thus, $\cos\alpha = \frac{1}{\sqrt{2}}$

Hence, $\alpha = 45°$ or $\frac{\pi}{4}$

2. Solution:

The given line is $\frac{x-1}{2} = \frac{y}{-1} = \frac{z+1}{2}$

Its direction-ratios are <2, -1, 2>.

Hence, its direction- cosine are:

$$< \frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{2}{\sqrt{4+1+4}} >$$

$$i.e. < \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} > or < \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} >.$$

3. Solution:

Since α, β, γ are direction-angles of a line,

∴ cos2 α + cos2 β + cos2γ = 1

$$< \frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{2}{\sqrt{4+1+4}} >$$

$$i.e. < \frac{2}{3}, \frac{-1}{3}, \frac{2}{3} > or < \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} >.$$

⇒ 1 + cos2α + 1 + cos2β + 1 + cos2γ = 2

⇒ cos 2α + cos 2β + cos 2γ + 1 = 0, which is true.

4. Solution:

The given plane is2x + y – z = 5

$$\Rightarrow \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$$

Its intercepts are $\frac{x}{5/2}$, 5 and -5.

Hence, the length of the intercept on the x-axis is $\frac{x}{5/2}$

Solution:

Length of the perpendicular from P (3, -4,5) on the z-axis

$= \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9+16} = \sqrt{25}$ = 5 units.

5. Solution:

Let $\vec{n} = 2\hat{i} - \hat{j} + 2\hat{k}$

Then, $|\vec{n}| = \sqrt{4+1+4} = \sqrt{9} = 3$.

Now, $\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - \hat{j} + 2\hat{k}}{3}$.

Hence, the reqd. equation of the plane is:

$\vec{r} \cdot \left(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 5$

$\Rightarrow \vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 15$.

6. Solution:

   Direction cosines of the line are:

   $< \cos 90°, \cos 135°, \cos 45° >$

   $<0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$

7. Solution:

   The equations of the line through A (3,4,1) and B (5,1,6) are:

   $\frac{x-3}{5-3} = \frac{y-4}{1-4} = \frac{z-1}{6-1}$

   $\Rightarrow \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ ...(1)

   Any point on (1) is (3 + 2k,4- 3k, 1 + 5k) .... (2)

   This lies on xy-plane (z = 0).

   $\therefore 1 + 5k = 0 \Rightarrow k = -\frac{1}{5}$

   Putting in (2), $[\, 3 - \frac{2}{5}, 4 + \frac{3}{5}, 1-1)$

   i.e. $(\frac{13}{5}, \frac{23}{5}, 0)$

   which are the reqd. co-ordinates of the point.

8. Solution:

The vector equation of the line is $\vec{r} = \vec{a} + \lambda\vec{m}$

i.e., $\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$

Short Answer:

1. Solution:

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{|(1)(-3) + (1)(-4) + (2)(1)|}{\sqrt{1+1+4}\sqrt{9+16+1}}$$

$$= \frac{|-3-4+2|}{\sqrt{6}\sqrt{26}} = \frac{5}{\sqrt{156}}.$$

Hence, $\theta = \cos^{-1}\left(\frac{5}{\sqrt{156}}\right)$.

2. Solution:

The given lines can be rewritten as:

$$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3} \;\ldots\ldots\ldots\ldots (1)$$

$$\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4} \;\ldots\ldots\ldots (2)$$

Here < 2,7, – 3 > and < -1,2,4 > are direction- ratios of lines (1) and (2) respectively.

$$\therefore \quad \cos\theta = \frac{(2)(-1) + (7)(2) + (-3)(4)}{\sqrt{4+49+9}\sqrt{1+4+16}}$$

$$= \frac{-2+14-12}{\sqrt{62}\sqrt{21}} = 0$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}.$$

Hence, the given lines are perpendicular.

3. Solution:

Vector equation of the line passing through

(1.2.3) and(-3,4,3)is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

where $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} + 4\hat{j} + 3\hat{k}$

$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-4\hat{i} + 2\hat{j})$ ...(1)

Equation of z-axis is $\vec{r} = \mu\hat{k}$ ...(2)

Since $(-4\hat{i} + 2\hat{j}) \cdot \hat{k} = 0$ = 0,

∴ Line (1) is perpendicular to z-axis.

4. Solution:

Let $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.

Then $|\vec{n}| = \sqrt{4+9+16} = \sqrt{29}$.

Now $\hat{n} = \dfrac{\vec{n}}{|\vec{n}|} = \dfrac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$.

Hence, the reqd. equation of the plane is :

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}.$$

In Cartesian Form :

$$(x\hat{i} + y\hat{j} + z\hat{k}).\left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow (x)\left(\frac{2}{\sqrt{29}}\right) + y\left(\frac{-3}{\sqrt{29}}\right) + z\left(\frac{4}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

$\Rightarrow 2x - 3y + 4z = 6.$

5. Solution:

The given plane is $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$

$\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) = 1$ .... (1)

Now $|-6\hat{i} + 3\hat{j} + 2\hat{k}| = \sqrt{36 + 9 + 4}$

$= \sqrt{49} = 7$

Dividing (1) by 7,

$\vec{r} \cdot \left(-\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right) = \frac{1}{7}$

which is the equation of the plane in the form $\vec{r} \cdot \hat{n} = p$

Thus, $\hat{n} = -\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}$

which is the unit vector perpendicular to the plane through the origin.

Hence, the direction-cosines of $\hat{n}$ are $< -\frac{6}{7}, \frac{3}{7}, \frac{2}{7} >$

6. Solution:

Vector in the direction of first line

$\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5}$,

$\vec{b} = (3\hat{i} + 4\hat{j} + 5\hat{k})$

Vector in the direction of second line

$\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$

$\vec{d} = 4\hat{i} - 3\hat{j} + 5\hat{k}$

∴ θ, the angle between two given lines is given by:

$$\cos\theta = \frac{\vec{b}\cdot\vec{d}}{|\vec{b}||\vec{d}|}$$

$$= \frac{(3\hat{i}+4\hat{j}+5\hat{k})\cdot(4\hat{i}-3\hat{j}+5\hat{k})}{|3\hat{i}+4\hat{j}+5\hat{k}||4\hat{i}-3\hat{j}+5\hat{k}|}$$

$$= \frac{(3)(4)+(4)(-3)+(5)(5)}{\sqrt{9+16+25}\sqrt{16+9+25}}$$

$$= \frac{12-12+25}{\sqrt{50}\sqrt{50}} = \frac{25}{50} = \frac{1}{2}.$$

Hence, $\theta = \frac{\pi}{3}$

7. Solution:

The given line is:

$$\vec{r} = (\hat{i}-\hat{j}+\hat{k}) + \lambda(2\hat{i}-\hat{j}+3\hat{k})$$

and the given plane is $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k}) = 4$.

Now the line is parallel to $2\hat{i}-\hat{j}+3\hat{k}$ and nor¬mal to the plane $2\hat{i}+\hat{i}-\hat{k}$

If 'θ' is the angle between the line and the plane,

then $\left(\frac{\pi}{2}-\theta\right)$ is the angle between the line and normal to the plane.

Then

$$\cos\left(\frac{\pi}{2}-\theta\right) = \frac{(2\hat{i}-\hat{j}+3\hat{k}).(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{4+1+9}\sqrt{4+1+1}}$$

$$\Rightarrow \quad \sin\theta = \frac{4-1-3}{\sqrt{14}\sqrt{6}} = 0$$

$$\Rightarrow \quad \theta = 0^\circ.$$

Hence, the line is parallel to the plane.

8. Solution:

(i) The given lines are

$$\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2} \text{ ....}(1)$$

$$\text{and } \frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5} \text{ ....} (2)$$

These are perpendicular if:

$$(-3)\left(-\frac{3\lambda}{7}\right)+\left(\frac{\lambda}{7}\right)(1)\ +2(-5)=0$$

$$\text{if } \frac{9\lambda}{7}+\frac{\lambda}{7}-10=0 \text{ if } \frac{10\lambda}{7}=10.$$

Hence λ = 1.

(ii) The direction cosines of line (1) are <-3,1,2>

The direction cosines of line (2) are < -3,1, -5 >

Clearly, the lines are intersecting.

Long Answer:

1. Solution:

   Comparing given equations with:

$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$, we have:

$\vec{b}_1 = \vec{i} + 2\vec{j} - 3\hat{k}$, $\vec{b}_2 = 2\hat{i} + 4\vec{j} - 5\hat{k}$.

and $\vec{a}_1 = (4\hat{i} - \hat{j})$, $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$.

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$= \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4) = 2\hat{i} - \hat{j}$.

$\therefore \left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-1)^2 + 0^2}$

$= \sqrt{4+1+0} = \sqrt{5}$.

Also, $\vec{a}_2 - \vec{a}_1 = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j})$

$= -3\hat{i} + 2\hat{k}$.

$\therefore d$, the S.D. $= \left| \dfrac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

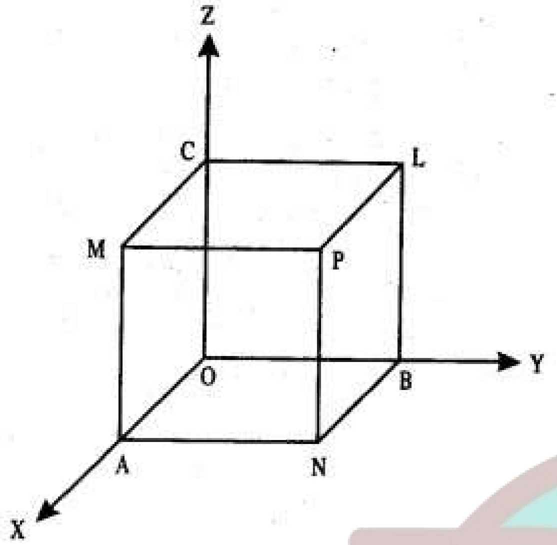
$= \left| \dfrac{(2\hat{i} - \hat{j}) \cdot (-3\hat{i} + 2\hat{k})}{\sqrt{5}} \right|$

$= \left| \dfrac{(2)(-3) + (-1)(0) + (0)(2)}{\sqrt{5}} \right|$

$= \left| \dfrac{-6-0+0}{\sqrt{5}} \right| = \dfrac{6}{\sqrt{5}} = \dfrac{6\sqrt{5}}{5}$ units.

2. Solution:

Let O be the origin and OA, OB, OC (each = a) be the axes.



Thus the co-ordinates of the points are :

O (0,0,0), A (a, 0,0), B (0, a, 0), C (0,0, a),

P (a, a, a), L (0, a, a), M (a, 0, a), N (a, a, 0).

Here OP, AL, BM and CN are four diagonals.

Let < l, m, n > be the direction-cosines of the given line.

Now direction-ratios of OP are:

<a-0,a-0,a-0>i.e.<a,a,a>

i.e. < 1,1,1 >,

direction-ratios of AL are:

<0-a, a-0, a-0> i.e. <-a,a,a>

i.e. <-l, 1,1 >,

direction-ratios of BM are:

<a-0,0-a, a-0>

i.e. <a,-a,a> i.e. < 1,-1, 1 >

and direction-ratios of CN are:

<a-0,a-0,0-a> i.e. <a,a,-a>

i.e. < 1,1,-1 >.

Thus the direction-cosines of OP are:

$$< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$$

the direction-cosines of AL are:

$$< -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$$

the direction-cosines of BM are:

$$< \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$$

and the direction-cosines of CN are:

$$< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} >$$

If the given line makes an angle 'a' with OP, then:

$$\cos\alpha = \left| l\left(\frac{1}{\sqrt{3}}\right) + m\left(\frac{1}{\sqrt{3}}\right) + n\left(\frac{1}{\sqrt{3}}\right) \right|.$$

$$\therefore \cos\alpha = \frac{|l+m+n|}{\sqrt{3}} \quad \text{...(1)}$$

If the given line makes an angle 'β' with AL, then :

$$\cos\beta = \left| l\left(-\frac{1}{\sqrt{3}}\right) + m\left(\frac{1}{\sqrt{3}}\right) + n\left(\frac{1}{\sqrt{3}}\right) \right|$$

$$\therefore \cos\beta = \frac{|-l+m+n|}{\sqrt{3}} \quad \text{...(2)}$$

Similarly, $\cos\gamma = \dfrac{|l-m+n|}{\sqrt{3}}$ ...(3)

and $\cos\delta = \dfrac{|l+m-n|}{\sqrt{3}}$ .... (4)

Squaring and adding (1), (2), (3) and (4), we get:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$= \frac{1}{3}\,[(l + m + n)^2 + (-l + m + n)^2$

$+ (l-m + n)^2 + (l + m — n)^2]$

$= \frac{1}{3} [4(l^2 + m^2 + n^2)] = \frac{1}{3} [4(1)].$

Hence, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2\delta = \frac{4}{3}$

3. Solution:

The two given lines are:

$\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ .... (1)

and $\frac{x+1}{2} = \frac{1-y}{4} = \frac{z+2}{1}$ .... (2)

Let <a, b, c> be the direction-ratios of the normal to the plane containing line (1).

∴ Equation of the plane is:

a(x- l) + b(y-4) + c(z-4) ...(3),

where 3a + 2b – 2c = 0 ...(4)

[∵ Reqd. plane contains line (1)] and 2a – 4b + 1.c = 0

[∵ line (1) a parallel to the reqd. plane] Solving (4) and (5),

$\Rightarrow \quad \frac{a}{-6} = \frac{b}{-7} = \frac{c}{-16}$

$\Rightarrow \quad \frac{a}{6} = \frac{b}{7} = \frac{c}{16} = k$, where $k \neq 0$.

$\therefore \quad a = 6k, b = 7k$ and $c = 16k$.

Putting in (3),

6k(x- 1) + 7k(y – 4) + 16k(z – 4) = 0

= 6(x – 1) + 7(y – 4) + 16(z – 4) =0

[∵k ≠ 0]

⇒ 6x + 7y+ 16z-98 = 0,

which is the required equation of the plane.

Now, S.D. between two lines = perpendicular distance of (-1,1, – 2) from the plane

$$i.e.\ \text{S.D.} = \left|\frac{6(-1)+7(1)+16(-2)-98}{\sqrt{(6)^2+(7)^2+(16)^2}}\right|$$

$$= \left|\frac{-6+7-32-98}{\sqrt{36+49+256}}\right| = \frac{129}{\sqrt{341}} \text{ units.}$$

6(-1) + 7(1) +16(-2) – 98

V$(6)^2 + (7)^2 + (16)^2$

-6 + 7-32-98 V^{36} + 49 + 256

4. Solution:

(i) Cartesian equations

Any plane through (2,2, -1) is:

a(x – 2) + b(y- 2) + c(z + 1) = 0 ... (1)

Since the plane passes through the points (3,4,2) and (7,0,6),

∴ a(3 – 2) + b(4 – 2) + c(2 +1) = 0

and a(7 – 2) + b(0 – 2) + c(6 + 1) = 0

⇒ a + 2b + 3c = 0 ...(2)

and 5a – 2b + 7c = 0 ...(3)

Solving (2) and (3), $\frac{a}{14+6} = \frac{b}{15-7} = \frac{c}{-2-10}$

$\Rightarrow \frac{a}{20} = \frac{b}{8} = \frac{c}{-12}$

$\Rightarrow \frac{a}{5} = \frac{b}{2} = \frac{c}{-3}$ = k (say), value k ≠ 0.

∴ a = 5k,b = 2k and c = -3k,

Putting the values of a, b, c in (1), we get:

5k(x – 2) + 2k(y – 2) – 3k(z + 1) = 0

⇒ 5(x-2) + 2(y-2)-3(z+ 1) =0[∵ k ≠ 0]

=» 5x- 10 + 2y-4-3z-3 = 0

=» 5x + 2y-3z-17 = 0, ...(4)

which is the reqd. Cartesian equation.

Its vector equation is $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ =17.

(ii) Any plane parallel to (4) is

5x + 2y – 3z + λ – 0 … (5)

Since it passes through (4, 3,1),

5(4) + 2(3) – 3(1) + λ = 0

⇒ 20 + 6 — 3 + λ = 0

⇒ λ = -23.

Putting in (5), 5x + 2y – 3z – 23 = 0, which is the reqd. equation.

Its vector equation is $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ = 23.

Case Study Answers:

1. Answer :

i. (b) < 1, -2, 2 >

Solution:

Equation of plane is x - 2y + 2z = 3

∴ D.R.'s of normal to the plane are , which is also the D.R.'s of perpendicular from the point (3, -2, 1) to the given plane.

ii. (c) 2 units

Solution:

Required length = Perpendicular distance from (3, -2, 1) to the plane x - 2y + 2z = 3

$$= \left| \frac{3-2(-2)+2(1)-3}{\sqrt{1^2+(-2)^2+2^2}} \right| = \frac{6}{3} = 2 \text{ units}$$

iii. (b) $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-1}{2}$

Solution:

The equation of perpendicular from the point (x_1, y_1, z_1) to the plane ax + by + cz = d is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Here, x_1= 3, y_1= -2, z_1 = 1 and a = 1, b = -2, c = 2

∴ Required equation is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-1}{2}$

iv. (d) Both (b) and (c)

Solution:

The equation of the plane parallel to the plane $x-2y+2z-3=0$ is $x-2y+2z+\lambda=0$

Now, distance of this plane from the point (3, -2, 1) is

$$=\left|\frac{3+4+2+\lambda}{\sqrt{1^2+(-2)^2+2^2}}\right|=\left|\frac{9+\lambda}{3}\right|$$

But, this distance is given to be unity

$$\therefore |9+\lambda|=3$$

$$\Rightarrow \lambda+9=\pm 3 \Rightarrow \lambda=-6$$

Or -12

Thus, required equation of planes are

x - 2y + 2z- 6 = 0 or x - 2y + 2z - 12 = 0

v. (a) $\left(\frac{5}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

Solution:

Let the coordinate of image of (3, -2, 1) be

Q(r + 3, -2r - 2, 2r + 1)

Let R be the mid-point of PQ, then coordinate of R be

$$\left(\frac{r+6}{2}, \frac{-2r-4}{2}, r+1\right)$$

Since, R lies on the plane x - 2y + 2z = 3

$$\therefore \left(\frac{r+6}{2}\right) - 2\left(\frac{-2r-4}{2}\right) + 2(r+1) = 3$$

$$\Rightarrow 9r = -12 \Rightarrow r = -\frac{4}{3}$$

Thus, the coordinates of Q be $\left(\frac{5}{3}, \frac{2}{3}, \frac{-5}{3}\right)$.

2. Answer :

i. (d) $\frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$

Solution:

Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$$\frac{x-0}{8-0} = \frac{y-0}{10-0} = \frac{30-z}{-30}$$

$$\Rightarrow \frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$$

ii. (b) $5\sqrt{61}\mathrm{m}$

Solution:

The coordinates of point Care (15, -20, 0) and D are (0, 0, 30)

∴ Length of the cable DC

$$= \sqrt{(0-15)^2 + (0-20)^2 + (30-0)^2}$$

$$= \sqrt{225 + 400 + 900}$$

$$= \sqrt{1525} = \sqrt{61}\mathrm{m}.$$

iii. (a) $-6\hat{i} + 4\hat{j} - 30\hat{k}$

Solution:

Since, the coordinates of point B are (-6, 4, 0) and D are (0, 0, 30), therefore vector DB is

$$(-6-0)\hat{i} + (4-0)\hat{j} + (0-30)\hat{k},$$

i.e., $-6\hat{i} + 4\hat{j} - 30\hat{k}$

iv. (b) $17\hat{i} - 6\hat{j} - 90\hat{k}$

Solution:

Required sum

$$(8\hat{i} + 10\hat{j} - 30\hat{k}) + (-6\hat{i} + 4\hat{j} - 30\hat{k}) + (15\hat{i} - 20\hat{j} - 30\hat{k})$$

$$17\hat{i} - 6\hat{j} - 90\hat{k}$$

v. (a) $\sqrt{164} + \sqrt{52} + \sqrt{625}$

Solution:

Clearly, $\mathrm{OA} = \sqrt{8^2 + 10^2} = \sqrt{164}$

$$\mathrm{OB} = \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52}$$

And $\mathrm{OC} = \sqrt{15^2 + 20^2} = \sqrt{225 + 400} = \sqrt{625}$