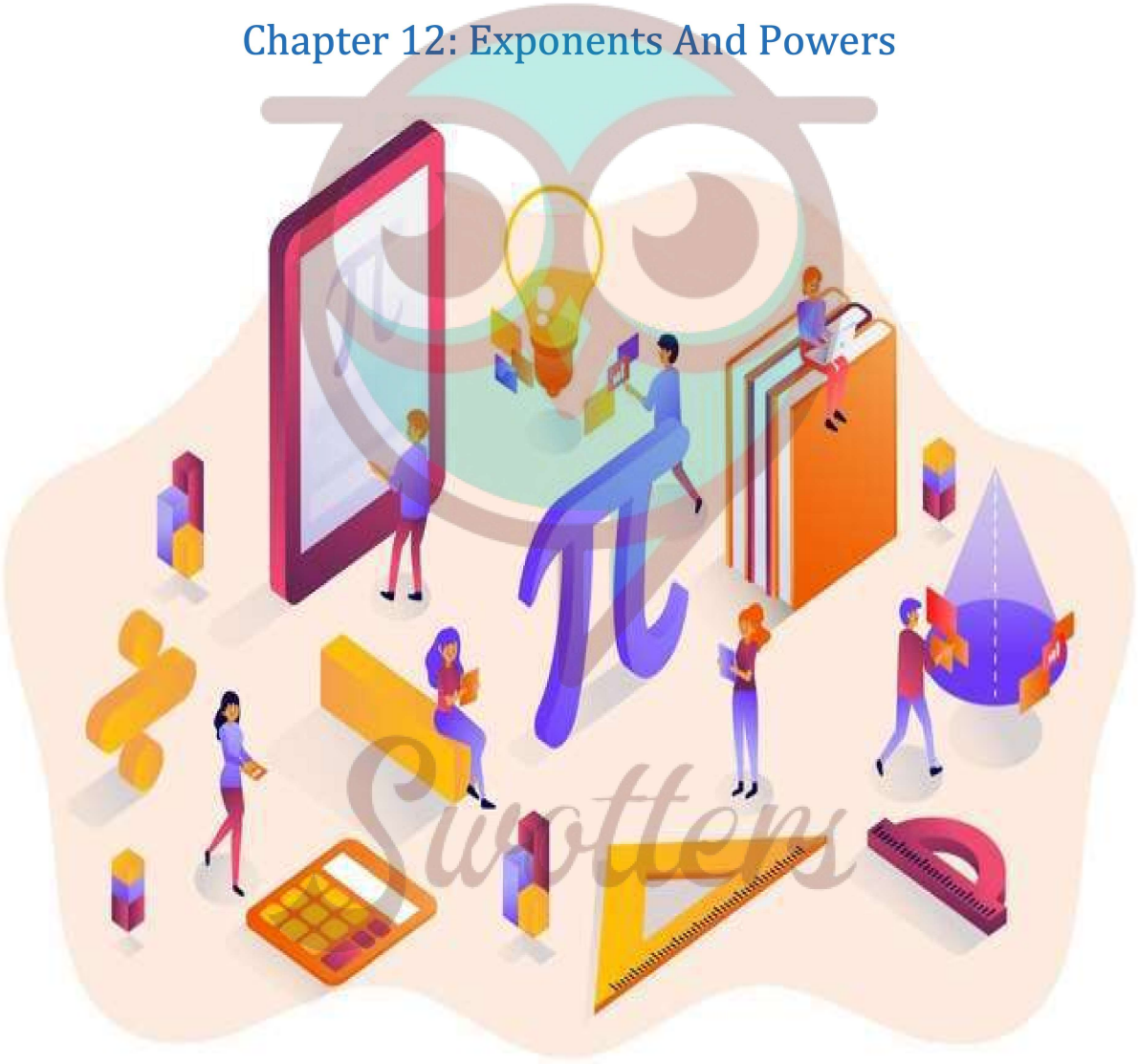
MATHEMATICS

Chapter 12: Exponents And Powers



Important Questions

Multiple Choice Questions-

Question 1. $a^m \times a^m$ is equal to

(a) a^{m+n}

(b) a^{m-n}

(c) a^{mn}

(d) a^{n-m}

Question 2. $a^m \div a^n$ is equal to

(a) a^{m-n}

(b) a^{m+n}

(c) a^{mn}

(d) a^{n-m}

Question 3. $(a^m)^n$ is equal to

(a) a^{m+n}

(b) a^{m-n}

(c) a^{mn}

(d) a^{n-m}

Question 4. $a^m \times b^m$ is equal to

(a) $(ab)^m$

(b) $(ab)^{-m}$

(c) a^m b

(d) ab^m

Question 5. a^0 is equal to

(a) 0

(b) 1

(c) -1

(d) a.

Question 6. $\frac{a^m}{b^m}$ is equal to bm

(a) $\left(\frac{a}{b}\right)^m$

(b) $\left(\frac{b}{a}\right)^m$

(c) $\left(\frac{a^m}{b}\right)^m$

(d) $\left(\frac{a}{b^m}\right)^m$

Question 7. 2 × 2 × 2 × 2 × 2 is equal to

(a) 2^4

(b) 2^3

(c) 2^2

(d) 2^5

Question 8. In 10^2, the exponent is

(a) 1

(b) 2

(c) 10

(d) 1.

Question 9. In 10^2 the base is

(a) 1

(b) 0

(c) 10

(d) 100.

Question 10. 10^{-1} is equal to

(a) 10

(b) -1

(c) $\frac{1}{10}$

(d) $\frac{-1}{10}$

Very Short Questions:

1. Find the multiplicative inverse of:

(i) 3^{-3}

(ii) 10^{-10}

2. Expand the following using exponents.

(i) 0.0523

(ii) 32.005

3. Simplify and write in exponential form.

(i) $(-5)^2 \times (-5)^{-3}$ (ii) $\left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2}$

4. Simplify the following and write in exponential form.

(i) $2^3 \times 3^3$ (ii) $\left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5$

5. Express 8-4 as a power with the base 2.

We have $8 = 2 \times 2 \times 2 = 2^3$

$8^{-4} = (2^3)^{-4} = 2^{3\times(-4)} = 2^{-12}$

Short Questions :

1. Simplify the following and write in exponential form.

(i) $(3^6 \div 3^8)^4 \times 3^{-4}$

(ii) $\frac{1}{27} \times 3^{-3}$

2. Find the value of k if $(-2)^{k+1} \times (-2)^3 = (-2)^7$

3. Simplify the following:

(i) $\left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii) $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$

4. Find the value of $\left[\left(-\frac{3}{4}\right)^{-2}\right]^2$

5. Write the following in standard form

(i) 0 0035

(ii) 365.05

6. Find the value of P if

$$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$$

Long Questions :

1.

If $\left(\frac{x}{y}\right) = \left(\frac{3}{2}\right)^{-2} \div \left(\frac{3}{7}\right)^0$, find the value of $\left(\frac{x}{y}\right)^{-3}$.

2. Find the value of x if

$$\left(\frac{125}{27}\right)\times\left(\frac{125}{27}\right)^{x}=\left(\frac{5}{3}\right)^{18}$$

3. Solve the following: $(81)^{-4} \div (729)^{2-x} = 9^{4x}$

4. Simplify: $\dfrac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$

5. Simplify: $\dfrac{(-2)^3 \times (-2)^7}{3 \times 4^6}$

6. Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Answer Key-

Multiple Choice questions-

1. (a) a^{m+n}
2. (a) a^{m-n}
3. (c) a^{mn}
4. (a) $(ab)^m$
5. (b) 1
6. (a) $\left(\frac{a}{b}\right)^m$
7. (d) 2^5
8. (b) 2
9. (c) 10
10. (c) $\frac{1}{10}$

Very Short Answer :

1. (i) Multiplicative inverse of $3^{-3} = \dfrac{1}{3^{-3}} = 3^3$

   (ii) Multiplicative inverse of $10^{-10} = \dfrac{1}{10^{-10}} = 10^{10}$

2.

$(i)\ 0.0523 = \frac{5}{100} + \frac{2}{1000} + \frac{3}{10000}$

$= 5 \times \frac{1}{100} + 2 \times \frac{1}{1000} + 3 \times \frac{1}{10000}$

$= 5 \times \frac{1}{10^2} + 2 \times \frac{1}{10^3} + 3 \times \frac{1}{10^4}$

$= 5 \times 10^{-2} + 2 \times 10^{-3} + 3 \times 10^{-4}$

$(ii)\ 32.005 = 3 \times 10 + 2 \times 1 + \frac{5}{1000}$

$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{1000}$

$= 3 \times 10 + 2 \times 1 + 5 \times \frac{1}{10^3}$

$= 3 \times 10 + 2 \times 1 + \times 5 \times 10^{-3}$

3.

$(i)\ (-5)^2 \times (-5)^{-3}$

$= (-5)^{2+(-3)} = (-5)^{2-3}$

$= (-5)^{-1} = -\frac{1}{5}$

$(ii)\ \left(\frac{1}{2}\right)^{-3} \times \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{(-3)+(-2)}$

$= \left(\frac{1}{2}\right)^{-3-2} = \left(\frac{1}{2}\right)^{-5}$

$= \frac{1}{2^{-5}} = 2^5$

4.

$(i)\ 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

$(ii)\ \left(\frac{4}{5}\right)^5 \times \left(\frac{5}{6}\right)^5 = \left(\frac{4}{5} \times \frac{5}{6}\right)^5$

$= \left(\frac{4}{6}\right)^5 = \left(\frac{2}{3}\right)^5$

5. We have $8 = 2 \times 2 \times 2 = 2^3$

$8^{-4} = (2^3)^{-4} = 2^{3\times(-4)} = 2^{-12}$

Short Answer :

1.

(i) $(3^6 \div 3^8)^4 \times 3^{-4}$

$= (3^{6-8})^4 \times 3^{-4} = 3^{-2\times4} \times 3^{-4}$

$= 3^{-8} \times 3^{-4} = 3^{-8-4} = 3^{-12}$

(ii) $\frac{1}{27} \times 3^{-3}$

$= \frac{1}{3^3} \times 3^{-3}$

$= 3^{-3} \times 3^{-3} = 3^{-3-3} = 3^{-6}$

2. $(-2)^{k+1} \times (-2)^3 = (-2)^7$

$\Rightarrow (-2)^{k+1+3} = (-2)^7$

$\Rightarrow (-2)^{k+4} = (-2)^7$

$\Rightarrow k + 4 = 7$

$\Rightarrow k = 3$

Hence, k = 3.

3.

(i) $\left\{\left(\frac{1}{4}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

$= \left\{\frac{1^{-3}}{4^{-3}} - \frac{1^{-3}}{3^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$\Rightarrow \left(\frac{4^3}{1^3} - \frac{3^3}{1^3}\right) \div \frac{4^2}{1^2}$

$\Rightarrow (64 - 27) \div 16$

$\Rightarrow 37 \div 16 = \frac{37}{16}$

(ii) $\left(\frac{2}{3}\right)^{-6} \times \left(\frac{3}{2}\right)^{-4}$

$= \frac{2^{-6}}{3^{-6}} \times \frac{3^{-4}}{2^{-4}} = \frac{3^6}{2^6} \times \frac{2^4}{3^4}$

$= \frac{3^{6-4}}{2^{6-4}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$

4.

$$\left[\left(-\frac{3}{4}\right)^{-2}\right]^2 = \left(-\frac{3}{4}\right)^{-4}$$

$$= (-1)^{-4} \times \left(\frac{3}{4}\right)^{-4}$$

$$= 1 \times \frac{3^{-4}}{4^{-4}} = \frac{4^4}{3^4} = \frac{256}{81}$$

5.

$$(i)\ 0.0035 = \frac{35}{10000}$$

$$= \frac{3.5 \times 10}{10^4} = 3.5 \times 10^{1-4} = 3.5 \times 10^{-3}$$

$$(ii)\ 365.05 = \frac{36505}{100} = \frac{3.6505 \times 10^4}{10^2}$$

$$= 3.6505 \times 10^2$$

6.

$$\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^{-6} = \left(\frac{2}{5}\right)^{2P-1}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{3-6} = \left(\frac{2}{5}\right)^{2P-1}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{-3} = \left(\frac{2}{5}\right)^{2P-1}$$

Equating the powers of the same base

$$2P - 1 = -3$$

$$2P = -3 + 1$$

$$2P = -2$$

$$\therefore \quad P = -1$$

Long Answer :

1.

$$\left(\frac{x}{y}\right) = \left(\frac{3^{-2}}{2^{-2}}\right) \div 1 \qquad \left[\because \left(\frac{a}{b}\right)^0 = 1\right]$$

$$= \frac{2^2}{3^2} = \frac{4}{9}$$

$$\therefore \quad \left(\frac{x}{y}\right)^{-3} = \left(\frac{4}{9}\right)^{-3}$$

$$= \frac{4^{-3}}{9^{-3}} = \frac{9^3}{4^3}$$

$$= \frac{729}{64}$$

2.

$$\left(\frac{125}{27}\right) \times \left(\frac{125}{27}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5^3}{3^3}\right) \times \left(\frac{5^3}{3^3}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^3 \times \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{18}$$

$$\Rightarrow \quad \left(\frac{5}{3}\right)^{3+x} = \left(\frac{5}{3}\right)^{18}$$

$\Rightarrow$ 3 + x = 18 [Equating the powers of same base]

x = 18 – 3 = 15

3.

$$(81)^{-4} \div (729)^{2-x} = (9)^{4x}$$

$$\Rightarrow \quad (9^2)^{-4} \div (9^3)^{2-x} = (9)^{4x}$$

$$\Rightarrow \quad 9^{-8} \div 9^{6-3x} = 9^{4x}$$

$$\Rightarrow \quad 9^{-8-(6-3x)} = 9^{4x}$$

$$\Rightarrow \quad 9^{-8-6+3x} = 9^{4x}$$

$$\Rightarrow \quad 9^{-14+3x} = 9^{4x}$$

Equating the power of same base, we have

$$-14 + 3x = 4x$$

$$\Rightarrow \quad 4x - 3x = -14$$

$$\therefore \quad x = -14$$

4.

$$\frac{(x^{m+n})^2 \times (x^{n+p})^2 \times (x^{p+m})^2}{(x^m \cdot x^n \cdot x^p)^3}$$

$$= \frac{x^{2m+2n} \times x^{2n+2p} \times x^{2p+2m}}{x^{3m} \cdot x^{3n} \cdot x^{3p}}$$

$$[\because \ (x^a)^b = (x^{ab})]$$

$$= \frac{x^{2m+2n+2n+2p+2p+2m}}{x^{3m+3n+3p}}$$

$$[\because \ x^a \times x^b = x^{a+b}]$$

$$= \frac{x^{4m+4n+4p}}{x^{3m+3n+3p}}$$

$$= x^{(4m+4n+4p)-(3m+3n+3p)}$$

$$= x^{4m+4n+4p-3m-3n-3p}$$

$$= x^{m+n+p} \qquad \left[\because \ \frac{x^a}{x^b} = x^{a-b}\right]$$

5.

$$\frac{(-2)^3 \times (-2)^7}{3 \times 4^6} = \frac{(-2)^{3+7}}{3 \times (2^2)^6} \ \{a^m \times a^n = a^{m+n}\}$$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} \ \{(a^m)^n = a^{m \times n}\}$$

$$= \frac{(-2)^{10}}{3 \times 2^{12}} = \frac{2^{10-12}}{3} \ \{a^m \div a^n = a^{m-n}, (-2)^{10} = 2^{10}\}$$

$$= \frac{2^{-2}}{3} = \frac{1}{3 \times 2^2} = \frac{1}{12}$$

6. $(-5)^{x+1} \times (-5)^5 = (-5)^7$

$(-5)^{x+1+5} = (-5)^7$ $\{a^m \times a^n = a^{m+n}\}$

$(-5)^{x+6} = (-5)^7$

On both sides, powers have the same base, so their exponents must be equal.

Therefore, $x + 6 = 7$

$x = 7 - 6 = 1$

$x = 1$.