MATHEMATICS

Chapter 12: Three Dimensional Geometry

Important Questions

Multiple Choice questions-

Question 1. The projections of a directed line segment on the coordinate axes are 12, 4, 3. The DCS of the line are:

(a) 12/13, -4/13, 3/13

(b) -12/13, -4/13, 3/13

(c) 12/13, 4/13, 3/13

(d) None of these

Question 2. The angle between the planes $r \cdot n_1 = d_1$ and $r \cdot n_1 = d_2$ is:

(a) $\cos \theta = \{|n_1| \times |n_2|\} / (n_1 \cdot n_2)$

(b) $\cos \theta = (n_1 \cdot n_2)/\{|n_1| \times |n_2|\}^2$

(c) $\cos \theta = (n_1 \cdot n_2)/\{|n_1| \times |n_2|\}$

(d) $\cos \theta = (n_1 \cdot n_2)^2 / \{|n_1| \times |n_2|\}$

Question 3. For every point P(x, y, z) on the xy-plane

(a) x = 0

(b) y = 0

(c) z = 0

(d) None of these

Question 4. The locus of a point P(x, y, z) which moves in such a way that x = a and y = b, is a.

(a) Plane parallel to xy-plane

(b) Line parallel to x-axis

(c) Line parallel to y-axis

(d) Line parallel to z-axis

Question 5. The equation of the plane containing the line 2x – 5y + z = 3, x + y + 4z = 5 and parallel to the plane x + 3y + 6z = 1 is

(a) x + 3y + 6z + 7 = 0

(b) x + 3y – 6z – 7 = 0

(c) x – 3y + 6z – 7 = 0

(d) x + 3y + 6z – 7 = 0

Question 6. The coordinate of foot of perpendicular drawn from the point A(1, 0, 3) to the join of the point B(4, 7, 1) and C(3, 5, 3) are

(a) (5/3, 7/3, 17/3)

(b) (5, 7, 17)

(c) (5/3, -7/3, 17/3)

(d) (5/7, -7/3, -17/3)

Question 7. The coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane is

(a) (0, 17/2, 13/2)

(b) (0, -17/2, -13/2)

(c) (0, 17/2, -13/2)

(d) None of these

Question 8. If P is a point in space such that OP = 12 and OP inclined at angles 45 and 60 degrees with OX and OY respectively, then the position vector of P is

(a) $6i + 6j \pm 6\sqrt{2}k$

(b) $6i + 6\sqrt{2}j \pm 6k$

(c) $6\sqrt{2}i + 6j \pm 6k$

(d) None of these

Question 9. The image of the point P(1,3,4) in the plane $2x – y + z = 0$ is

(a) (-3, 5, 2)

(b) (3, 5, 2)

(c) (3, -5, 2)

(d) (3, 5, -2)

Question 10. There is one and only one sphere through

(a) 4 points not in the same plane

(b) 4 points not lie in the same straight line

(c) none of these

(d) 3 points not lie in the same line

Very Short Questions:

1. Name the octants in which the following lie. (5,2,3)
2. Name the octants in which the following lie. (-5,4,3)

3. Find the image of (-2,3,4) in the y z plane.
4. Find the image of (5,2,-7) in the plane xy.
5. A point lie on X –axis what are co ordinate of the point
6. Write the name of plane in which x axis and y- axis taken together.
7. The point (4, -3, 6) lie in which octants.
8. The point (2, 0, 8) lie in which palne.
9. A point is in the XZ plane. What is the value of y co-ordinates?
10. What is the coordinates of XY plane?

Short Questions:

1. Given that P(3,2,-4), Q(5,4,-6) and R(9,8,-10) are collinear. Find the ratio in which Q divides PR.
2. Determine the points in xy plane which is equidistant from these point A (2,0,3) B(0,3,2) and C(0,0,1).
3. Find the locus of the point which is equidistant from the point A(0,2,3) and B(2,-2, 1)
4. Show that the points A(0,1,2) B(2,-1,3) and C(1,-3,1) are vertices of an isosceles right angled triangle.
5. Using section formula, prove that the three points A(-2,3,5), B(1,2,3), and C(7,0,-1) are collinear.

Long Questions:

1. Prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.
2. The mid points of the sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices.
3. Let P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) be two points in space find coordinate of point R which divides P and Q in the ratio $m_1 : m_2$ by geometrically.
4. Show that the plane ax + by + cz + d = 0 divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratios $\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$.
5. Prove that the points 0(0, 0, 0), A(2, 0, 0), B(1, $\sqrt{3}$, 0), and c $\left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right)$ are the vertices of a regular tetrahedron.

Answer Key:

MCQ:

1. (c) 12/13, 4/13, 3/13
2. (c) $\cos\theta = (n_1 . n_2)/\{|n_1| \times |n_2|\}$
3. (c) z = 0
4. (b) Line parallel to x-axis
5. (d) x + 3y + 6z – 7 = 0
6. (a) (5/3, 7/3, 17/3)
7. (c) (0, 17/2, -13/2)
8. (c) 6√2i + 6j ± 6k
9. (a) (-3, 5, 2)
10. (a) 4 points not in the same plane

Very Short Answer:

1. I
2. II
3. (2, 3, 4)
4. (5, 2, 7)
5. (a, 0, 0)
6. XY Plane
7. VIII
8. XZ
9. Zero
10. (x, y, 0)

Short Answer:

1. Suppose Q divides PR in the ratio λ:1. Then coordinator of Q are.

$$\left(\frac{9\lambda+3}{\lambda+1}, \frac{8\lambda+2}{\lambda+1}, \frac{-10\lambda-4}{\lambda+1}\right)$$

But, coordinates of Q are (5,4,-6). Therefore

$$\frac{9\lambda+3}{\lambda+1}=5, \frac{8\lambda+2}{\lambda+1}=4, \frac{-10\lambda-4}{\lambda+1}=6$$

These three equations give

$= \frac{1}{2}$

So Q divides PR in the ratio $\frac{1}{2}$: 1 or 1:2

2. We know that Z- coordinate of every point on xy-plane is zero. So, let P(x, y, 0) be a point in xy-plane such that PA = PB = PC

Now, PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y-0)^2 + (0-3)^2 = (x-0)^2 + (y-3)^2 + (0-2)^2$

$\Rightarrow 4x - 6y = 0 \text{ or } 2x - 3y = 0 .... (i)$

$PB = PC$

$\Rightarrow PB^2 = PC^2$

$\Rightarrow (x-0)^2 + (y-3)^2 + (0-2)^2 = (x-0)^2 + (y-0)^2 + (0-1)^2$

$\Rightarrow -6y + 12 = 0 \Rightarrow y = 2 .... (ii)$

Putting y = 2 in (i) we obtain x = 3

Hence the required points (3,2,0).

3. Let P(x, y, z) be any point which is equidistant from A(0,2,3) and B(2,-2,1). Then

PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow \sqrt{(x-0)^2 + (y-2)^2 + (2-3)^2} = \sqrt{(x-2)^2 + (y+2)^2 + (z-1)^2}$

$\Rightarrow 4x - 8y - 42 + 4 = 0 \text{ or } x - 2y - 2 + 1 = 0$

4. We have

$AB = \sqrt{(2-0)^2 + (-1-1)^2 (+3-2)^2} = \sqrt{4+4+1} = 3$

$BC = \sqrt{(1-2)^2 + (-3+1)^2 + (1-3)^2} = \sqrt{1+4+4} = 3$

And $CA = \sqrt{(1-0)^2 + (-3-1)^2 + (1-2)^2} = \sqrt{1+16+1} = 3\sqrt{2}$

Clearly AB = BC and $AB^2 + BC^2 = AC^2$

Hence, triangle ABC is an isosceles right angled triangle.

5. Suppose the given points are collinear and C divides AB in the ratio λ : 1.

Then coordinates of C are

$\left(\frac{\lambda-2}{\lambda+1}, \frac{2\lambda+3}{\lambda+1}, \frac{3\lambda+5}{\lambda+1}\right)$

But, coordinates of C are (3,0,-1) from each of there equations, we get $\lambda = \frac{3}{2}$

Since each of there equation give the same value of V. therefore, the given points are collinear and C divides AB externally in the ratio 3:2.

Long Answer:

1. Let ABCD be tetrahedron such that the coordinates of its vertices are A (x_1, y_1, z_1), B(x_2, y_2, z_2) C (x_3, y_3, z_3) and D (x_4, y_4, z_4)

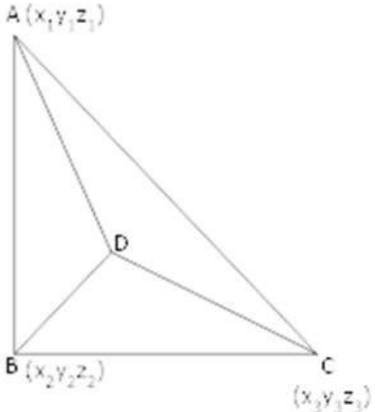
   The coordinates of the centroids of faces ABC, DAB, DBC and DCA respectively

   $G_1\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right]$

   $G_2\left[\frac{x_1+x_2+x_4}{3}, \frac{y_1+y_2+y_4}{3}, \frac{z_1+z_2+z_4}{3}\right]$

   $G_3\left[\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3}\right]$

   $G_4\left[\frac{x_4+x_3+x_1}{3}, \frac{y_4+y_3+y_1}{3}, \frac{z_4+z_3+z_1}{3}\right]$



   Now, coordinates of point G dividing DG1 in the ratio 3:1 are

$$\left[\frac{1.x_4+3\left(\frac{x_1+x_2+x_3}{3}\right)}{1+3}, \frac{1.y_4+3\left(\frac{y_1+y_2+y_3}{3}\right)}{1+3}, \frac{1.z_4+3\left(\frac{z_1+z_2+z_3}{3}\right)}{1+3}\right]$$

$$=\left[\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right]$$

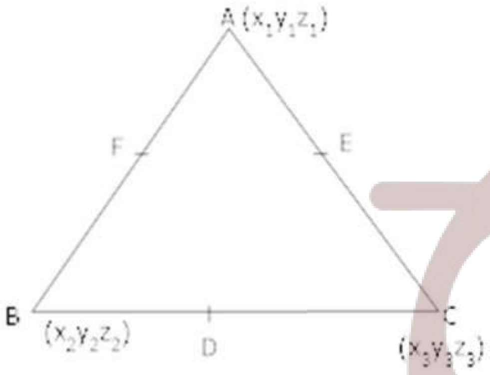
   Similarly the point dividing CG2, AG3 and BG4 in the ratio 3:1 has the same coordinates.

Hence the point $G\left[\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right]$ is common to DG1, CG2, AG3 and BG4.

Hence they are concurrent.

2. Suppose vertices of ΔABC are A (x_1, y_1, z_1), B(x_2, y_2, z_2) and C (x_3, y_3, z_3) respectively

Given coordinates of mid point of side BC, CA, and AB respectively are D(1,5,-1), E(0,4,-2) and F(2,3,4)



$\therefore \frac{x_2+x_3}{2}=1 \quad \frac{y_2+y_3}{2}=5 \quad \frac{z_2+z_3}{2}=-1$

$x_2+x_3=2......(i)$

$\frac{x_1+x_3}{2}=0$

$y_2+y_3=10......(ii)$

$\frac{y_1+y_3}{2}=4$

$z_2+z_3=2......(iii)$

$\frac{z_1+z_3}{2}=-2$

$x_1+x_3=0.......(iv)$

$\frac{x_1+x_2}{2}=2$

$y_1+y_2=8.......(v)$

$\frac{y_1+y_2}{2}=3$

$z_1+z_3=-4.......(vi)$

$\frac{z_1+z_2}{2}=4$

$x_1+x_2=4.......(vii)$

$y_1 + y_2 = 6$.......(viii)

$z_1 + z_2 = 8$.......(ix)

Adding eq. (i), (iv), (viii)

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3$.......(x)

Subtracting eq. (i), (iv), (vii) from (x) we get

$x_1 = 1,\ x_2 = 3,\ x_3 = -1$

Similarly, adding eq. (ii), (v) and (viii)

$y_1 + y_2 + y_3 = 12$......(xi)

Subtracting eq. (ii), (v) and (viii) from (ix)

$y_1 = 2,\ y_2 = 4,\ y_3 = 6$

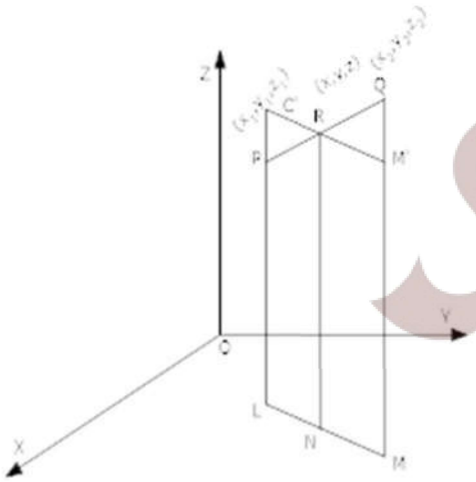
Similarly $z_1 + z_2 + z_3 = 3$

$z_1 = 1,\ z_2 = 7,\ z_3 = -5$

∴ Coordinates of vertices of ΔABC are A(1,3,-1), B(2,4,6) and C(1,7,-5)

3. Let co-ordinate of Point R be (x, y, z) which divider line segment joining the point PQ in the ratio $m_1 : m_2$

Clearly $\Delta PRL' \sim \Delta QRM'$ [By AA sinilsrity]



$$\therefore \frac{PL'}{MQ'} = \frac{PR}{RQ}$$

$$\Rightarrow \frac{LL' - LP}{MQ - MM'} = \frac{m_1}{m_2}$$

$$\Rightarrow \frac{NR - LP}{MQ - NR} = \frac{m_1}{m_2} \qquad \left[\begin{array}{l}\because LL' = NR \\ \text{and } MM' = NR\end{array}\right]$$

$$\Rightarrow \frac{z-z_1}{z_2-z} = \frac{m_1}{m_2}$$

$$\Rightarrow z = \frac{m_1 . z_2 + m_2 z_1}{m_1 + m_2}$$

Similarly, $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ and

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

4. Suppose the plane ax + by + cz + d = 0 divides the line joining the points (x_1, y_1, z_1) and (x_1, y_2, z_2) in the ratio λ : 1

$$\therefore x = \frac{\lambda x_2 + x_1}{\lambda + 1}, \quad y = \frac{\lambda y_2 + y_1}{\lambda + 1}, \quad z = \frac{\lambda z_2 + z_1}{\lambda + 1}$$

$\because$ Plane $ax + by + cz + d = 0$ Passing through (x, y, z)

$$\therefore Q\frac{(\lambda x_2 + x_1)}{\lambda + 1} + b\frac{(\lambda y_2 + y_1)}{\lambda + 1} + c\frac{(\lambda z_2 + z_1)}{\lambda + 1} + d = 0$$

$$a(\lambda x_2 + x_1) + b(\lambda y_2 + y_1) + c(\lambda z_2 + z_1) + d(\lambda + 1) = 0$$

$$\lambda(ax_2 + by_2 + cz_2 + d) + (ax_1 + by_1 + cz_1 + d) = 0$$

$$\lambda = -\frac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}$$

Hence Proved.

5. To prove O, A, B, C are vertices of regular tetrahedron.

We have to show that

|OA|=|OB|=|OC|=|AB|=|BC|=|CA|

$|OA| = \sqrt{(0-2)^2 + 0^2 + 0^2} = 2$ unit

$|OB| = \sqrt{(0-1)^2 + (0-\sqrt{3})^2 + 0^2} = \sqrt{1+3} = \sqrt{4} = 2$ unit

$|OC| = \sqrt{(0-1)^2 + \left(0-\frac{1}{\sqrt{3}}\right) + \left(0-\frac{2\sqrt{2}}{3}\right)^2}$

$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$ unit

$|AB| = \sqrt{(2-1)^2 + (0-\sqrt{3})^2 + (10-0)^2} = \sqrt{1+3+0}$

$= \sqrt{4} = 2$ unit

$|BC| = \sqrt{(1-1)^2 + \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)^2 + \left(0-\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}$

$= \sqrt{0+\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

$|CA| = \sqrt{(1-2)^2 + \left(\frac{1}{\sqrt{3}}-0\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}-0\right)^2}$

$= \sqrt{1+\frac{1}{3}+\frac{8}{3}}$

$= \sqrt{\frac{12}{3}} = 2$ unit

∴ |AB| = |BC| = |CA| = |OA| = |OB| = |OC| = 2 unit

∴ O, A, B, C are vertices of a regular tetrahedron.