MATHEMATICS

Important Questions

Multiple Choice questions-

1. The point which does not lie in the half plane $2x + 3y - 12 < 0$ is

(a) (1, 2)

(b) (2, 1)

(c) (2, 3)

(d) (-3, 2).

2. The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \le 10$, $x + 3y \le 15$, $x, y \ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5).

Let $Z = px + qy$, where $p, q > 0$. Conditions on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(a) $p = 3q$

(b) $p = 2q$

(c) $p = q$

(d) $q = 3p$.

3. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is

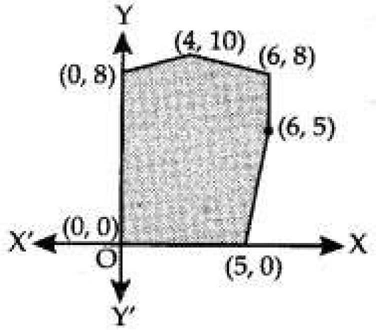
(a) $p = q$

(b) $p = 2q$

(c) $q = 2p$

(d) $q = 3p$.

4. The feasible solution for a LPP is shown in the following figure. Let $Z = 3x – 4y$ be the objective function.



Minimum of Z occurs at:

(a) (0, 0)

(b) (0, 8)

(c) (5, 0)

(d) (4, 10).

5. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let Z = px + qy, where p, q> 0. Condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20) is Maximum of Z occurs at:

(a) (5, 0)

(b) (6, 5)

(c) (6, 8)

(d) (4, 10).

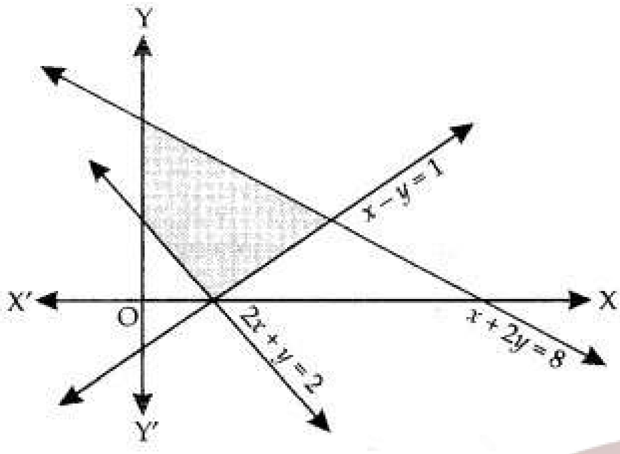
Very Short Questions:

1. Draw the graph of the following LPP:

   $5x + 2y \leq 10, x \geq 0, y \geq 0.$

2. Solve the system of linear inequations: $x + 2y \leq 10$; $2x + y \leq 8$.

3. Find the linear constraints for which the shaded area in the figure below is the solution set:



4. A small firm manufactures neclaces and bracelets. The total number of neclaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a neclace. The maximum number of hours available per day is 16. If the profit on a neclace is ₹100 and that on a bracelet is ₹300. Formulate an LPP for finding how many of each should be produced daily to maximize the profit?

   It is being given that at least one of each must be produced. (C.B.S.E. 2017)

5. Old hens can be bought for? 2.00 each and young ones at? 5.00 each. The old hens lay 3 eggs per week and the young hens lay 5 eggs per week, each egg being worth 30 paise. A hen costs ₹1.00 per week to feed. A man has only ₹80 to spend for hens. Formulate the problem for maximum profit per week, assuming that he cannot house more than 20 hens.

Long Questions:

1. Maximize Z-5x + 3y

   subject to the constraints:

   $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0, y \ge 0$. (N.C.E.R.T)

2. Minimize $Z = 3x + 2y$ subject to the constraints:

   $x + y \ge 8$, $3x + 5y \le 15$, $x \ge 0$, $y \ge 0$. (N.C.E.R.T.)

3. Determine graphically the minimum value of the objective function :

   $Z = -50x + 20y$

   subject to the constraints:

   $2x - y \ge -5$, $3x + y \ge 3$, $2x - 3y \le 12$, $x, y \ge 0$. (N.C.E.R.T.)

   Hence, find the shortest distance between the lines. (C.B.S.E. Sample Paper 2018-19)

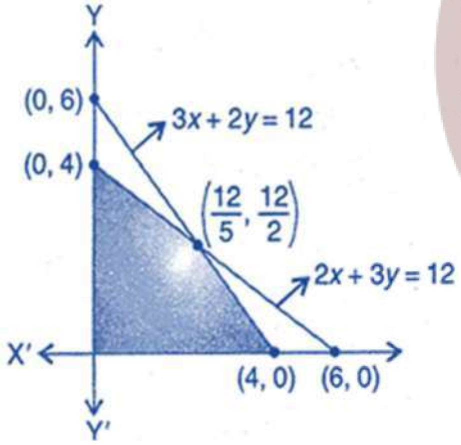
2. Minimize and Maximize Z = 5x + 2y subject to the following constraints: $x - 2y \le 2$, $3x + 2y < 12$, $-3x + 2y \le 3$, $x \ge 0$, $y \ge 0$. (A.I.C.B.S.E. 2015)

Assertion and Reason Questions:

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Consider the graph of $2x+3y\le12$, $3x+2y\le12$, $x\ge0$, $y\ge0$.



Assertion(A): (5, 1) is an infeasible solution of the problem.

Reason (R): Any point inside the feasible region is called an infeasible solution.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Consider the graph of constraints

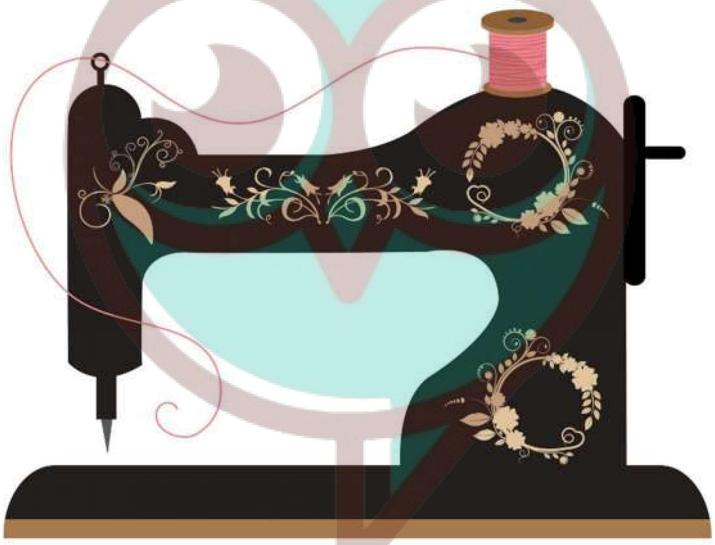
$5x + y \le 100$, $x + y \le 60$, $x, y \ge 0$

Assertion (A): The points (10, 50), (0, 60) and (20, 0) are feasible solutions.

Reason(R): Points within and on the boundary of the feasible region represents infeasible solutions.

Case Study Questions:

1. Suppose a dealer in rural area wishes to purpose a number of sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items for storage. An electronic sewing machine costs him ₹ 360 and a manually operated sewing machine ₹ 240. He can sell an electronic sewing machine at a profit of ₹ 22 and a manually operated sewing machine at a profit of ₹ 18.



Based on the above information, answer the following questions.

(i) Let x and y denotes the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assume that the dealer purchased atleast one of the the given machines, then:

a. $X + y \geq 0$
b. $X + y < 0$
c. $X + y > 0$
d. $X + y \leq 0$

(ii) Let the constraints in the given problem is represented by the following inequalities.

$$x + y \leq 20$$

$$360x + 240y \leq 5760$$

$$x, y \geq 0$$

Then which of the following point lie in its feasible region.

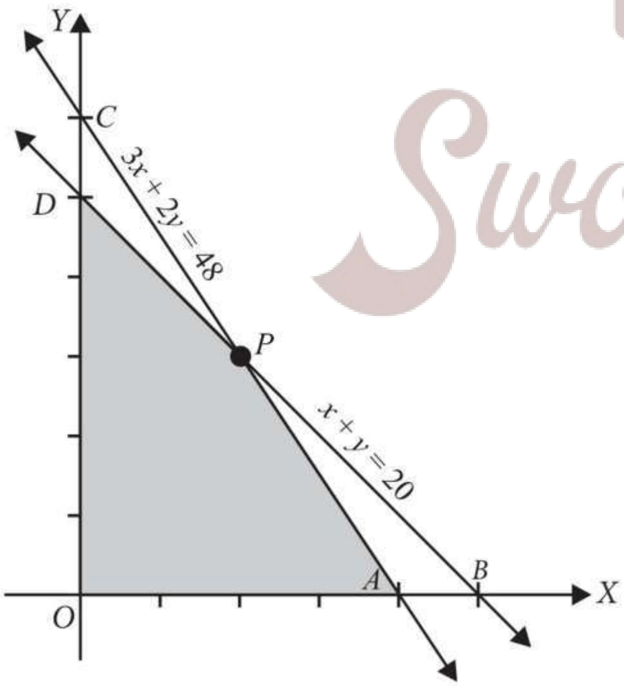
a. (0, 24)
b. (8, 12)
c. (20, 2)
d. None of these

(iii) If the objective function of the given problem is maximise z = 22x + 18y, then its optimal value occur at:

a. (0, 0)
b. (16, 0)
c. (8, 12)
d. (0, 20)

(iv) Suppose the following shaded region APDO, represent the feasible region corresponding to mathematical formulation of given problem.

Then which of the following represent the coordinates of one of its corner points



a. (0, 24)

b. (12, 8)
c. (8, 12)
d. (6, 14)

(v) If an LPP admits optimal solution at two consecutive vertices of a feasible region, then:

a. The required optimal solution is at the midpoint of the tine joining two points.
b. The optimal solution occurs at every point on the tine joining these two points.
c. The LPP under consideration is not solvable.
d. The LPP under consideration must be reconstructed.

2. Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = 4x - 6y be the objective function.

Based on the above information, answer the following questions.

(i) The minimum value of Z occurs at:

a. (6, 8)
b. (5, 0)
c. (0, 3)
d. (0, 8)

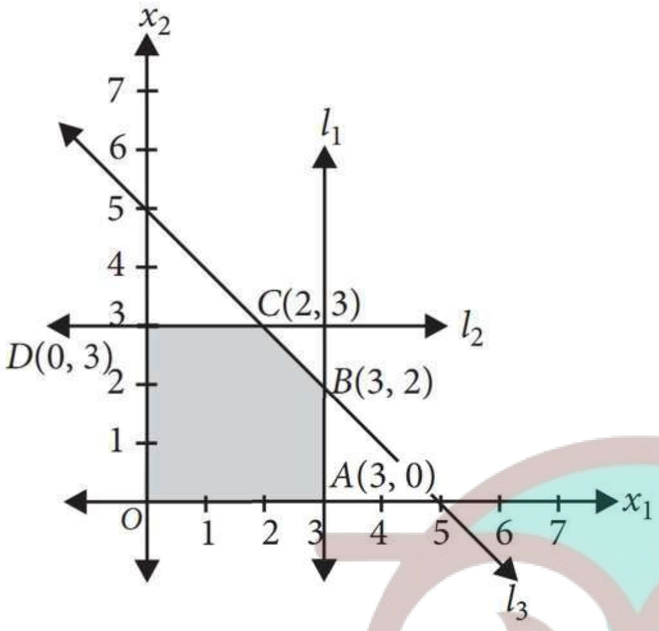
(ii) Maximum value of Z occurs at:

a. (5, 0)
b. (0, 8)
c. (0, 3)
d. (6, 8)

(iii) Maximum of Z - Minimum of Z =

a. 58
b. 68
c. 78
d. 88

(iv) The corner points of the feasible region determined by the system of linear inequalities are:



a. (0, 0), (-3, 0), (3, 2). (2, 3)
b. (3, 0), (3, 2), (2, 3), (0, -3)
c. (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)
d. None of these

(v) The feasible solution of LPP belongs to:

a. First and second quadrant.
b. First and third quadrant.
c. Only second quadrant.
d. Only first quadrant.

Answer Key-

Multiple Choice questions-

1. Answer: (c) (2, 3)
2. Answer: (d) q = 3p.
3. Answer: (d) q = 3p.
4. Answer: (b) (0, 8)
5. Answer: (a) (5, 0)

Very Short Answer:

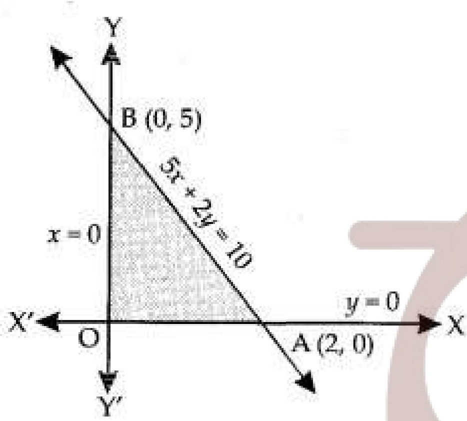
1. Solution:

Draw the line AB: 5.v + 2y = 10 ...(1),

which meets x-axis at A (2, 0) and y-axis at B (0,5).

Also, x = 0 is y-axis and y = 0 is x-axis.

Hence, the graph of the given LPP is as shown (shaded):

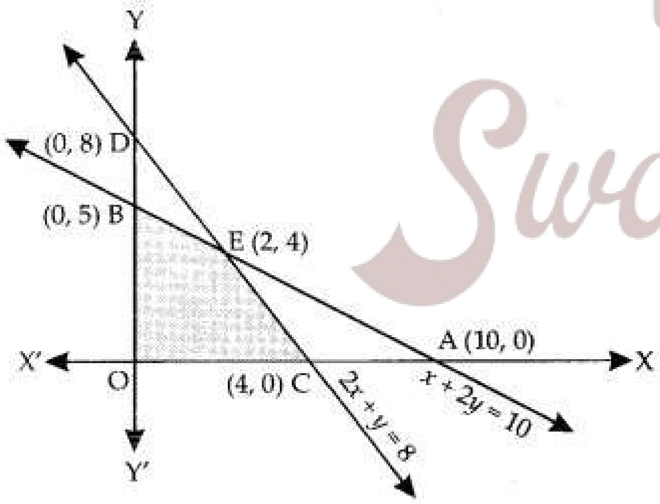


2. Solution:

Draw the st. lines x + 2y = 10 and 2x + y = 8.

These lines meet at E (2,4).

Hence, the solution of the given linear inequations is shown as shaded in the following figure :



3. Solution:

From the above shaded portion, the linear constraints are :

$2x + y \geq 2, x - y \leq 1$,

$x + 2y \le 8, x \ge 0, y \ge 0.$

4. Solution:

Let 'x' neclaces and 'y' bracelets be manufactured per day.

Then LPP problem is:

Maximize Z = 100x+300y

Subject to the constraints : $x + y \le 24$,

(1) $(x) + \frac{1}{2}y \le l6$,

i.e. $2x + y \le 32$

and $x \ge 1$

and $y \ge 1$

i.e. $x - 1 \ge 0$

and $y - 1 \ge 0$.

5. Solution:

Let 'x' be the number of old hens and 'y' the number of young hens.

Profit = $(3x + 5y)\frac{30}{100} - (x + y)$ (1)

$= \frac{9x}{10} + \frac{3}{2}yx - y$

$= \frac{y}{2} - \frac{x}{10} = \frac{5y-x}{10}$

∴ LPP problem is:

Maximize Z = $\frac{5y-x}{10}$ subject to:

$x \ge 0$,

$y \ge 0$,

$x + y \le 20$ and

$2x + 5y \le 80$.

Long Answer:

1. Solution:

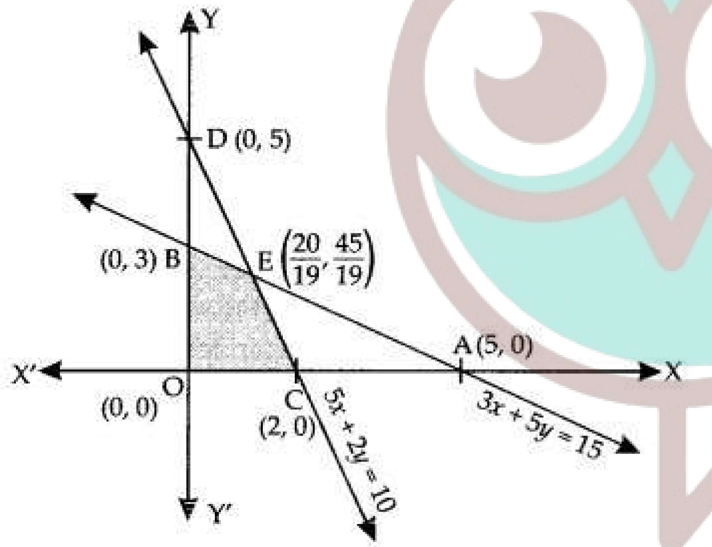
The system of constraints is :

3x + 5y ≤ 15 ...(1)

5x + 2y ≤ 10 ...(2)

and x ≥ 0, y≥ 0 ...(3)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (3):



It is observed that the feasible region OCEB is bounded. Thus we use Corner Point Method to determine the maximum value of Z, where:

Z = 5x + 3y ...(4)

The co-ordinates of O, C, E and B are (0, 0), (2,0), $\left(\frac{20}{19}, \frac{45}{19}\right)$ (Solving 3x + 5y = 15 and 5x + 2y – 10) and (0, 3) respectively.

We evaluate Z at each comer point:

Comer Point	Corresponding Value of Z
O: (0,0)	0
C: (2,0)	10
E($\left(\frac{20}{19}, \frac{45}{19}\right)$)	$\frac{20}{19}$ (Maximum)
B(0.3)	9

Hence' Z_{max} = at the Point $\left(\frac{20}{19}, \frac{45}{19}\right)$

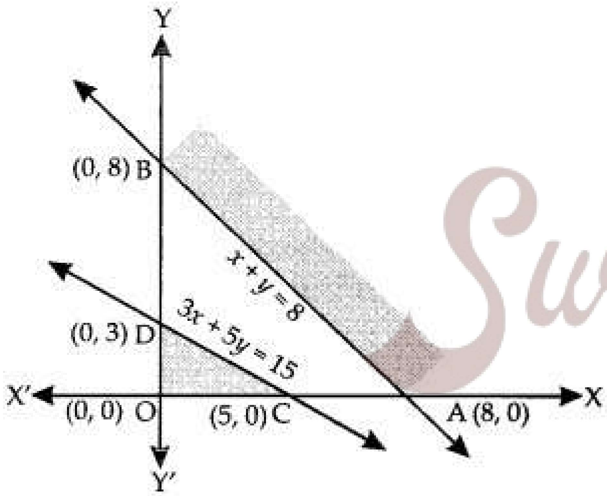
2. Solution:

The system of constraints is :

x +y ≥ 8, , x ≥ 0, y ≥ 0...(1)

3x + 5y ≤ 15 ...(2)

and x ≥ 0, y ≥ 0 ...(3)



It is observed that there is no point, which satisfies all (1) – (3) simultaneously.

Thus there is no feasible region.

Hence, there is no feasible solution.

Solution:

The system of constraints is :

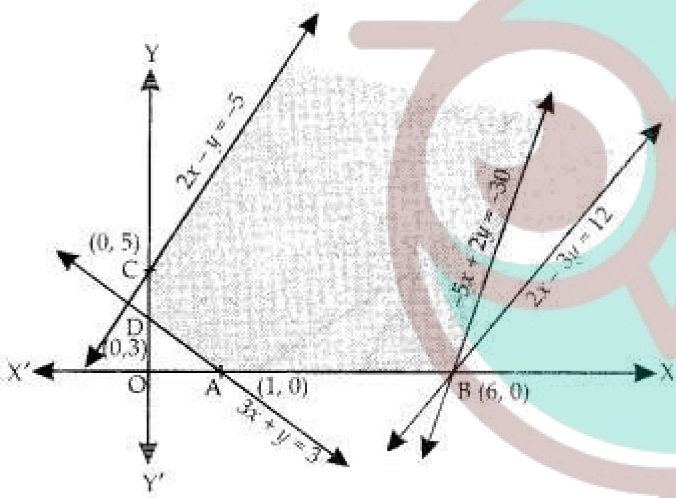
2x-y ≥ – 5 ...(1)

3x +y ≥ 3 ...(2)

2x – 3y ≤ 12 ...(3)

and x,y ≥ 0 ...(4)

The shaded region in the following figure is the feasible region determined by the system of constraints (1) – (4).



t is observed that the feasible region is unbounded.

We evaluate Z = – 50x + 20y at the corner points:

A (1, 0), B (6, 0), C (0, 5) and D (0, 3):

Corner Point	Corresponding Value of Z
A: (1,0)	-50
B : (6, 0)	– 300 (Minimum)
C : (0, 5)	100
D : (0, 3)	60

From the table, we observe that – 300 is the minimum value of Z.

But the feasible region is unbounded.

∴ – 300 may or may not be the minimum value of Z. ”

For this, we draw the graph of the inequality.

$-50x + 20y < -300$

i.e. $-5x + 2y < -30$.

Since the remaining half-plane has common points with the feasible region,

∴ $Z = -50x + 20y$ has no minimum value.

3. Solution:

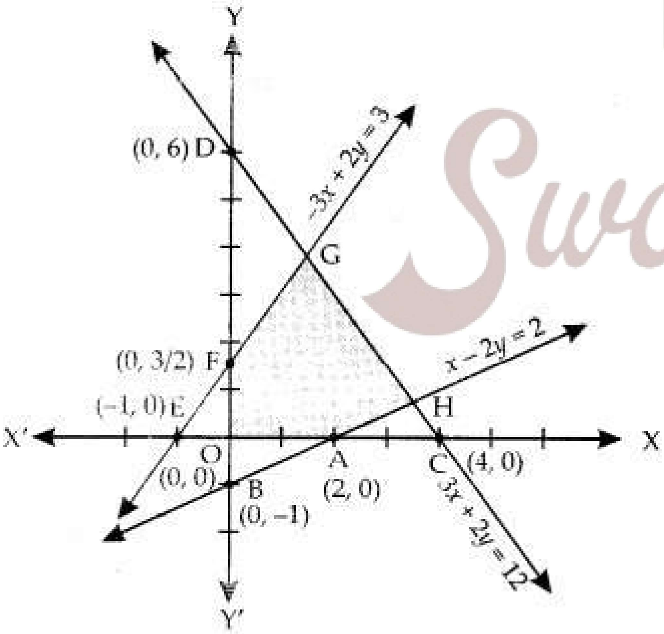
The given system of constraints is:

$x - 2y \leq 2$ ...(1)

$3x + 2y < 12$ ...(2)

$-3x + 2y \leq 3$ ...(3)

and $x \geq 0, y \geq 0$.



The shaded region in the above figure is the feasible region determined by the system of constraints (1) – (4). It is observed that the feasible region OAHGF is bounded. Thus we use Corner Point Method to determine the maximum and minimum value of Z, where

Z = 5x + 2y ...(5)

The co-ordinates of O, A, H, G and F are :

(0, 0). (2. 0), $\left(\frac{7}{2}, \frac{3}{4}\right)$ and $\left(\frac{3}{2}, \frac{15}{4}, \frac{3}{2}\right)$

respectively. [Solving x

2y = 2 and 3x + 2y = 12 for

H and -3x + 2y = 3 and

3x + 2y = 12 for G]

We evaluate Z at each corner point:

Corner Point	Corresponding value of Z
O: (0,0)	0 (Minimum)
A: (2,0)	10
H$\left(\frac{7}{2}, \frac{3}{4}\right)$	19 (Maximum)
G$\left(\frac{3}{2}, \frac{15}{4}\right)$	15
F: (0, $\frac{3}{2}$))	3

Hence, Z_{max} = 19 at $\left(\frac{7}{2}, \frac{3}{4}\right)$ and
Z_{max} = 0at (0,0)

Case Study Answers:

1. Answer :

(i) (c) x+y>0

(ii) (b) (8, 12)

Solution:

Since (8, 12) satisfy all the inequalities, therefore (8, 12) is the point in its feasible region.

(iii) (c) (8, 12)

Solution:

At (0, 0), z = 0

At (16, 0), z = 352

At (8, 12), z = 392

At (0, 20), z = 360

It can be observed that max z occur at (8, 12). Thus, z will attain its optimal value at (8, 12).

(iv) (c) (8, 12)

Solution:

We have, x + y = 20 (i)

And 3x + 2y = 48 (ii)

On solving (i) and (ii), we get

x = 8, y = 12.

Thus, the coordinates of Pare (8, 12) and hence (8, 12) is one of its corner points.

(v) (b) The optimal solution occurs at every point on the tine joining these two points.

Solution:

The optimal solution occurs at every point on the line joining these two points.

2. Answer :

Construct the following table of values of objective function:

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	4 × 0 - 6 × 3 = -18
(5, 0)	4 × 5 - 6 × 0 = 20
(6, 8)	4 × 6 - 6 × 8 = -24
(0, 8)	4 × 0 - 6 × 8 = -48

(i) (d) (0, 8)

Solution:

Minimum value of Z is -48 which occurs at (0, 8).

(ii) (a) (5, 0)

Solution:

Maximum value of Z is 20, which occurs at (5, 0).

(iii) (b) 68

Solution:

Maximum of Z - Minimum of Z = 20 - (-48) = 20 + 48 = 68

(iv) (c) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)

Solution:

The comer points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).

(v) (d) Only first quadrant.

Assertion and Reason Answers:

1. c) A is true but R is false.

2. c) A is true but R is false.