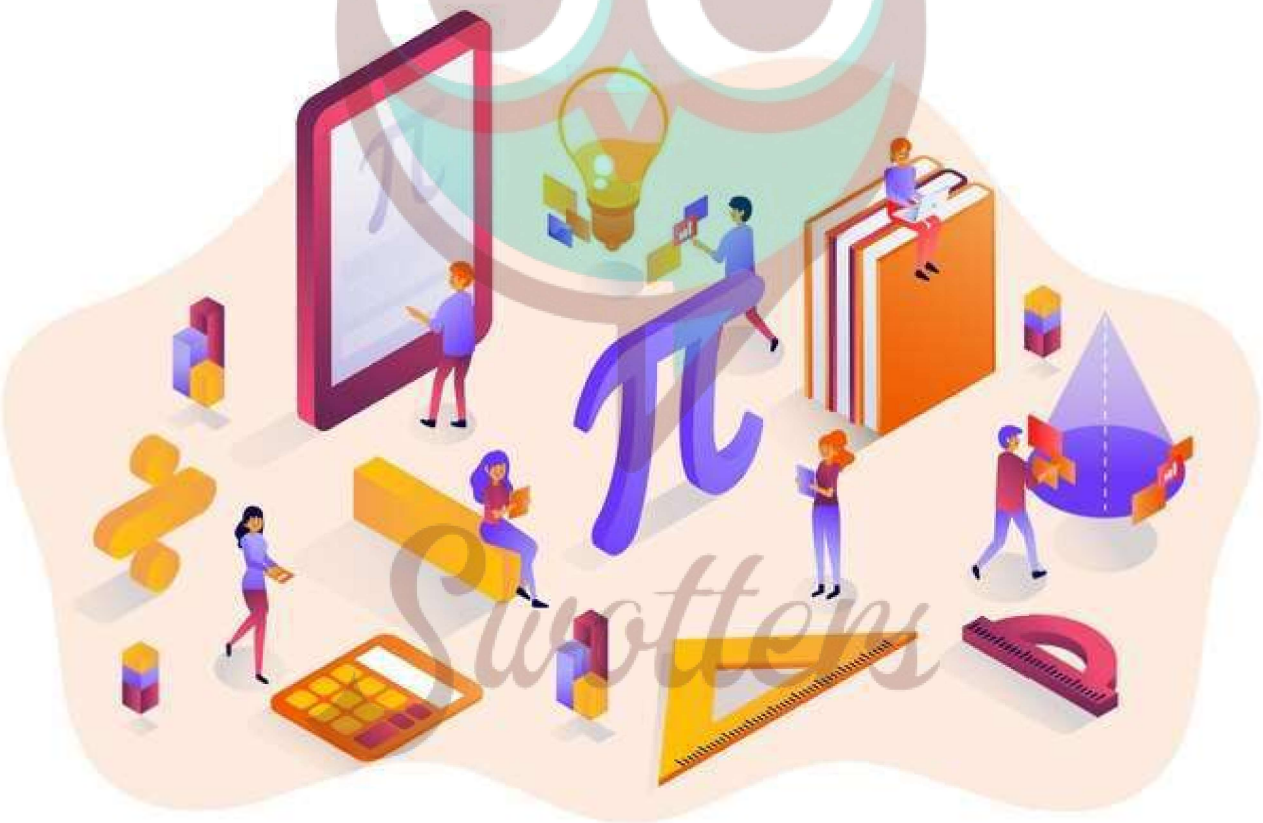
MATHEMATICS

Chapter 13: Exponents and Powers



Important Questions

Multiple Choice Questions-

Question 1. The exponential form of 10000 is

(a) 10^3

(b) 10^4

(c) 10^5

(d) none of these

Question 2. The exponential form of 100000 is

(a) 10^3

(b) 10^4

(c) 10^5

(d) none of these

Question 3. The exponential form of 81 is

(a) 3^4

(b) 3^3

(c) 3^2

(d) none of these

Question 4. The exponential form of 125 is

(a) 5^4

(b) 5^3

(c) 5^2

(d) none of these

Question 5. The exponential form of 32 is

(a) 2^3

(b) 2^4

(c) 2^5

(d) none of these

Question 6. The exponential form of 243 is

(a) 3^5

(b) 3^4

(c) 3^3

(d) 3^2

Question 7. The exponential form of 64 is

(a) 2^5

(b) 2^6

(c) 2^7

(d) 2^8

Question 8. The exponential form of 625 is

(a) 5^2

(b) 5^3

(c) 5^4

(d) 5^5

Question 9. The exponential form of 1000 is

(a) 10^1

(b) 10^2

(c) 10^3

(d) 10^4

Question 10. The value of $(-2)^3$ is

(a) 8

(b) -8

(c) 16

(d) -16

Question 11. The value of $(-2)^4$ is

(a) 8

(b) -8

(c) 16

(d) -16

Question 12. What is the base in 8^2?

(a) 8

(b) 2

(c) 6

(d) 10

Question 13. What is the exponent in 8^2?

(a) 8

(b) 2

(c) 16

(d) 6

Question 14. (-1) even number =

(a) -1

(b) 1

(c) 0

(d) none of these

Question 15. (-1)odd number =

(a) -1

(b) 1

(c) 0

(d) none of these

Very Short Questions:

1. Express 343 as a power of 7.
2. Which is greater 3^2 or 2^3?
3. Express the following number as a powers of prime factors:

   (i) 144

   (ii) 225
4. Find the value of:

   (i) $(-1)^{1000}$

   (ii) $(1)^{250}$

   (iii) $(-1)^{121}$

   (iv) $(10000)^0$
5. Express the following in exponential form:

   (i) $5 \times 5 \times 5 \times 5 \times 5$

   (ii) $4 \times 4 \times 4 \times 5 \times 5 \times 5$

   (iii) $(-1) \times (-1) \times (-1) \times (-1) \times (-1)$

   (iv) $a \times a \times a \times b \times c \times c \times c \times d \times d$

Short Questions :

1. Express each of the following as product of powers of their prime factors:

   (i) 405

(ii) 504

(iii) 500

2. Simplify the following and write in exponential form:

(i) $(5^2)^3$

(ii) $(2^3)^3$

(iii) $(a^b)^c$

(iv) $[(5)^2]^2$

3. Verify the following:

$(i)\ \left(-\frac{3}{4}\right)^3 = -\frac{27}{64}$ $(ii)\ \left(-\frac{2}{3}\right)^6 = \frac{64}{729}$

4. Simplify:

$(i)\ \frac{2^2 \times 3^4 \times 2^5}{2^4 \times 9}$ $(ii)\ 2^3 \times k^3 \times 5k^4$

5. Simplify and write in exponential form:

$(i)\ \left(\frac{3^5}{3^2}\right) \times 3^{10}$ $(ii)\ 8^2 \div 2^3$

6. Express each of the following as a product of prime factors is the exponential form:

(i) 729 × 125

(ii) 384 × 147

7. Simplify the following:

(i) $10^3 \times 9^0 + 3^3 \times 2 + 7^0$

(ii) $6^3 \times 7^0 + (-3)^4 - 9^0$

8. Write the following in expanded form:

(i) 70,824

(ii) 1,69,835

Long Questions :

1. Find the number from each of the expanded form:

(i) $7 \times 10^8 + 3 \times 10^5 + 7 \times 10^2 + 6 \times 10^1 + 9$

(ii) $4 \times 10^7 + 6 \times 10^3 + 5$

2. Find the value of k in each of the following:

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^6 = \left(\frac{4}{9}\right)^{2k-3}$

(ii) $\left(-\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1}$

3. Find the value of

(a) $3^0 \div 4^0$

(b) $(8^0 - 2^0) \div (8^0 + 2^0)$

(c) $(2^0 + 3^0 + 4^0) - (4^0 - 3^0 - 2^0)$

4. Express the following in standard form:

(i) 8,19,00,000

(ii) 5,94,00,00,00,000

(iii) 6892.25

5. Evaluate:

(i) $\frac{5^4 \times 7^5 \times 2^9}{8 \times 49 \times 5^2}$ (ii) $\frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2}$

6. Find the value of x, if

7. If $\frac{x}{y} = \left(\frac{3}{2}\right)^2 \div \left(\frac{5}{7}\right)^0$, find the value of $\left(\frac{y}{x}\right)^3$

Answer Key-

Multiple Choice questions-

1. (b) 10^4

2. (c) 10^5

3. (a) 3^4

4. (b) 5^3

5. (c) 2^5

6. (a) 3^5

7. (b) 2^6

8. (c) 5^4

9. (c) 10^3

10. (b) -8

11. (c) 16

12. (a) 8

13. (b) 2

14. (b) 1

15. (a) -1

Very Short Answer :

1. We have $343 = 7 \times 7 \times 7 = 7^3$

Thus, $343 = 7^3$

7	343
7	49
7	7
	1

2. We have $3^2 = 3 \times 3 = 9$

$2^3 = 2 \times 2 \times 2 = 8$

Since $9 > 8$

Thus, $3^2 > 2^3$

3. (i) We have

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

Thus, $144 = 2^4 \times 3^2$

2	144
2	72
2	36
2	18
3	9
3	3
	1

(ii) We have

$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$

Thus, $225 = 3^2 \times 5^2$

3	225
3	75
5	25
5	5
	1

4. (i) $(-1)^{1000} = 1$ [$\because (-1)^{\text{even number}} = 1$]

(ii) $(1)^{250} = 1$ [$\because (1)^{\text{even number}} = 1$]

(iii) $(-1)^{121} = -1$ [$\because (-1)^{\text{odd number}} = -1$]

(iv) $(10000)^0 = 1$ [$\because a^0 = 1$]

5. (i) $5 \times 5 \times 5 \times 5 \times 5 = (5)^5$

(ii) $4 \times 4 \times 4 \times 5 \times 5 \times 5 = 4^3 \times 5^3$

(iii) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) = (-1)^5$

(iv) $a \times a \times a \times b \times c \times c \times c \times d \times d = a^3b^1c^3d^2$

Short Answer :

1. (i) We have

$405 = 3 \times 3 \times 3 \times 3 \times 5 = 3^4 \times 5^1$

Thus, $405 = 3^4 \times 5^1$

3	405
3	135
3	45
3	15
5	5
	1

(ii) We have

$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7^1$

Thus, $504 = 2^3 \times 3^2 \times 7^1$

2	504
2	252
2	126
3	63
3	21
7	7
	1

(iii) We have

$500 = 2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$

Thus, $500 = 2^2 \times 5^3$

2	500
2	250
5	125
5	25
5	5
	1

2. (i) $(5^2)^3 = 5^{2\times3} = 5^6$

(ii) $(2^3)^3 = 2^{3\times3} = 2^9$

(iii) $(a^b)^c = a^{b\times c} = a^{bc}$

(iv) $[(5)^2]^2 = 5^{2\times2} = 5^4$

3.

$$(i)\ \left(-\frac{3}{4}\right)^3 = \left(-\frac{3}{4}\right)\times\left(-\frac{3}{4}\right)\times\left(-\frac{3}{4}\right)$$

$$= -\frac{3\times3\times3}{4\times4\times4} = -\frac{27}{64}$$

$$(ii)\ \left(-\frac{2}{3}\right)^6 = \left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)\times\left(-\frac{2}{3}\right)$$

$$= \frac{64}{729}$$ Hence verified.

4.

$$(i)\ \frac{2^2\times3^4\times2^5}{2^4\times9} = \frac{2^{2+5}\times3^4}{2^4\times3^2} = \frac{2^7\times3^4}{2^4\times3^2}$$

$$= 2^{7-4}\times3^{4-2} = 2^4\times3^2$$

$$= 16\times9 = 144$$

$$(ii)\ 2^3\times k^3\times5k^4 = 8\times5\times k^{3+4} = 40k^7$$

5.

$$(i)\ \left(\frac{3^5}{3^2}\right)\times3^{10} = 3^{5-2}\times3^{10} = 3^3\times3^{10} = 3^{3+10} = 3^{13}$$

$$(ii)\ 8^2\div2^3 = (2^3)^2\div2^3 = 2^{3\times2}\div2^3$$

$$= 2^6\div2^3 = 2^{6-3} = 2^3 = 8$$

6. (i) $729\times125 = 3\times3\times3\times3\times3\times3\times5\times5\times5 = 3^6\times5^3$

Thus, $729\times125 = 3^6\times5^3$

3	729
3	243
3	81
3	27
3	9
3	3
	1

5	125
5	25
5	5
	1

(ii) $384 \times 147 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 = 2^7 \times 3^2 \times 7^2$

Thus, $384 \times 147 = 2^7 \times 3^2 \times 7^2$

2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

3	147
7	49
7	7
	1

7. (i) $10^3 \times 9^0 + 3^3 \times 2 + 7^0$

$= 1000 + 54 + 1$

$= 1055$

(ii) $6^3 \times 7^0 + (-3)^4 - 9^0$

$= 216 \times 1 + 81 - 1$

$= 216 + 80$

$= 296$

8. (i) 70,824

$= 7 \times 10000 + 0 \times 1000 + 8 \times 100 + 2 \times 10 + 4 \times 10^0$

$= 7 \times 10^4 + 8 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$

(ii) 1,69,835

$= 1 \times 100000 + 6 \times 10000 + 9 \times 1000 + 8 \times 100 + 3 \times 10 + 5 \times 10^0$

$= 1 \times 10^5 + 6 \times 10^4 + 9 \times 10^3 + 8 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$

Long Answer :

1. (i) $7 \times 10^8 + 3 \times 10^5 + 7 \times 10^2 + 6 \times 10^1 + 9$

$= 7 \times 100000000 + 3 \times 100000 + 7 \times 100 + 6 \times 10 + 9$

= 700000000 + 300000 + 700 + 60 + 9

= 700300769

(ii) $4 \times 10^7 + 6 \times 10^3 + 5$

= $4 \times 10000000 + 6 \times 1000 + 5$

= 40000000 + 6000 + 5

= 40006005

2.

(i) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^6 = \left(\frac{4}{9}\right)^{2k-3}$

$\left(\frac{2}{3}\right)^{3+6} = \left[\left(\frac{2}{3}\right)^2\right]^{2k-3}$

$\Rightarrow \left(\frac{2}{3}\right)^9 = \left(\frac{2}{3}\right)^{2(2k-3)}$

$\Rightarrow \left(\frac{2}{3}\right)^9 = \left(\frac{2}{3}\right)^{4k-9}$

Comparing the powers of similar base, we have

$4k - 9 = 9$

$4k = 9 + 9$

$4k = 18$

$\Rightarrow k = \frac{\cancel{21}^{\cancel{18}^{9}}}{\cancel{4}_2} = \frac{9}{2}$

(ii) $\left(-\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1}$

$\Rightarrow \left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 = \left(\frac{4}{5}\right)^{6k+1}$ $[\because (-a)^2 = a^2]$

$\Rightarrow \left(\frac{4}{5}\right)^{2+5} = \left(\frac{4}{5}\right)^{6k+1}$

$\Rightarrow \left(\frac{4}{5}\right)^7 = \left(\frac{4}{5}\right)^{6k+1}$

Comparing the powers of similar base

$6k + 1 = 7$

$\Rightarrow 6k = 7 - 1$

$\Rightarrow 6k = 6$

$\therefore k = 1$

3. (a) We have $3^0 \div 4^0 = 1 \div 1 = 1$ $[\because a^0 = 1]$

(b) $(8^0 - 2^0) \div (8^0 + 2^0) = (1 - 1) \div (1 + 1) = 0 \div 2 = 0$

(c) $(2^0 + 3^0 + 4^0) - (4^0 - 3^0 - 2^0)$

$= (1 + 1 + 1) - (1 - 1 - 1)$ [$\because a^0 = 1$]

$= 3 - 1$

$= 2$

4. (i) $8,19,00,000 = 8.19 \times 10^7$

(ii) $5,94,00,00,00,000 = 5.94 \times 10^{11}$

(iii) $6892.25 = 6.89225 \times 10^3$

5.

$$(i)\ \frac{5^4 \times 7^5 \times 2^9}{8 \times 49 \times 5^2} = \frac{5^4 \times 7^5 \times 2^9}{2^3 \times 7^2 \times 5^2}$$

$$= 5^{4-2} \times 7^{5-2} \times 2^{9-3}$$

$$= 5^2 \times 7^3 \times 2^6$$

$$= 25 \times 323 \times 64$$

$$= 516800$$

$$(ii)\ \frac{15^4 \times 18^3}{3^3 \times 5^2 \times 12^2}$$

$$= \frac{(3 \times 5)^4 \times (2 \times 3 \times 3)^3}{3^3 \times 5^2 \times (2 \times 2 \times 3)^2} = \frac{3^4 \times 5^4 \times (2 \times 3^2)^3}{3^3 \times 5^2 \times (2^2 \times 3)^2}$$

$$= \frac{3^4 \times 5^4 \times 2^3 \times (3^2)^3}{3^3 \times 5^2 \times (2^2)^2 \times 3^2} = \frac{3^4 \times 5^4 \times 2^3 \times 3^6}{3^3 \times 5^2 \times 2^4 \times 3^2}$$

$$= \frac{3^{4+6} \times 5^4 \times 2^3}{3^{3+2} \times 5^2 \times 2^4} = \frac{3^{10} \times 5^4 \times 2^3}{3^5 \times 5^2 \times 2^4}$$

$$= \frac{3^{10-5} \times 5^{4-2}}{2^{4-3}} = \frac{3^5 \times 5^2}{2}$$

$$= \frac{243 \times 25}{2} = \frac{6075}{2} = 3037.50$$

6.

We have

$$\frac{2^{2x} \times 4 \times 2^x - 8^x}{(2^5)^3 \times 9} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{2x} \times 2^2 \times 2^x - (2^3)^x}{2^{5\times 3} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{2x+2+x} - 2^{3x}}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x+2} - 2^{3x}}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x}(2^2 - 1)}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x}(4 - 1)}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x} \times 3}{2^{15} \times 3^2} = \frac{1}{24}$$

$$\Rightarrow \frac{2^{3x-15}}{3} = \frac{1}{24}$$

$$\Rightarrow 2^{3x-15} = \frac{3}{24}$$

$$\Rightarrow 2^{3x-15} = \frac{1}{8}$$

$$\Rightarrow 2^{3x-15} = \frac{1}{2^3}$$

$$\Rightarrow 2^{3x-15} = 2^{-3}$$

Comparing the powers of the similar base, we have

$$3x - 15 = -3$$

$$\Rightarrow 3x = -3 + 15$$

$$\Rightarrow 3x = 12$$

$$\Rightarrow x = \frac{12}{3} = 4$$

Thus, $x = 4$

7.

$$\frac{x}{y} = \frac{3^2}{2^2} \div \left(\frac{5}{7}\right)^0$$

$$\frac{x}{y} = \frac{9}{4} \div 1$$

$$\Rightarrow \quad \frac{x}{y} = \frac{9}{4}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{4}{9}$$

$$\therefore \quad \left(\frac{y}{x}\right)^3 = \left(\frac{4}{9}\right)^3 = \frac{64}{729}$$

Thus, $\left(\frac{y}{x}\right)^3 = \frac{64}{729}$