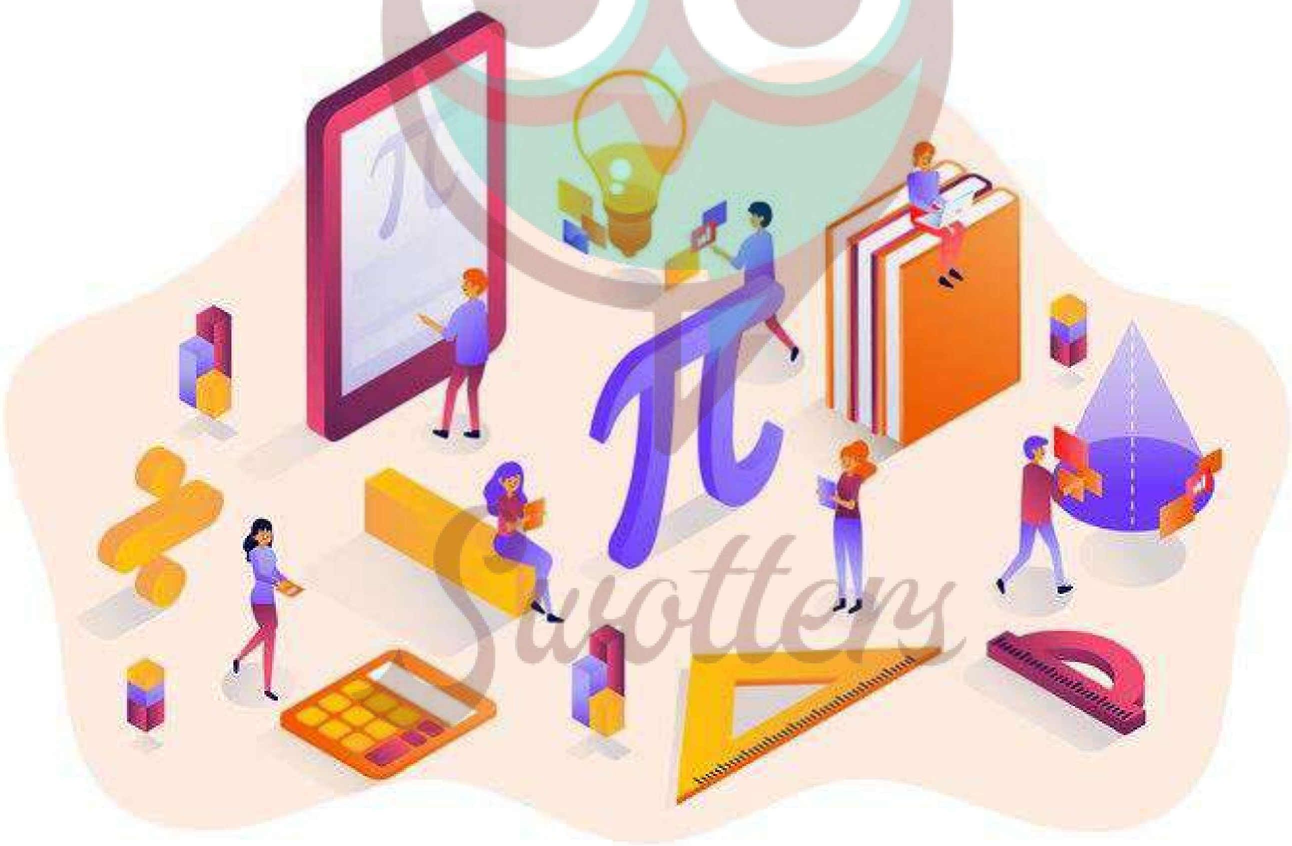
MATHEMATICS



Important Questions

Multiple Choice questions-

1. If P(A) = $\frac{1}{2}$, P(B) = 0, then P (A/B) is

(a) 0

(b) $\frac{1}{2}$

(c) not defined

(d) 1.

2. If A and B are events such that P (A/B) = P(B/A), then

(a) A ⊂ B but A ≠ B

(b) A = B

(c) A ∩ B = ø

(d) P (A) = P (B).

3. The probability of obtaining an even prime number on each die when a pair of dice is rolled is

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{1}{12}$

(d) $\frac{1}{36}$

4. Two events A and B are said to be independent if:

(a) A and B are mutually exclusive

(b) P (A'B') = [1 – P(A)] [1 – P(B)]

(c) P (A) = P (B)

(d) P (A) + P (B) = 1.

5. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is:

(a) $\frac{4}{5}$

(b) $\frac{1}{2}$

(c) $\frac{1}{5}$

(d) $\frac{2}{5}$

6. If A and B are two events such that A ⊂ B and P (B) ≠ 0, then which of the following is correct

(a) P (A / B) = $\frac{p(B)}{p(A)}$

(b) P (A/B) < P (A)

(c) P (A/B) ≥ P (A)

(d) None of these.

7. If A and B are two events such that P (A) ≠ 0 and P (B/A) = 1, then

(a) A ⊂ B

(b) B ⊂ A

(c) B = ø

(d) A = ø

8. If P (A/B) > P (A), then which of the following is correct?

(a) P (B/A) < P (B)

(b) P (A ∩ B) < P (A).P(B)

(c) P (B/A) > P (B)

(d) P (B/A) = P (B).

9. If A and B are any two events such that

P (A) + P (B) – P (A and B) = P (A), then:

(a) P (B/A) = 1

(b) P (A/B) = 1

(c) P (B/A) = 0

(d) P (A/B) = 0

10. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. What is the value of E (X)?

(a) $\frac{37}{221}$

(b) $\frac{5}{13}$

(c) $\frac{1}{13}$

(d) $\frac{2}{13}$

Very Short Questions:

1. If A and B are two independent event, prove that A’ and B are also independent. (C.B.S.E. Sample Paper 2018-19)
2. One card is drawn from a pack of 52 cards so that each card is equally likely to be se-lected. Prove that the following cases are in-dependent:
   - A: “The card drawn is a spade”

     B: “The card drawn is an ace.” (N.C.E.R.T.)
   - A: “The card drawn is black”

     B: “The card drawn is a king.” (.N.C.E.R.T.)
3. A pair of coins is tossed once. Find the probability of showing at least one head.
4. P(A) = 0.6, P(B) = 0. 5 and P(A/B) = 0.3, then find P(A∪ B) (C.B.S.E. Sample Paper 2018-19)
5. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red. (C.B.S.E. Sample Paper 2018-19)
6. Evaluate P(A ∪ B), if 2P(A) = P(B) = $\frac{5}{13}$ and P(A/B) = $\frac{2}{5}$ (C.B.S.E. 2018 C)

Short Questions:

1. Given that A and B are two independent events such that P(A) = 0.3 and P(B) = 0.5. Find P(A/B). (C.B.S.E. 2019 C)

2. A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag.

   Find the probability that the balls drawn are one white and one red. (C.B.S.E. 2019 C)

3. The probabilities of A, B and C solving a problem independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. If all the three try to solve the problem independently, find the probability that the problem is solved. (C.B.S.E. 2019 C)

4. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not. {Delhi 2019)

5. A die is thrown 6 times. If "getting an odd number" is a success, what is the probability of (i) 5 successes (ii) at most 5 successes? (Delhi 2019)

6. The random variable 'X' has a probability distribution P(X) of the following form, where 'k' is some number:

$$\mathbf{P}(\mathbf{X}=\boldsymbol{x})=\begin{cases}\boldsymbol{k}, \text{ if } \boldsymbol{x}=\mathbf{0}\\ 2k, \text{ if } x=1\\ 3k, \text{ if } x=2\\ 0, \text{ otherwise.}\end{cases}$$

   Determine the value of 'P. (Outside Delhi 2019)

7. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boy and 2 girls are selected. (Outside Delhi 2019)

8. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number. {Outside Delhi 2019)

Long Questions:

1. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. (C.B.S.E. 2018)

2. Two numbers are selected at random (with-out replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

3. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. (A.I.C.B.S.E. 2013)

4. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact? (C.B.S.E. Sample Paper 2019-20)

Case Study Questions:

1. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms, Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information, answer the following questions.

(i) The conditional probability that an error is committed in processing given that Sonia processed the form is:

a. 0.0210
b. 0.04
c. 0.47

d. 0.06

(ii) The probability that Sonia processed the form and committed an error is:

a. 0.005
b. 0.006
c. 0.008
d. 0.68

(iii) The total probability of committing an error in processing the form is:

a. 0
b. 0.047
c. 0.234
d. 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is:

a. 1

b. $\frac{30}{47}$

c. $\frac{20}{47}$

d. $\frac{17}{47}$

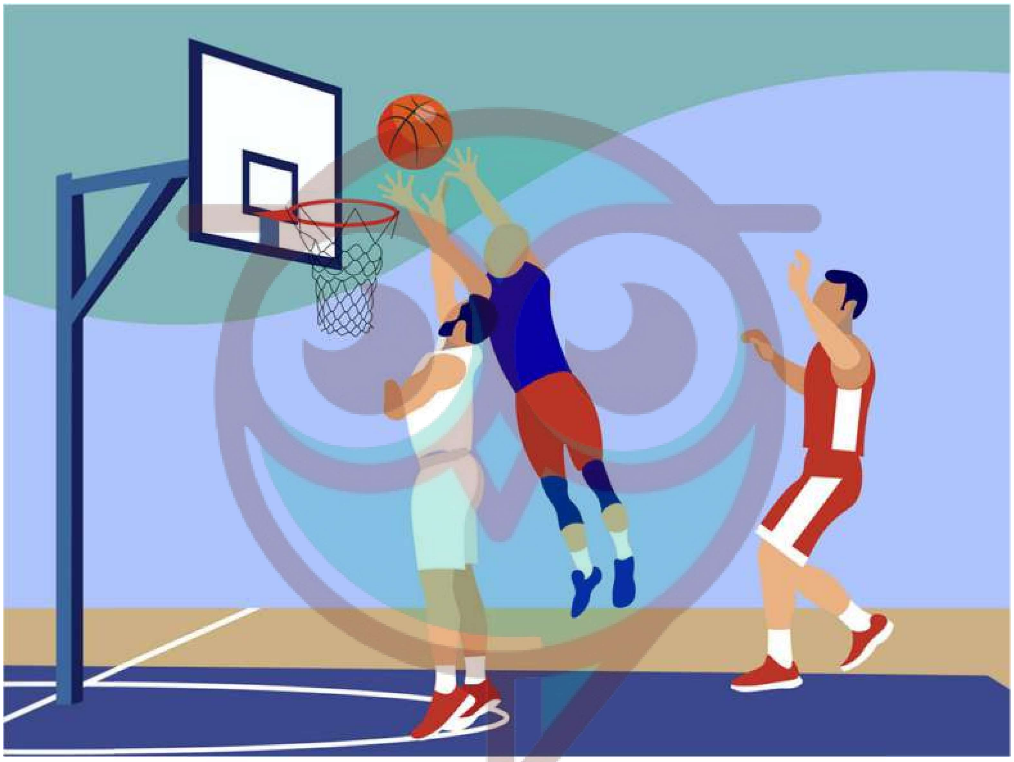
(v) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$ is:

a. 0
b. 0.03
c. 0.06
d. 1

2. Between students of class XII of two schools A and B basketball match is organised. For which, a team from each school is chosen, say T_1 be the team of school A and T_2 be the team of school B. These teams have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probability of T_1 winning, rawmg an osrng a game against T_2 are $\frac{1}{2}$, $\frac{3}{10}$ and $\frac{1}{5}$ respectively.

Each team gets 2 points for a win, 1 point for a draw and 0 point for a loss in a game.

Let X and Y denote the total points scored by team A and B respectively, after two games.



Based on the above information, answer the following questions.

i. P(T_2 winning a match against T_1) is equal to:

a. $\frac{1}{5}$

b. $\frac{1}{6}$

c. $\frac{1}{3}$

d. None of these

ii. P(T_2 drawing a match against T_1) is equal to:

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{6}$

d. $\frac{3}{10}$

iii. P(X > Y) is equal to:

a. $\frac{1}{4}$

b. $\frac{5}{12}$

c. $\frac{1}{20}$

d. $\frac{11}{20}$

iv. P(X = Y) is equal to:

a. $\frac{11}{100}$

b. $\frac{1}{3}$

c. $\frac{29}{100}$

d. $\frac{1}{2}$

v. P(X + Y = 8) is equal to:

a. 0

b. $\frac{5}{12}$

c. $\frac{13}{36}$

d. $\frac{7}{12}$

Answer Key-

Multiple Choice questions-

1. Answer: (c) not defined
2. Answer: (d) P (A) = P (B).
3. Answer: (d) $\frac{1}{36}$
4. Answer: (b) P (A'B') = [1 – P(A)] [1 – P(B)]
5. Answer: (a) $\frac{4}{5}$
6. Answer: (c) P (A/B) ≥ P (A)
7. Answer: (a) A ⊂ B
8. Answer: (c) P (B/A) > P (B)
9. Answer: (b) P (A/B) = 1
10. Answer: (d) $\frac{2}{13}$

Very Short Answer:

1. Solution:

   Since A and B are independent events, [Given]

   .-. P (A ∩ B) = P (A) . P(B) ...(1)

   Now P(A'∩B) = P (B) – P (A ∩ B)

   = P(B) – P (A) P(∩ B) [Using (1)]

   = (1 – P(A)) P(B) = P(A') P(B).

   Hence, A' and B are independent events.

2. Solution:

   (a) $P(A) = \frac{13}{52} = \frac{1}{4}$, $P(B) = \frac{4}{52} = \frac{1}{13}$

   $P(A \cap B) = \frac{1}{52} = \frac{1}{4} \cdot \frac{1}{13} = p(A).p(B)$

   Hence, the events A and B are independent

(b) $P(A) = \frac{26}{52} = \frac{1}{2}$, $P(B) = \frac{4}{52} = \frac{1}{13}$

$P(A \cap B) = \frac{2}{52} = \frac{1}{26} = \frac{1}{2} \cdot \frac{1}{13} = P(A) . P(B)$

Hence, the events A and B are independent

3. Solution:

S, Sample space = {HH, HT, TH, TT}

where H ≡ Head and T ≡ Tail.

∴ P (at least one head) = $\frac{3}{4}$.

4. Solution:

We have: P(A/B) = 0.3

$\frac{P(A \cap B)}{P(B)} = 0.3$

$\frac{P(A \cap B)}{0.5} = 0.3$

P (A ∩ B) = 0.5 x 0.3 = 0.15.

Now, P(A∪B) = P(A) + P(B) – P(A ∩ B)

= 0.6 + 0.5 – 0. 15

Hence, P (A ∪ B) = 1.1 – 0.15 = 0.95.

5. Solution:

P (Red transferred and red drawn or black transferred red drawn)

$= \frac{3}{8} \times \frac{7}{11} + \frac{5}{8} \times \frac{6}{11}$

$= \frac{21}{88} + \frac{30}{88} = \frac{51}{88}$

Solution:

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = \frac{2}{11}$

P(A∪B) = P(A) + P(B) – (A ∩ B)

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$

Short Answer:

1. Solution:

   We have:

   P(A)= 0.3 and P(B) = 0.5.

   Now P (A ∩ B) = P(A). P(B)

   [∵A and B are independent events]

   = (0.3) (0.5) = 0.15.

   Hence, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.5} = 0.3.$

2. Solution:

   Reqd. probability

   = P (White, Red) + P (Red, White)

   $\frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{4}{7} = \frac{9}{35} + \frac{8}{35} = \frac{17}{35}$

3. Solution:

   Given: $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

   $\therefore P(\overline{A}) = 1 - \frac{1}{2} = \frac{1}{2}, P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

   and $P(\overline{C}) = 1 - \frac{1}{4} = \frac{3}{4}.$

   Probability that the problem is solved

   = Probability that the problem is solved by at least one person

   $= 1 - P(\overline{A})P(\overline{B})P(\overline{C})$

   $= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

4. Solution:

Here, A: number is even i.e.,

A = {2,4,6}

and B: number is red i.e.,

B = {1,2,3}

$\therefore$ P(A) = $\frac{3}{6} = \frac{1}{2}$ and P(B) = $\frac{3}{6} = \frac{1}{2}$

And,

P (A ∩ B) = P(Number is even and red) = $\frac{1}{6}$.

Thus, P (A ∩ B) ≠ P(A). P(B)

[$\because \frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}$]

Hence, the events A and B are not independent.

5. Solution:

Probability of getting an odd number is one 3 1

trial = $\frac{3}{6} = \frac{1}{2}$ = p (say)

Probability of getting an even number in one 3

trial = $\frac{3}{6} = \frac{1}{2}$ = g(say) o l

Also, n = 6.

(i) P (5 successes) = P(5) = $^6C_5\, q^1\, p^5$

(ii) P (at most 5 successes)

= P (0) + P(1) + … + P(5) = 1 – P(6)

= 1 - $^6C_6\, q^0\, p^6$

= $1 - \frac{1}{64} = \frac{63}{64}$

6. Solution:

   We have: P (X = 0) + P (X = 1) + P (X = 2) = 1

   ⇒ k + 2k + 3k = 1

   ⇒ 6k = 1.

   Hence, k = $\frac{1}{6}$.

7. Solution:

   Read, probability = $\frac{^3C_2 \times {^5C_2}}{^8C_4}$

   = $\frac{3\times10}{70} = \frac{3}{7}$

8. Solution:

   Let the events be as :

   A: Card bears an odd number.

   B: Number on the card is greater than 5.

   A∩B = {7, 9, 11}.

   Hence, P(A/B) = $\frac{P(A\cap B)}{P(B)}$

   = $\frac{3/12}{7/12} = \frac{3}{7}$

Long Answer:

1. Solution:

   Let the events be as:

   E: Sum of numbers is 8

   F: Number of red dice less than 4.

   E: {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

   F = {(1, 1), (2, 1), … (6, 1), (1, 2), (2, 2), … (6, 2), (1, 3), (2, 3), … (6, 2) (6, 3)} and E ∩ F = {(5, 3), (6, 2)}

$P(E) = \frac{5}{36}$, $P(F) = \frac{18}{36}$

and $P(E \cap F) = \frac{2}{36}$.

Hence, $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{18/36}$

$= \frac{2}{18} = \frac{1}{9}$

2. Solution:

The first five positive integers are 1, 2, 3, 4 and 5.

We select two positive numbers in 5 x 4 = 20 way.

Out of three, two numbers are selected at ran-dom.

Let 'X' denote the larger of the two numbers.

X can be 2, 3, 4 or 5.

∴ P (X = 2) = P (Larger number is 2)

{(1, 2), (2,1)} = $\frac{2}{20}$

Similarly, P (X = 3) = $\frac{4}{20}$,

P (X = 4) = $\frac{6}{20}$

and P (X = 5) = $\frac{8}{20}$

Hence, the probability distribution is:

X	2	3	4	5
P (X)	1/10	2/10	3/10	4/10
X. P(X)	2/10	6/10	12/10	20/10
X^2 P(X)	4/10	18/10	48/10	100/10

∴ Mean = Σ X P(X)

$$= 2 \times \frac{2}{20} + 3 \times \frac{4}{20} + 4 \times \frac{6}{20} + 5 \times \frac{8}{20}$$

$$= \frac{4+12+24+40}{20} = \frac{80}{20} = 4$$

and variance = $\Sigma X^2P(x) - [\Sigma P(x)]^2$

$$= \frac{170}{10} - (1)^2 = 17 - 1 = 16.$$

3. Solution:

We have: P(A) = Probability of student A coming to school in time = $\frac{3}{7}$

P(B) = Probability of student B coming to school in time = $\frac{5}{7}$

$\therefore P(\overline{A}) = 1 - \frac{3}{7} = \frac{4}{7}$

and $P(\overline{B}) = 1 - \frac{5}{7} = \frac{2}{7}$

∴ Probability that only one of the students coming to school in time

$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$
$= P(A)P(\overline{B}) + P(\overline{B})PB)$

[∵ A and B are independent => A and $\overline{B}$ and $\overline{A}$ and B are also independent]

$= \left(\frac{3}{7}\right)\left(\frac{2}{7}\right) + \left(\frac{4}{7}\right)\left(\frac{5}{7}\right) = \frac{26}{49}$

4. Solution:

$P(A) = \frac{80}{100} = \frac{4}{5}$

and $P(B) = \frac{90}{100} = \frac{9}{10}$

$P(\overline{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$

$P(\overline{B}) = 1 - P(B) = 1 - \frac{9}{10} = \frac{1}{10}$

∴ P(Agree) = P(Both speak the truth or both tell a lie)

$= P(AB \text{ or } \bar{A}\bar{B})$

$= P(A)\,P(B) \text{ or } P(\bar{A})\,P(\bar{B})$

$= \left(\frac{4}{5}\right)\left(\frac{9}{10}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{10}\right)$

$= \frac{36}{50}+\frac{1}{50}=\frac{37}{50}=\frac{74}{100}.$

Hence, the reqd. percentage = 74%.

Case Study Answers:

1. Answer :

Let A be the event of commiting an error and E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form.

(i) (b) 0.04

Solution:

Required probability = $P(A|E_2)$\

$$= \frac{P(A \cap E_2)}{P(E_2)}$$

$$= \frac{\left(0.04 \times \frac{20}{100}\right)}{\left(\frac{20}{100}\right)} = 0.04$$

(ii) (c) 0.008

Solution:

Required probability = $P(A \cap E2)$

$$= 0.04 \times \frac{20}{100} = 0.008$$

(iii) (b) 0.047

Solution:

Total probability is given by

$P(A) = P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2) + P(E_3) \times P(A|E_3)$.

$$= \frac{50}{100} \times 0.06 + \frac{20}{100} \times 0.04 + \frac{30}{100} \times 0.03$$

$$= 0.047$$

(iv) (d) 17471747

Solution:

Using Bayes' theorem, we have

$$P(E_1 \mid A) = \frac{P(E_1) \times P(A \mid E_1)}{P(E_1) \times P(A \mid E_1) + P(E_2) \times P(A \mid E_2) + P(E_3) \times P(A \mid E_3)}$$

$$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$$

$\therefore$ Required probability $= P(\bar{E} \mid A)$

$$= 1 - P(E_1 \mid A) = 1 - \frac{30}{47} = \frac{17}{47}$$

(v) (d) 1

Solution:

$$\sum_{i=1}^{3} P(E_i \mid A) = P(E_1 \mid A) + P(E_2 \mid A) + P(E_3 \mid A) = 1$$

[$\therefore$ Sum of posterior probabilities is 1)

2. Answer :

(i) (a) $\frac{1}{2}$

Solution:

Clearly, P(T_2 winning a match against T_1)

= P(T_1 losing) = $\frac{1}{5}$

(ii) (d) $\frac{3}{10}$

Solution:

Clearly, P(T_2 drawing a match against T_1)

= P(T_1 drawing) = $\frac{3}{10}$

(iii) (d) $\frac{11}{20}$

Solution:

According to given information, we have the following possibilities for the values of X and Y.

X	4	3	2	1	0
Y	0	1	2	3	4

Now, P(X > Y) = P(X = 4, Y = 0) + P(X = 3, Y = 1)

= P(T_1 win) P(T_1 win) + P(T_1 win) P(match draw) + P(match draw) P(T_1 win)

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{3}{10} + \frac{3}{10} \times \frac{1}{2}$$

$$= \frac{5+3+3}{20} = \frac{11}{20}$$

(iv) (c) $\frac{29}{100}$

Solution:

P(X = Y) = P(X = 2, Y = 2)

= P(T_1 win) P(T_2 win) + P(T_2 win) P(T_1 win) + P(match draw) P(match draw)

$$= \frac{1}{2} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{5} + \frac{3}{10} \times \frac{3}{10}$$

$$= \frac{1}{10} + \frac{1}{10} + \frac{9}{100} = \frac{29}{100}$$

(v) (a) 0

Solution:

From the given information, it is clear that maximum sum of X and Y can be 4, therefore P(X + Y = 8) = 0.