MATHEMATICS

Important Questions

Multiple Choice questions-

1. If the surface areas of two spheres are in ratio 16 : 9, then their volumes will be in the ratio:

(a) 27 : 64

(b) 64 : 27

(c) 4 : 3

(d) 3 : 4

2. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio of their volumes?

(a) 3 : 1 : 2

(b) 3 : 2 : 1

(c) 1 : 2 : 3

(d) 1 : 3 : 2

3. If the area of three adjacent faces of cuboid are X, Y and Z respectively, then the volume of cuboid is:

(a) XYZ

(b) 3XYZ

(c) $\sqrt{xyz}$

(d) $\sqrt{3xyz}$

4. The volumes of two spheres are in the ratio 27 : 8. The ratio of their curved surface is:

(a) 9 : 4

(b) 4 : 9

(c) 3 : 2

(d) 2 : 3

5. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of spheres respectively, then (R – r): r is:

(a) 1 : 2

(b) 1 : 3

(c) 2 : 3

(d) 4 : 9

6. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is:

(a) 27 : 20

(b) 20 : 27

(c) 9 : 4

(d) 4 : 9

7. If the radius of base of a right circular cylinder is halved, keeping the height same, the ratio of the volume of the reduced cylinder to that of the original cylinder is:

(a) 2 : 3

(b) 3 : 4

(c) 1 : 4

(d) 4 : 1

8. If the volumes of a cube is 1728 cm^3, the length of its edge is equal to:

(a) 7 cm

(b) 12 cm

(c) 18 cm

(d) 19 cm

9. The volume (in cm^3) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm is: .

(a) 9.7

(b) 72.6

(c) 58.2

(d) 19.4

10. The circumference of the edge of hemispherical bowl is 132 cm. When π is taken as $\frac{22}{7}$, the capacity of bowl in cm³ is:

(a) 2772

(b) 924

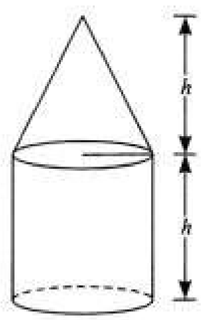
(c) 19404

(d) 9702

Very Short Questions:

1. What is the capacity of a cylindrical vessel with a hemispherical portion raised upward at the bottom?

2. A solid cone of radius r and height h is placed over a solid cylinder having same base radius and height as that of a cone. What is the total surface area of the combined solid?



3. Two identical solid hemispheres of equal base radius r cm are struck together along their bases. What will be the total surface area of the combination?

4. A solid ball is exactly fitted inside the cubical box of side a. What is the volume of the ball?

5. If two cubes of edge 5 cm each are joined end to end, find the surface area of the resulting cuboid.

6. A solid piece of iron in the form of a cuboid of dimension 49 cm × 33 cm × 24 cm is melted to form a solid sphere. Find the radius of sphere.

7. A mason constructs a wall of dimensions 270 cm × 300 cm × 350 cm with the bricks each of size 22.5 cm × 11.25 cm × 8.75 cm and it is assumed that space is covered by the mortar. Find the number of bricks used to construct the wall.

8. The radii of the ends of a frustum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.

9. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

10. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

Short Questions :

1. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?

2. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, find the height of the frustum.

3. If the slant height of the frustum of a cone is 10 cm and the perimeters of its circular base are 18 cm and 28 cm respectively. What is the curved surface area of the frustum?

4. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

5. A vessel is in the form of a hollow hemisphere mounted by a hollow 7 cm cylinder. The diameter of the hemisphere is 14 cm and the total height T of the vessel is 13 cm. Find the inner surface area of the vessel.

6. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

7. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

8. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

9. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

OR

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

10. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Long Questions :

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. (Use $\pi = \frac{22}{7}$)

2. A hemispherical tank, full of water, is emptied by a pipe at the rate of $\frac{25}{7}$ litres per sec. How much time will it take to empty half the tank if the diameter of the base of the tank is 3 m?

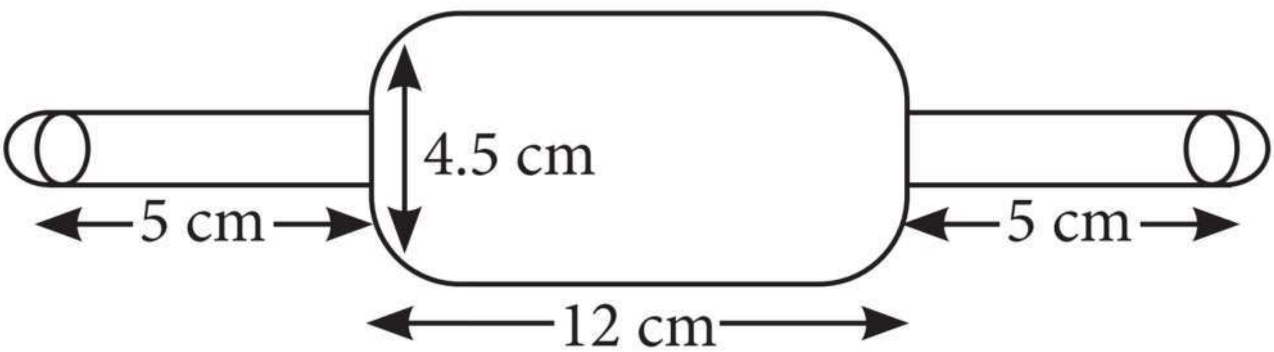
3. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

4. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

5. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = \frac{22}{7}$)

Case Study Questions:

1. Arp an a is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.



i. Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5cm and length of each part is 5cm.

a. 475cm^2

b. 78.57cm^2

c. 877cm^2

d. 259.19cm^2

ii. Find the volume of big cylindrical part.

190.93cm^3

75cm^3

77cm^3

83.5cm^3

iii. Volume of two hemispherical ends having diameter 2.5cm, is:

4.75cm^3

8.18cm^3

2.76cm^3

75cm^3

iv. Curved surface area of two hemispherical ends, is:

17.5cm^2

7.9cm^2

19.64cm^2

15.5cm^2

v. Find the difference of volumes of bigger cylindrical part and total volume of the two small hemispherical ends.

175.50cm^3

182.75cm^3

76.85cm^3

96cm^3

2. Isha's father brought an ice-cream brick, empty cones and scoop to pour the ice-cream into cone for all the family members. Dimensions of the ice-cream brick are (30 × 25 × 10)cm? and radius of hemi-spherical scoop is 3.5cm. Also, the radius and height of cone are 3.5cm and 15cm respectively.

i. The quantity of ice-cream in the brick (in litres) is:

a. 3

b. 7.5

c. 2.5

d. 4.5

ii. Volume of hemispherical scoop is:

a. 40.6cm^3

b. 2509cm^3

c. 89.83cm^3

d. 20cm^3

iii. Volume of a cone is:

a. 148cm^3

b. 250.05cm^3

c. $145.83cm^3$

d. $192.5cm^3$

iv. The minimum number of scoops required to fill one cone up to brim is:

a. 2

b. 3

c. 4

d. 5

v. The number of cones that can be filled up to brim using the whole brick is:

a. 15

b. 39

c. 40

d. 42

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) Both A and R is false.

Assertion: If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75%.

Reason: Curved surface area is increased when diameter decreases

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) Both A and R is false.

Assertion: The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 15 mm, then the internal volume is 765 cm^3.

Reason: If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x , then its internal volume is (l – 2x) (b – 2x) (h – 2x).

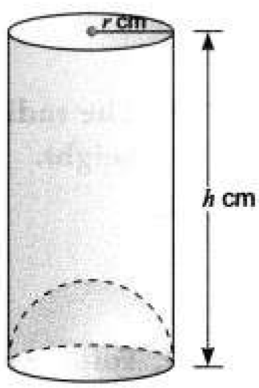
Answer Key-

Multiple Choice questions-

1. (b) 64 : 27
2. (a) 3 : 1 : 2
3. (c) $\sqrt{xyz}$
4. (a) 9 : 4
5. (a) 1 : 2
6. (b) 20 : 27
7. (c) 1 : 4
8. (b) 12 cm
9. (d) 19.
10. (a) 2772

Very Short Answer :

1.



Capacity of the given vessel

= capacity of cylinder – capacity of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}(3h - 2r)$$

2. The total surface area of the combined solid in Fig.

= curved surface area of cone + curved surface area of cylinder + area of the base.

$$= \pi rl + 2\pi rh + \pi r^2 = \pi r(l + 2h + r) = \pi r\left(\sqrt{r^2 + h^2} + 2h + r\right)$$

3. The resultant solid will be a sphere of radius r whose total surface area is $4\pi r^2$.

4. Diameter of the solid ball = edge of the cube = a

$$\therefore \quad \text{Volume of the ball} = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \frac{4}{3} \times \frac{1}{8}\pi a^3 = \frac{1}{6}\pi a^3$$

5. Total length (l) = 5 + 5 = 10 cm

Breadth (b) = 5 cm, Height (h) = 5 cm

Surface Area = 2 (lb + bh + lh)

= 2(10 × 5 + 5 × 5 + 5 × 10) = 2 × 125 = 250 cm^2

6. Volume of iron piece = Volume of the sphere formed

$= 49 \times 33 \times 24 = \frac{4}{3}\pi r^2$

$$r^3 = \frac{49 \times 33 \times 24 \times 3 \times 7}{4 \times 22}$$

r = 21 cm

7. Space occupied with bricks = $\frac{7}{8}$ × volume of the wall

$= \frac{7}{8} \times 270 \times 300 \times 350$

$$\therefore \text{ Number of bricks} = \frac{\text{Space occupied with bricks}}{\text{Volume of one brick}} = \frac{\frac{7}{8} \times 270 \times 300 \times 350}{22.5 \times 11.25 \times 8.75} = 11{,}200$$

8.

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{40^2 + (20-11)^2} = \sqrt{1600+81} = \sqrt{1681} = 41 \text{ cm}$$

9. As per question

Volume of hemisphere = Surface area of hemisphere

$= \frac{2}{3}\pi r^2 = 3\pi r^2 =$, units $r = \frac{9}{2}$ units.

10. Volume of a cone: Volume of a hemisphere: Volume of a cylinder

$$= \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$$

$$= \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \qquad (\because r = h)$$

$$= 1:2:3$$

Short Answer :

1. Let edge of the cube be ‘a’.

Then, diameter of the sphere that will fit inside the given cube = a

∴ Volume of the cube : Volume of the sphere.

$$= a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = a^3 : \frac{4}{3} \times \frac{1}{8}\pi a^3 = a^3 : \frac{1}{6}\pi a^3 = 6 : \pi$$

2. Let r and R be radii of the circular ends of the frustum of the cone.

Then, R – r = 4, l = 5

We know, $l^2 = (R-r)^2 + h^2$

$\Rightarrow 5^2 = 4^2 + h^2$ or $h^2 = 25 - 16 = 9$

$\Rightarrow h = 3$ cm

3. Let r and R be the radii of the two circular ends of the frustum of the cone.

Then, $2\pi r = 18$ and $2\pi R = 28$

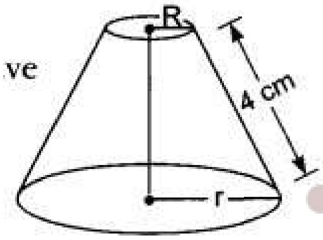
$\Rightarrow \quad r = \frac{18}{2\pi} \quad$ and $\quad R = \frac{28}{2\pi} \quad \Rightarrow \quad r = \frac{9}{\pi} \quad$ and $\quad R = \frac{14}{\pi}$

Now, curved surface area of the frustum $= \pi(r + R)l$

$= \pi\left(\frac{9}{\pi} + \frac{14}{\pi}\right) \times 10 = 23 \times 10 = 230 \text{ cm}^2$

4.

ve



We have, slant height, l = 4 cm

Let R and r be the radii of two circular ends respectively. Therefore, we have

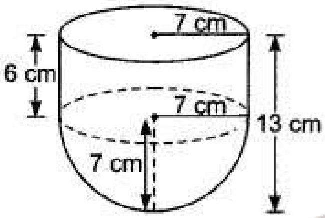
$\Rightarrow 2\pi R = 18 = \pi R = 9$

$\Rightarrow 2\pi r = 6 = \pi r = 3$

$\therefore$ Curved surface area of the frustum = $(\pi R + \pi r)l$

$= (9 + 3) \times 4 = 12 \times 4 = 48 \text{ cm}^2$

5.



Here, radius of hemisphere = radius of cylinder = r cm = 7 cm

and height of cylinder, h = (13 – 7) cm = 6 cm

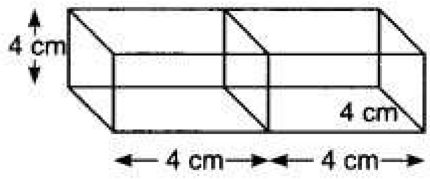
Now, inner surface area of the vessel

= Curved surface area of the cylindrical part + Curved surface area of hemispherical part = $(2\pi rh + 2\pi r2) = 2\pi r\,(h + r)$

$= 2 \times \frac{22}{7} \times 7\,(6 + 7)$

$= 2 \times 22 \times 13 = 572\ cm^2$

6.



Let the length of each edge of the cube of volume 64 cm3 be x cm.

Then, Volume = $64\ cm^3$

$\Rightarrow x^2 = 64$

$\Rightarrow x^2 = 43$

$\Rightarrow x = 4$ cm

4 cm The dimensions of cuboid so formed are

l = Length = (4 + 4) cm = 8 cm

b = Breadth = 4 cm and h = Height = 4 cm

∴ Surface area of the cuboid = 2 (lb + bh + lh)

$= 2 (8 \times 4 + 4 \times 4 + 8 \times 4)$

$= 2 (32 + 16 + 32)$

$= 160\ cm^2$

7. The greatest diameter that a hemisphere can have = 7 cm = l

Radius of the hemisphere (R) = $\frac{7}{2}$ cm

∴ Surface area of the solid after surmounting hemisphere

$= 6l^2 - \pi R^2 + 2\pi R^2 = 6l^2 + \pi R^2$

$$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 294 + 38.5 = 332.5 \text{ cm}^2$$

8. Let the length of pipe by h m.

Volume of cuboid = 4.4 × 2.6 × 1 m^2

Inner and outer radii of cylindrical pipe are 30 cm, (30 + 5) cm = 35 cm

$\therefore$ Volume of material used $= \frac{\pi}{100^2}(35^2 - 30^2) \times h \text{ m}^3$

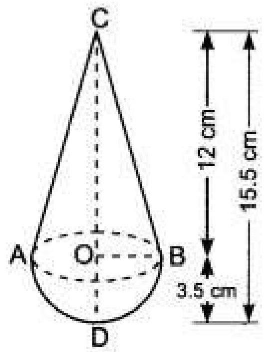
$= \frac{\pi}{100^2} \times 65 \times 5h$ [using $a^2 - b^2 = (a + b)(a - b)$]

Now $\frac{\pi}{100^2} \times 65 \times 5h = 4.4 \times 2.6$

$\Rightarrow h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$

$\Rightarrow h = 112$ m

9.



We have,

CD = 15.5 cm and OB = OD = 3.5 cm

Let r be the radius of the base of cone and h be the height of conical part of the toy.

Then, r = OB = 3.5 cm

h = OC = CD – OD = (15.5 – 3.5) cm = 12 cm

$$l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2}$$

$$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ cm}$$

Also, radius of the hemisphere, $r = 3.5$ cm

$\therefore$ Total surface area of the toy

= Surface area of cone + Surface area of hemisphere

$$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times 3.5\,(12.5 + 2 \times 3.5)$$

$$= \frac{22}{7} \times 3.5 \times 19.5 = 214.5 \text{ cm}^2$$

10. Here, we have

Edge of the cube = l = Diameter of the hemisphere

Therefore, radius of the hemisphere = $\frac{l}{2}$

$\therefore$ Surface area of the remaining solid after cutting out the hemispherical

$$\text{depression} = 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \pi \times \frac{l^2}{4} = \frac{l^2}{4}(24 + \pi)$$

Long Answer :

1. Radius, r = 7 cm

Height of cone, h = 2(7) = 14 cm

Volume of solid = Vol. of hemisphere + Volume of cone

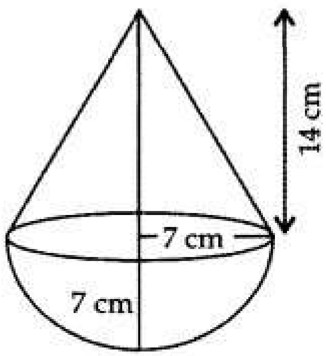
$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (2r + h)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7(2(7) + 14)$$

$$= \frac{22 \times 7}{3} \times 28 = \frac{4312}{3}$$

$$= \mathbf{1437.\bar{3}\ cm^3}$$



2.

Here, $r = \frac{3}{2}$ m

$\frac{25}{7}$ lt $= \frac{1}{1000} \times \frac{25}{7} m^3 = \frac{1}{280} m^3$ ...[$\because$ 1 lt. $= \frac{1}{1000} m^3$

Required time $= \dfrac{\frac{1}{2}\text{ Vol. of hemispherical tank}}{\text{Vol. of pipe}}$

$$= \frac{\frac{1}{2} \times \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}{\frac{1}{280}}$$

$$= \frac{22 \times 9}{7 \times 8} \times \frac{280}{1} = 990 \text{ secs.}$$

= 16 mins. 30 secs.

$\therefore$ **Required time is $16\frac{1}{2}$ mins.**

3. Radius of tank, r1 = 40 cm

Internal radius of cylindrical pipe, $r_2 = \frac{2}{2} = 1$ cm

Let the height of rises water, h_1 = ?

Length of water flow in 1 second = 0.4 m

$= \frac{4}{10} \times 100 = 40$ cm

$\therefore$ Length of water flow in 30 minutes, h2

= 40 × 60 × 30 = 72,000 cm

Volume of water in cylinder tank

= Volume of water flow from cylindrical pipe in half an hour

As $\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\therefore$ $40 \times 40 \times h_1 = 1 \times 1 \times 72{,}000$

$h_1 = \frac{72{,}000}{40 \times 40} = 45$ cm

$\therefore$ Level of water in cylinder tank rises in half an hour, h_1 = 45 cm

4.

Radius of a marble, $r = \frac{1.4}{2} = \frac{7}{10}$ cm,

Radius of cylinder, $R = \frac{7}{2} = 3.5$ cm

No. of spherical marbles

$$= \frac{\text{Vol. of water rise in cylinder}}{\text{Vol. of one marble (sphere)}}$$

$$150 = \frac{\pi\left(\frac{7}{2}\times\frac{7}{2}\times h\right)}{\frac{4}{3}\times\pi\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10}}$$

$$\ldots\left[\begin{array}{l}\text{Vol. of cylinder} = \pi r^2 h \\ \text{Vol. of sphere} = \frac{4}{3}\pi r^3\end{array}\right.$$

$\Rightarrow \quad \frac{7}{2}\times\frac{7}{2}\times h = 150 \times \frac{4}{3}\times\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10}$

$h = \frac{50\times 2\times 2\times 4\times 7}{10\times 10\times 10} = \frac{56}{10}$ cm

$\therefore$ **Rise in water level, $h = \frac{56}{10} = 5.6$ cm**

5.

Given: $r = \frac{4.2}{2} = 2.1$ cm, $\quad h = 2.8$ cm

$l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (2.8)^2}$

$= \sqrt{4.41 + 7.84} = \sqrt{12.25}$

$= 3.5$ cm

T.S. area of the remaining solid

= C.S. ar. of cyl. + area of base + C.S. ar. of cone

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r(2h + r + l)$

$= \frac{22}{7} \times 2.1\ (5.6 + 2.1 + 3.5)$

$= 22 \times 0.3(11.2)$

$= 73.92\ \text{cm}^2$

Case Study Answer:

1. **Answer:**

i. (b) $78.57 cm^2$

Solution:

Curved surface area of two identical cylindrical parts,

$$= 2 \times 2\pi rh = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times 5$$

$$= 78.57 cm^2$$

ii. (a) $190.93 cm^3$

Solution:

Volume of big cylindrical part $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{4.5}{2} \times \frac{4.5}{2} \times 12190.93 cm^3$$

iii. (b) $8.18 cm^3$

Solution:

Volumeoftwo hemispherical ends $= 2 \times \frac{2}{3}\pi r^3$

$$= \frac{2\times2}{3} \times \frac{22}{7} \times (\frac{2.5}{2})^3 = 8.18 cm^3$$

iv. (c) $19.64 cm^2$

Solution:

Curved surface area of two hemispherical ends,

$$= 2 \times 2\pi r^2 = 2 \times 2 \times \frac{22}{7} \times \frac{2.5}{2} \times \frac{2.5}{2} = 19.64 cm^3$$

v. (b) $182.75 cm^3$

Solution:

Difference of volume of bigger cylinder to two small hemispherical ends = 190.93 - 8.18 = $182. 75 cm^3$

2. Answer:

i. (b) 7.5

Solution:

Quantity of ice-cream in the brick = volume of the brick = $(30 \times 25 \times 10)cm^3 = 7500cm^3$

$$= \frac{7500}{1000} l \qquad [\because 1l = 1000cm^3]$$

$$= 7.5l$$

ii. (c) $89.83cm^3$

Solution:

Volume of hemispherical scoop $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = \frac{1886.5}{21} = 89.83cm^3$$

iii. (d) $192.5cm^3$

Solution:

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 15 = \frac{4042.5}{21} = 192.5cm^3$$

iv. (a) 2

Solution:

Number of scoops required to fill one cone

$$= \frac{\text{Volume of a cone}}{\text{Volume of a scoop}} = \frac{192.5}{89.83} = 2.14 \approx 2$$

v. (b) 39

Solution:

Number of cones that can be filled using the,

$$\text{Whole brick} = \frac{\text{Volume of brick}}{\text{Volume of 1 cone}}$$

$$= \frac{7500}{192.5} = 38.960 \approx 39$$

Assertion Reason Answer-

(c) A is true but R is false.

(a) Both A and R are true and R is the correct explanation of A.