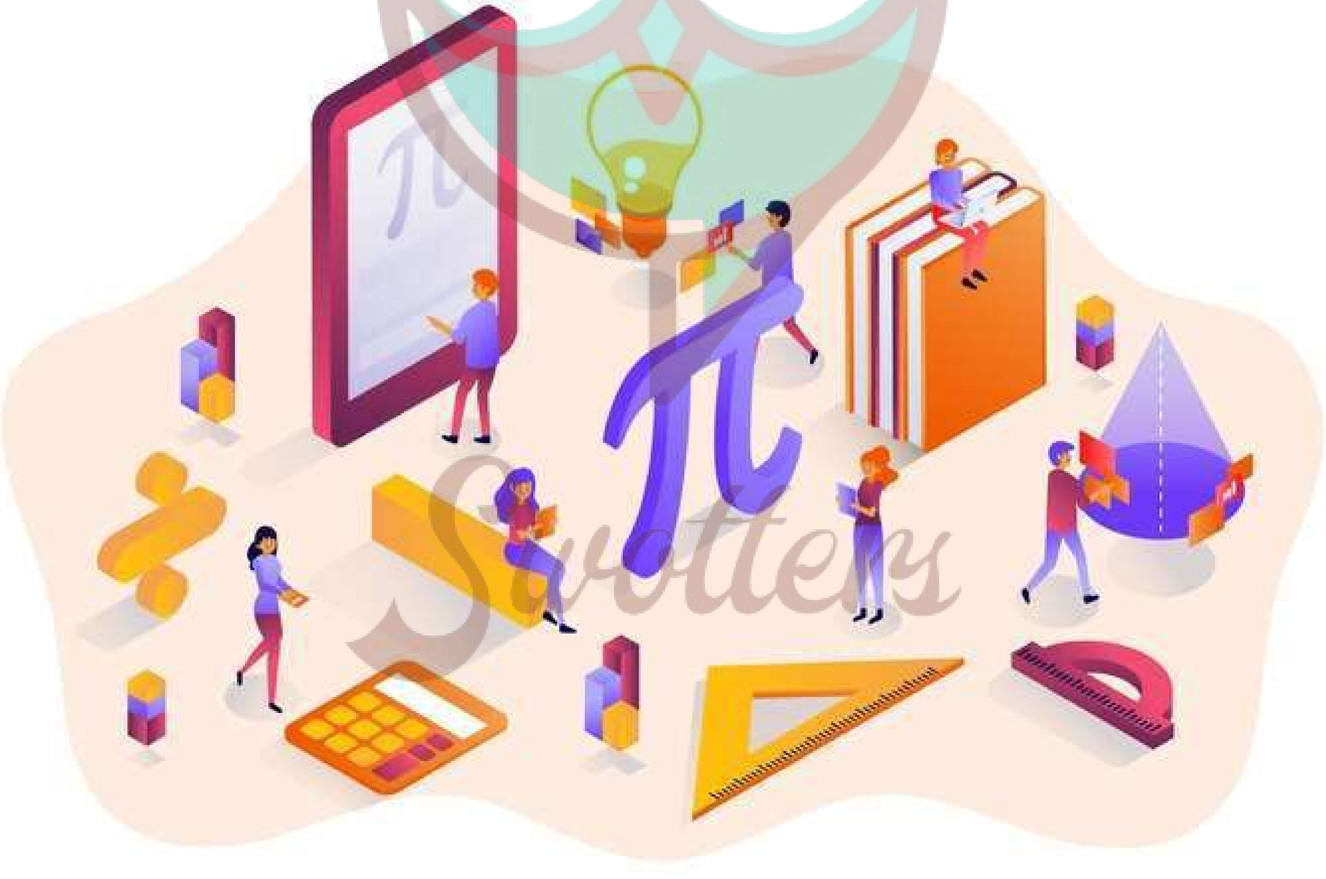
MATHEMATICS

Chapter 13: LIMITS AND DERIVATIVES



Important Questions

Multiple Choice questions-

Question 1. The expansion of log(1 – x) is:

(a) $x - x^2/2 + x^3/3 - \ldots\ldots..$

(b) $x + x^2/2 + x^3/3 + \ldots\ldots..$

(c) $-x + x^2/2 - x^3/3 + \ldots\ldots..$

(d) $-x - x^2/2 - x^3/3 - \ldots\ldots..$

Question 2. The value of $\text{Lim}_{x\to a}$ (a × sin x – x × sin a)/(ax² – xa²) is

(a) = $(a \times \cos a + \sin a)/a^2$

(b) = $(a \times \cos a - \sin a)/a^2$

(c) = $(a \times \cos a + \sin a)/a$

(d) = $(a \times \cos a - \sin a)/a$

Question 3. $\text{Lim}_{x\to -1} [1 + x + x^2 + \ldots\ldots\ldots + x^{10}]$ is

(a) 0

(b) 1

(c) -1

(d) 2

Question 4. The value of the limit $\text{Lim}_{x\to 0} \{\log(1 + ax)\}/x$ is

(a) 0

(b) 1

(c) a

(d) 1/a

Question 5. The value of the limit $\text{Lim}_{x\to 0} (\cos x)\cot^{2x}$ is

(a) 1

(b) e

(c) $e^{1/2}$

(d) $e^{-1/2}$

Question 6. Then value of $\text{Lim}_{x\to 1} (1 + \log x - x)\}/(1 - 2x + x^2)$ is

(a) 0

(b) 1

(c) 1/2

(d) -1/2

Question 7. The value of $\lim_{y\to 0}$ {(x + y) × sec (x + y) – x × sec x}/y is

(a) x × tan x × sec x

(b) x × tan x × sec x + x × sec x

(c) tan x × sec x + sec x

(d) x × tan x × sec x + sec x

Question 8. $\lim_{x\to 0}$ $(ex^2 - \cos x)/x^2$ is equals to

(a) 0

(b) 1

(c) 2/3

(d) 3/2

Question 9. The expansion of a^x is:

(a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \ldots\ldots\ldots.$

(b) $a^x = 1 - x/1! \times (\log a) + x^2/2! \times (\log a)^2 - x^3/3! \times (\log a)^3 + \ldots\ldots\ldots.$

(c) $a^x = 1 + x/1 \times (\log a) + x^2/2 \times (\log a)^2 + x^3/3 \times (\log a)^3 + \ldots\ldots\ldots.$

(d) $a^x = 1 - x/1 \times (\log a) + x^2/2 \times (\log a)^2 - x^3/3 \times (\log a)^3 + \ldots\ldots\ldots.$

Question 10. The value of the limit $\lim_{n\to 0} (1 + an)^{b/n}$ is:

(a) e^a

(b) e^b

(c) e^{ab}

(d) $e^{a/b}$

Very Short Questions:

1. Evaluate $\lim_{x\to 3}\left[\frac{x^2-9}{x-3}\right]$

2. Evaluate $\lim_{x\to 0}\frac{\sin 3x}{5x}$

3. Find derivative of 2x.

4. Find derivative of $\sqrt{\sin 2x}$

5. Evaluate $\lim_{x\to 0}\frac{\sin^2 4x}{x^2}$

6. What is the value of $\lim_{x\to a}\left(\frac{x^2-a^n}{x-a}\right)$

7. Differentiate $\frac{2x}{x}$

8. If y = $e^{\sin x}$ Find $\frac{dy}{dx}$

9. Evaluate $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

10. Differentiate x sin x with respect to x.

Short Questions:

1. Prove that $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1$

2. Evaluate $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x^2+x-3)}=1$

3. Evaluate $\lim_{x\to 0}\frac{x\tan 4x}{1-\cos 4x}$

4. It $y=\frac{(1-\tan\ )}{(1+\tan\ )}$. Show that $\frac{dy}{dx}=\frac{-2}{(1+\sin 2x)}$

5. Differentiate $e^{\sqrt{\cot x}}$

Long Questions:

1. Differentiate tan x from first principle.

2. Differentiate $(x + 4)^6$ From first principle.

3. Find derivative of cosec x by first principle.

4. Find the derivatives of the following fuchsias:

$(i)\ \left(x-\frac{1}{x}\right)^3 \quad (ii)\ \frac{(3x+1)(2\sqrt{x}-1)}{\sqrt{x}}$

5. Find the derivative of sin (x + 1) with respect to x from first principle.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$ is equal to 1, where $a+b+c\neq 0$.

Reason (R) $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is equal to $\frac{1}{4}$.

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) $\lim_{x \to 0} \frac{\sin ax}{bx}$ is equal to $\frac{a}{b}$.

Reason (R) $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

Answer Key:

MCQ:

1. (d) $-x - x^2/2 - x^3/3 - \ldots\ldots$

2. (b) $= (a \times \cos a - \sin a)/a^2$

3. (b) 1

4. (c) a

5. (d) $e^{-1/2}$

6. (d) -1/2

7. (d) x × tan x × sec x + sec x

8. (d) 3/2

9. (a) $a^x = 1 + x/1! \times (\log a) + x^2/2! \times (\log a)^2 + x^3/3! \times (\log a)^3 + \ldots\ldots\ldots$

10. (c) e^{ab}

Very Short Answer:

1.

$$\lim_{x\to 3}\frac{x^2-9}{x-3}\ \frac{0}{0}\ form$$

$$\lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}=3+3=6$$

2.

$$\lim_{x\to 0}\frac{\sin 3x}{5x}$$

$$=\lim_{3x\to 0}\frac{\sin 3x}{3x}\times\frac{3}{5}$$

$$=1\times\frac{3}{5}=\frac{3}{5}\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

3. Let y = 2^x

$$\frac{dy}{dx}=\frac{d}{dx}2=2^x 10g2$$

4.

$$\frac{d}{dx}\sqrt{\sin 2x}=\frac{1}{2\sqrt{\sin 2x}}\frac{d}{dx}\sin 2x$$

$$=\frac{1}{\cancel{2}\sqrt{\sin 2x}}\times\cancel{2}\cos 2x$$

$$=\frac{\cos 2x}{\sqrt{\cos 2x}}$$

5.

$$\lim_{x\to 0}\frac{\sin^2 4x}{x^2 4^2}\times 4^2=\lim_{4x\to 0}\left(\frac{\sin 4x}{4x}\right)^2\times 16$$

$$=1\times 16=16$$

6.

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=1$$

7.

$$\frac{d}{dx}\frac{2^x}{x}=\frac{x\frac{d}{dx}2^x-2^x\frac{d}{dx}x}{x^2}$$

$$=\frac{x\times 2^x 10g2-2^x\times 1}{x^2}$$

$$= 2x\frac{[x+10g2-1]}{x^2}$$

8.

$$y = e^{\sin x}$$

$$\frac{dy}{dx} = \frac{d}{dx}e^{\sin x}$$

$$= e^{\sin x} \times \cos x = \cos x e^{\sin x}$$

9.

$$\lim_{x\to1}\frac{x^{15}-1}{x^{10}-1}$$

$$= \frac{\lim_{x\to1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to1}\frac{x^{10}-1^{10}}{x-1}} = \frac{15\times1^{14}}{10\times1^{9}}$$

$$= \frac{15}{10} = \frac{3}{2}$$

10.

$$\frac{d}{dx}x\sin x = x\cos x + \sin x\,1$$

$$= x\cos x + \sin x$$

Short Answer:

1. We have

$$\lim_{x\to0}\frac{e^x-1}{x}$$

$$\lim_{x\to0}\left\{\frac{\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+---\right]-1}{x}\right\}\left[\because e^x = 1+x+\frac{x^2}{2!}+---\right]$$

$$\lim_{x\to0}\left\{\frac{x+\frac{x^2}{2!}+\frac{x^3}{3!}+---}{x}\right\}$$

$$\lim_{x\to0} x\left\{\frac{1+\frac{x}{2!}+\frac{x^2}{3!}+---}{x}\right\}$$

= 1 + 0 = 1

2.

$$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x^2+x-3)}$$

$$=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}\times\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$$

$$\lim_{x\to1}\frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$$

$$\lim_{x\to1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}=\frac{2\times1-3}{(2\times1+3)(\sqrt{1}+1)}$$

$$=\frac{-1}{10}$$

3.

$$\lim_{x\to0}\frac{x\tan 4x}{1-cas4x}$$

$$=\lim_{x\to0}\frac{x\sin 4x}{\cos 4x\left[2\sin^2 2x\right]}$$

$$=\lim_{x\to0}\frac{2x\sin 2x\cos 2x}{\cos 4x(2\sin^2 2x)}$$

$$=\lim_{x\to0}\left(\frac{\cos 2x}{\cos 4x}\cdot\frac{2x}{\sin 2x}\times\frac{1}{2}\right)$$

$$=\frac{1}{2}\frac{\lim_{2x\to0}\cos 2x}{\lim_{4x\to0}\cos 4x}\times\lim_{2x\to0}\left(\frac{2x}{\sin 2x}\right)=\frac{1}{2}\times1=\frac{1}{2}$$

4.

$$y=\frac{(1-\tan x)}{(1+\tan x)}$$

$$\frac{dy}{dx}=\frac{(1+\tan x)\frac{d}{dx}(1-\tan x)-(1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)(-\sec^2 x)-(1-\tan x)\sec^2 x}{(1+\tan x)^2}$$

$$=\frac{-\sec^2 x-\tan x\sec^2 x-\sec^2+\tan x\sec^2 x}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2}=\frac{-2}{\cos^2 x\left[1+\frac{sicx}{\cos x}\right]^2}$$

$$=\frac{-2}{\cos^2 x\left[\frac{\cos x+\sin x}{\cos^2 x}\right]^2}$$

$$=\frac{-2}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{-2}{1+\sin^2 x}$$

$$\therefore \frac{dy}{dx}=\frac{-2}{1+\sin 2x}$$

Hence Proved.

5.

Let $y=e^{\sqrt{\cot x}}$

$$\frac{dy}{dx}=\frac{d}{dx}e^{\sqrt{\cot x}}=e^{\sqrt{\cot x}}\frac{d}{dx}\sqrt{\cot x}$$

$$=e^{\sqrt{\cot x}}\times\frac{1}{2\sqrt{\cot x}}.\frac{d}{dx}\cot x$$

$$=\frac{e^{\sqrt{\cot x}}}{2\sqrt{\cot x}}-\cos ec^2 x$$

$$=\frac{-\cos cec^2 e^{\sqrt{\cot x}}}{2\sqrt{\cot x}}$$

Long Answer:

1.

$$f(x)=\tan x$$

$$f(x+h)=\tan(x+h)$$

$$f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\tan(x+h)-\tan x}{h}$$

$$= \lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x+h)\cos x-\cos(x+h)\sin x}{h\cos(x+h)\cos x}$$

$$= \lim_{h\to 0}\frac{\sin[x+h-x]}{h\cos(x+h)\cos x}\left[\because \sin(A-B)= \sin A\cos B-\cos A\sin B\right]$$

$$= \lim_{h\to 0}\frac{\sinh}{h\cos(x+h)\cos x}$$

$$= \frac{\lim_{h\to 0}\frac{\sinh}{h}}{\lim_{h\to 0}\cos(x+h)\cos x}=\frac{1}{\cos x.\cos x}\left[\because \lim_{h\to 0}\frac{\sinh}{h}=1\right]$$

$$= \frac{1}{\cos^2 x}=\sec^2 x$$

2.

let $f(x)=(x+4)^6$

$f(x+h)=(x+h+4)^6$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(x+h+4)^6-(x+4)^6}{h}$$

$$= \lim_{(x+h+4)\to(x+4)}\frac{(x+h+4)^6-(x+4)^6}{(x+h+4)-(x-4)}$$

$$= 6(x+4)^{(6-1)}\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$= 6(x+4)^5$$

3.

proof let $f(x)=\operatorname{cosec} x$

By def, $f(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(h)}{h}$

$$= \underset{h\to 0}{Lt}\frac{\operatorname{cosec}(x+h)-\operatorname{cosec} x}{h}$$

$$= \underset{h\to 0}{Lt}\frac{\frac{1}{\sin(x+h)}-\frac{1}{\sin x}}{h} = \underset{h\to 0}{Lt}\frac{\sin x-\sin(x+h)}{h\sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt}\frac{2\cos\frac{x+x+h}{2}\sin\frac{x-x+h}{2}}{h\sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt}\frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{h\sin(x+h)\sin x}$$

$$= \frac{\underset{\frac{h}{2}\to 0}{Lt}\cos\left(x+\frac{h}{2}\right)}{\cos x.\underset{h\to 0}{L}+\sin(x+h)}.\underset{\frac{h}{2}\to 0}{Lt}\frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -\frac{\cos x}{\sin x.\sin x}.1 = -\cos ecx\cot x$$

4.

(i) let $f(x)=\left(x-\frac{1}{x}\right)^3 = x^3-\frac{1}{x^3}-3x+\frac{1}{x}\left(x-\frac{1}{x}\right)$

$= x^3 - x^{-3} - 3x + 3x^{-1}.d.ff\ wr.t4, we$ get

$f(x) = 3\times x^2 - (-3)x^{-4} - 3\times 1 + 3\times(-1)x^{-2}$

$= 3x^2 + \frac{3}{x^4} - 3 - \frac{3}{x^2}$

(ii) let $f(x) = \frac{(3x+1)(2\sqrt{x}-1)}{\sqrt{x}} = \frac{6x^{\frac{3}{2}}-3x+2\sqrt{x}-1}{\sqrt{x}}$

$= 6x - 3x^{\frac{1}{2}} + 2 - x^{-\frac{1}{2}}.d:ff$ w.r.t. x.weget

$f(x) = 6\times 1 - 3\times\frac{1}{2}\times x^{-\frac{1}{2}} + 0 - \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= 6 - \frac{3}{2\sqrt{x}} + \frac{1}{2.x^{\frac{3}{2}}}$.

5.

let $f(x)=\sin(x+1)$

$f(x+h)=\sin(x+h+1)$

$$f^{1}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sin(x+h+1)-\sin(x+1)}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left[\frac{x+h+1+x+1}{2}\right]\sin\left[\frac{x+h+1-x-1}{2}\right]}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left[x+1+\frac{h}{2}\right]\sin\frac{h}{2}}{h}$$

$$=\lim_{h\to 0}2\cos\left(x+1+\frac{h}{2}\right)\times\lim_{h\to 0}\frac{\sin\frac{h}{2}}{2\frac{h}{2}}$$

$$=\cos(x+1)\times 1=\cos(x+1)$$

Assertion Reason Answer:

1. (iii) Assertion is true but reason is false.

2. (i) Both assertion and reason are true and reason is the correct explanation of assertion.