MATHEMATICS

Chapter 14: Factorisation

Important Questions

Multiple Choice Questions-

Question 1. The common factor of x^2y^2 and x^3y^3 is

(a) x^2y^2

(b) x^3y^3

(c) x^2y^3

(d) x^3y^2.

Question 2. The common factor of x^3y^2 and x^4y is

(a) $x^{43}y^2$

(b) x^4y

(c) x^3y^2

(d) x^3y.

Question 3. The common factor of $a^2 m^4$ and a^4m^2 is

(a) a^4m^4

(b) a^2m^2

(c) a^2m^4

(d) a^4m^2

Question 4. The common factor of p^3q^4 and p^4q^3 is

(a) p^4q^4

(b) p^4q^3

(c) p^3q^3

(d) p^3q^4

Question 5. The common factor 12y and 30 is

(a) 6

(b) 12

(c) 30

(d) 6y.

Question 6. The common factor of 2x, $3x^3$, 4 is

(a) 1

(b) 2

(c) 3

(d) 4.
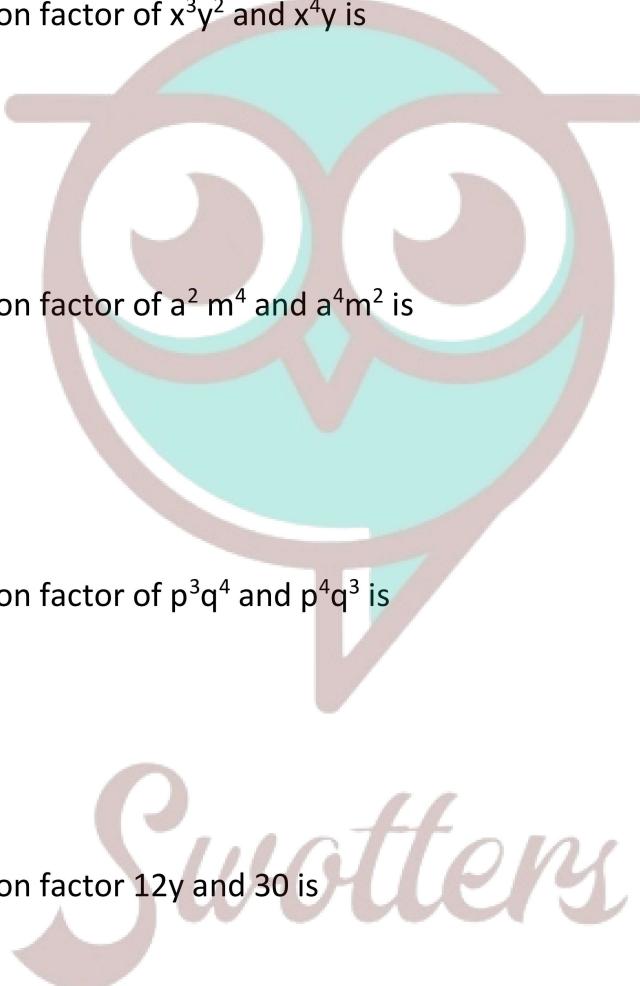
Question 7. The common factor of 10ab, 30bc, 50ca is

(a) 10

(b) 30

(c) 50

(d) abc.

Question 8. The common factor of $14a^2b$ and $35a^4b^2$ is

(a) a^4b^2

(b) $35a^4b^2$

(c) $14a^2b$

(d) $7a^2b$.

Question 9. The common factor of $8a^2b^4c^2$, $12a^4bc^4$ and $20a^3b^4$ is

(a) a^4b^4

(b) a^2b^2

(c) $4a^2b^2$

(d) $4a^2b$.

Question 10. The common factor of $6a^3b^4c^2$, $21a^2b$ and $15a^3$ is

(a) $3a^2$

(b) $3a^3$

(c) $6a^3$

(d) $6a^2$
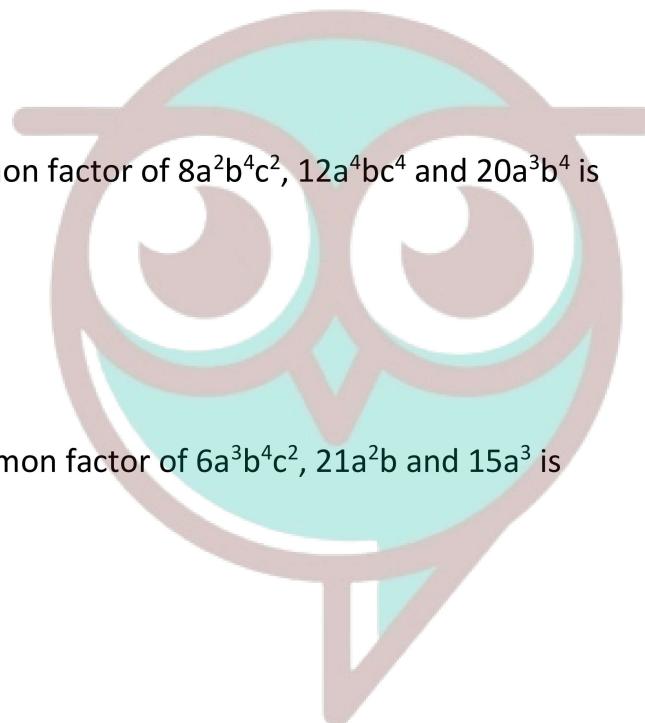
Very Short Questions:

1. Find the common factors of the following terms.
   (a) $25x^2y$, $30xy^2$
   (b) $63m^3n$, $54mn^4$

2. Factorise the following expressions.
   (a) $54m^3n + 81m^4n^2$
   (b) $15x^2y^3z + 25x^3y^2z + 35x^2y^2z^2$

3. Factorise the following polynomials.
   (a) $6p(p - 3) + 1 (p - 3)$
   (b) $14(3y - 5z)^3 + 7(3y - 5z)^2$

4. Factorise the following:
   (a) $p^2q - pr^2 - pq + r^2$

(b) $x^2 + yz + xy + xz$

5. Factorise the following polynomials.

(a) $xy(z^2 + 1) + z(x^2 + y^2)$

(b) $2axy^2 + 10x + 3ay^2 + 15$

Short Questions :

1. Factorise the following expressions.

(a) $x^2 + 4x + 8y + 4xy + 4y^2$

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

2. Factorise:

(a) $a^2 + 14a + 48$

(b) $m^2 - 10m - 56$

3. Factorise:

(a) $x^4 - (x - y)^4$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

4. Factorise the following polynomials.

(a) $16x^4 - 81$

(b) $(a - b)^2 + 4ab$

5. Factorise:

(a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$
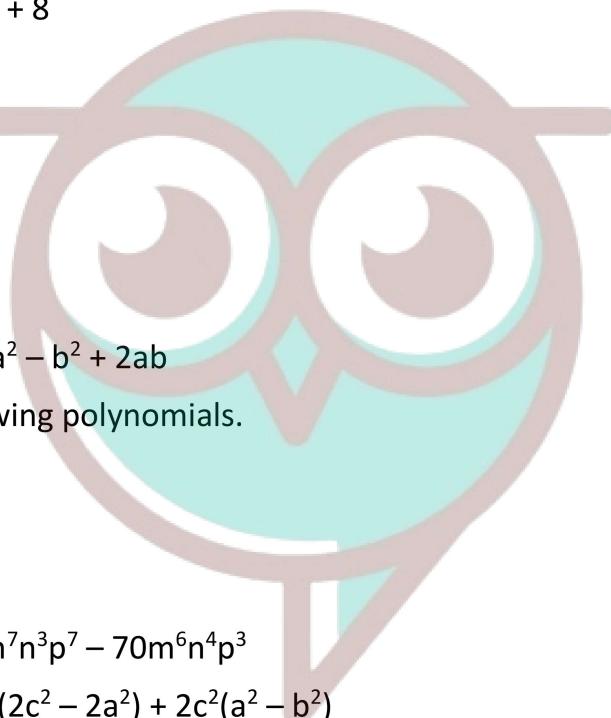
Long Questions :

1. Factorise:

(a) $(x + y)^2 - 4xy - 9z^2$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

2. Evaluate the following divisions:

(a) $(3b - 6a) \div (30a - 15b)$

(b) $(4x2 - 100) \div 6(x + 5)$

3. Simplify the following expressions:

(a) $\dfrac{(x-1)(x-2)\left(x^2-9x+14\right)}{(x-7)\left(x^2-3x+2\right)}$

(b) $\dfrac{\left(x^2-8x+12\right)\left(x^2-16\right)}{\left(x^2-36\right)\left(x^2-4\right)}$

4. Factorise the given expressions and divide that as indicated.

(a) $39n^3(50n^2 - 98) \div 26n^2(5n - 7)$

(b) $44(p^4 - 5p^3 - 24p^2) \div 11p(p - 8)$

5. If one of the factors of $(5x^2 + 70x - 160)$ is $(x - 2)$. Find the other factor.

Answer Key-

Multiple Choice questions-

1. (a) x^2y^2
2. (d) x^3y.
3. (b) a^2m^2
4. (c) p^3q^3
5. (a) 6
6. (a) 1
7. (a) 10
8. (d) $7a^2b$.
9. (d) $4a^2b$.
10. (a) $3a^2$
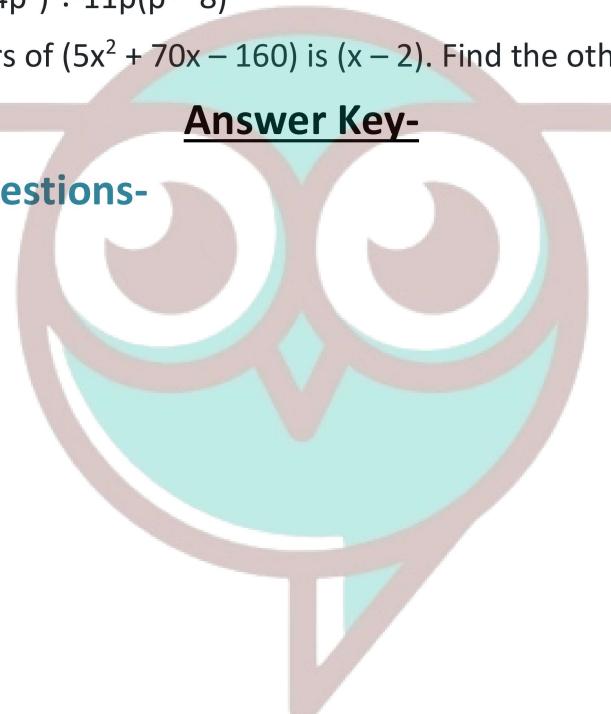
Very Short Answer :

1. (a) $25x^2y$, $30xy^2$

   $25x^2y = 5 \times 5 \times x \times x \times y$

   $30xy^2 = 2 \times 3 \times 5 \times x \times y \times y$

   Common factors are $5 \times x \times y = 5xy$

   (b) $63m^3n$, $54mn^4$

   $63m^3n = 3 \times 3 \times 7 \times m \times m \times m \times n$

   $54mn^4 = 2 \times 3 \times 3 \times 3 \times m \times n \times n \times n \times n$

   Common factors are $3 \times 3 \times m \times n = 9mn$

2. (a) $54m^3n + 81m^4n^2$

   $= 2 \times 3 \times 3 \times 3 \times m \times m \times m \times n + 3 \times 3 \times 3 \times 3 \times m \times m \times m \times m \times n \times n$

= 3 × 3 × 3 × m × m × m × n × (2 + 3 mn)

= $27m^3n$ (2 + 3mn)

(b) $15x^2y^3z + 25 x^3y^2z + 35x^2y^2z^2 = 5x^2y^2z$ (3y + 5x + 7)

3. (a) 6p(p − 3) + 1 (p − 3) = (p − 3) (6p + 1)

(b) $14(3y − 5z)^3 + 7(3y − 5z)^2$

= $7(3y − 5z)^2$ [2(3y − 5z) +1]

= $7(3y − 5z)^2$ (6y − 10z + 1)

4. (a) $p^2q − pr^2 − pq + r^2$

= $(p^2q − pq) + (-pr^2 + r^2)$

= $pq(p − 1) − r^2(p − 1)$

= $(p − 1) (pq − r^2)$

(b) $x^2 + yz + xy + xz$

= $x^2 + xy +xz + yz$

= x(x + y) + z(x + y)

= (x + y) (x + z)

5. (a) $xy(z^2 + 1) + z(x^2 + y^2)$

= $xyz^2 + xy + zx^2 + zy^2$

= $(xyz^2 + zx^2) + (xy + zy^2)$

= zx(yz + x) + y(x + yz)

= zx(x + yz) + y(x + yz)

= (x + yz) (zx + y)

(b) $2axy^2 + 10x + 3ay^2 + 15$

= $(2axy^2 + 3ay^2) + (10x + 15)$

= $ay^2(2x + 3) + 5(2x + 3)$

= $(2x + 3) (ay^2 + 5)$
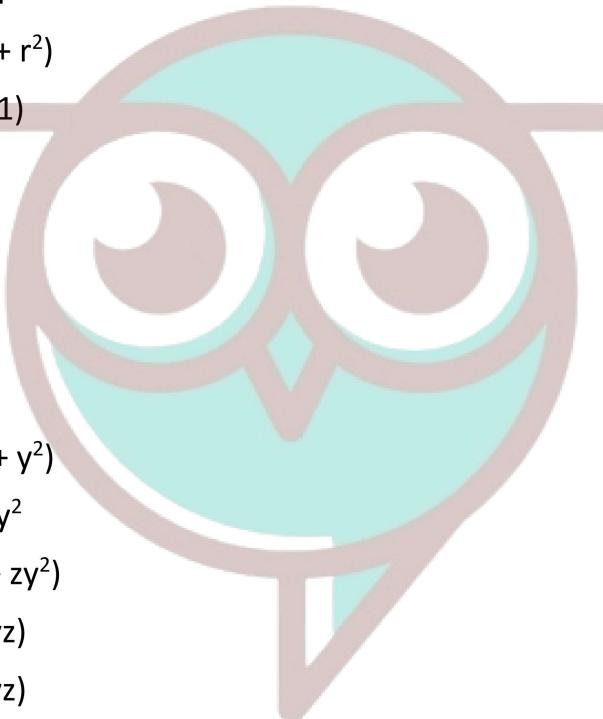
Short Answer :

1. (a) $x^2 + 4x + 8y + 4xy + 4y^2$

= $(x^2 + 4xy + 4y^2) + (4x + 8y)$

= $(x + 2y)^2 + 4(x + 2y)$

= (x + 2y)(x + 2y + 4)

(b) $4p^2 + 2q^2 + p^2q^2 + 8$

= $(4p^2 + 8) + (p^2q^2 + 2q^2)$

$= 4(p^2 + 2) + q^2(p^2 + 2)$

$= (p^2 + 2)(4 + q^2)$

2. (a) $a^2 + 14a + 48$

$= a^2 + 6a + 8a + 48$

$[6 + 8 = 14 ; 6 \times 8 = 48]$

$= a(a + 6) + 8(a + 6)$

$= (a + 6) (a + 8)$

(b) $m^2 - 10m - 56$

$= m^2 - 14m + 4m - 56$

$[14 - 4 = 10; 4 \times 4 = 56]$

$= m(m - 14) + 6(m - 14)$

$= (m - 14) (m + 6)$

3. (a) $x^4 - (x - y)^4$

$= (x^2)^2 - [(x - y)^2]^2$

$= [x^2 - (x - y)^2] [x^2 + (x - y)^2]$

$= [x + (x - y] [x - (x - y)] [x^2 + x^2 - 2xy + y^2]$

$= (x + x - y) (x - x + y)[2x^2 - 2xy + y^2]$

$= (2x - y) y(2x^2 - 2xy + y^2)$

$= y(2x - y) (2x^2 - 2xy + y^2)$

(b) $4x^2 + 9 - 12x - a^2 - b^2 + 2ab$

$= (4x^2 - 12x + 9) - (a^2 + b^2 - 2ab)$

$= (2x - 3)^2 - (a - b)^2$

$= [(2x - 3) + (a - b)] [(2x - 3) - (a - b)]$

$= (2x - 3 + a - b)(2x - 3 - a + b)$

4. (a) $16x^4 - 81$

$= (4x^2)^2 - (9)^2$

$= (4x^2 + 9)(4x^2 - 9)$

$= (4x^2 + 9)[(2x)^2 - (3)^2]$

$= (4x^2 + 9)(2x + 3) (2x - 3)$

(b) $(a - b)^2 + 4ab$

$= a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a + b)^2$
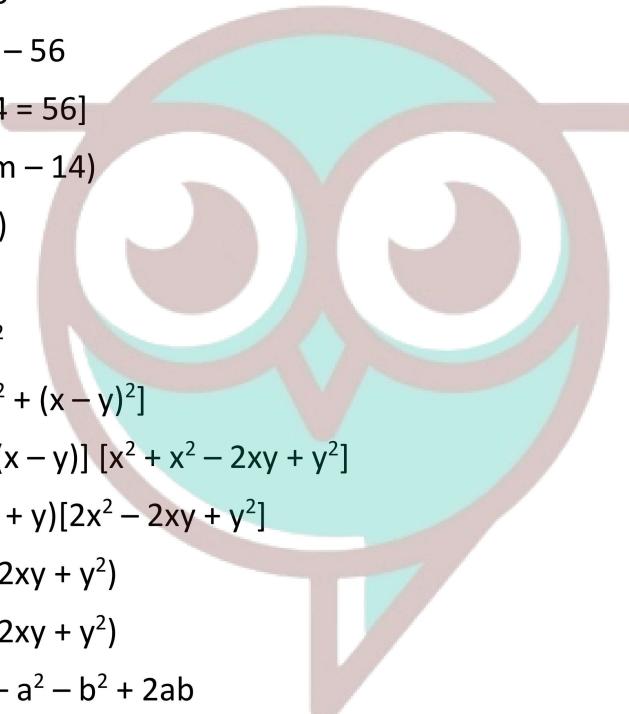
5. (a) $14m^5n^4p^2 - 42m^7n^3p^7 - 70m^6n^4p^3$

$= 14m^5n^3p^2(n - 3m^2p^5 - 5mnp)$

(b) $2a^2(b^2 - c^2) + b^2(2c^2 - 2a^2) + 2c^2(a^2 - b^2)$

$= 2a^2(b^2 - c^2) + 2b^2(c^2 - a^2) + 2c^2(a^2 - b^2)$

$= 2[a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)]$

$$= 2 \left[\begin{array}{l} \cancel{a^2b^2} - \cancel{a^2c^2} + \cancel{b^2c^2} - \cancel{a^2b^2} \\ \qquad\quad + \cancel{a^2c^2} - \cancel{b^2c^2} \end{array} \right]$$

$= 2 \times 0$

$= 0$

Long Answer :

1. (a) $(x + y)^2 - 4xy - 9z^2$

$= x^2 + 2xy + y^2 - 4xy - 9z^2$

$= (x^2 - 2xy + y^2) - 9z^2$

$= (x - y)^2 - (3z)^2$

$= (x - y + 3z)(x - y - 3z)$

(b) $25x^2 - 4y^2 + 28yz - 49z^2$

$= 25x^2 - (4y^2 - 28yz + 49z^2)$

$= (5x)^2 - (2y - 7)^2$

$= (5x + 2y - 7)[5x - (2y - 7)]$

$= (5x + 2y - 7)(5x - 2y + 7)$
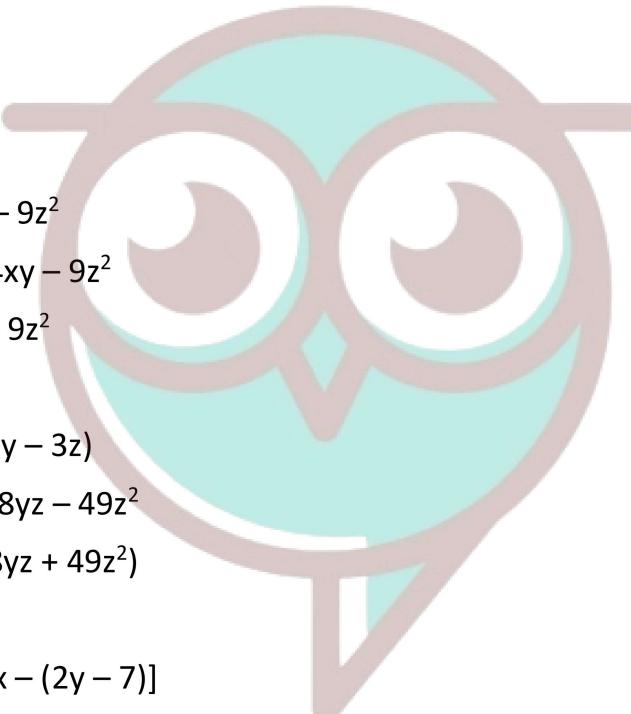
2.

(a) $(3b - 6a) \div (30a - 15b)$

$$= \frac{3b - 6a}{30a - 15b} = \frac{-\cancel{3}\,(2a - b)}{\underset{5}{\cancel{15}}\,(2a - b)}$$

$$= \frac{-1}{5}$$

(b) $(4x^2 - 100) \div 6(x + 5)$

$$= \frac{4x^2 - 100}{6(x + 5)} = \frac{4\left(x^2 - 25\right)}{6(x + 5)}$$

$$= \frac{\overset{2}{\cancel{4}}\,(x - 5)\,(x + 5)}{\underset{3}{\cancel{6}}\,(x + 5)}$$

$$= \frac{2}{3}(x - 5)$$

3.

(a) $\dfrac{(x - 1)(x - 2)\left(x^2 - 9x + 14\right)}{(x - 7)\left(x^2 - 3x + 2\right)}$

$$= \frac{(x - 1)(x - 2)\left(x^2 - 7x - 2x + 14\right)}{(x - 7)\left(x^2 - 2x - x + 2\right)}$$

$$= \frac{(x - 1)(x - 2)\left[x(x - 7) - 2(x - 7)\right]}{(x - 7)\left[x(x - 2) - 1(x - 2)\right]}$$

$$= \frac{(x - 1)\,(x - 2)\,(x - 7)\,(x - 2)}{(x - 7)\,(x - 2)\,(x - 1)} = (x - 2)$$

(b) $\dfrac{\left(x^2 - 8x + 12\right)\left(x^2 - 16\right)}{\left(x^2 - 36\right)\left(x^2 - 4\right)}$

$$= \frac{\left(x^2 - 6x - 2x + 12\right)(x - 4)(x + 4)}{(x - 6)(x + 6)(x - 2)(x + 2)}$$

$$= \frac{\left[x(x - 6) - 2(x - 6)\right](x - 4)(x + 4)}{(x - 6)(x + 6)(x - 2)(x + 2)}$$

$$= \frac{(x - 2)\,(x - 6)\,(x - 4)(x + 4)}{(x - 6)(x + 6)\,(x - 2)(x + 2)}$$

$$= \frac{(x - 4)(x + 4)}{(x + 6)(x + 2)}$$

4.

(a) $39n^3(50n^2 - 98) \div 26n^2(5n + 7)$

$= \dfrac{39n^3 \left(50n^2 - 98\right)}{26n^2 \left(5n - 7\right)}$

$= \dfrac{39n^3 \times 2 \left(25n^2 - 49\right)}{26n^2 \left(5n - 7\right)}$

$= \dfrac{3 \times 13n^3 \times 2 \left[(5n)^2 - (7)^2\right]}{2 \times 13n^2 \left(5n - 7\right)}$

$= \dfrac{3 \times \cancel{13} \; \cancel{n^3}^{n} \times \cancel{2} \left(5n + 7\right) \cancel{(5n - 7)}}{\cancel{2} \times \cancel{13} \; \cancel{n^2} \; \cancel{(5n - 7)}}$

$= 3n(5n + 7)$

(b) $44(p^4 - 5p^3 - 24p^2) \div 11p(p - 8)$

$= \dfrac{44 \left(p^4 - 5p^3 - 24p^2\right)}{11p \left(p - 8\right)}$

$= \dfrac{44 \times p^2 \left(p^2 - 5p - 24\right)}{11p \left(p - 8\right)}$

$= \dfrac{44p^2 \left(p^2 - 8p + 3p - 24\right)}{11p \left(p - 8\right)}$

$= \dfrac{44p^2 \left[p(p - 8) + 3(p - 8)\right]}{11p \left(p - 8\right)}$

$= \dfrac{\cancel{44}^{4} \; \cancel{p^2}^{p} \; \cancel{(p - 8)} (p + 3)}{\cancel{11} \; \cancel{p} \; \cancel{(p - 8)}}$

$= 4p(p + 3)$

5. Let the other factor be m.

$(x - 2) \times m = 5x^2 + 70x - 160$

$$\Rightarrow \quad m = \frac{5x^2 + 70x - 160}{x - 2}$$

$$= \frac{5\left(x^2 + 14x - 32\right)}{x - 2}$$

$$= \frac{5\left(x^2 + 16x - 2x - 32\right)}{x - 2}$$

$$= \frac{5\left[x(x + 16) - 2(x + 16)\right]}{x - 2}$$

$$= \frac{5(x + 16)\,(x - 2)}{(x - 2)}$$

$$= 5(x + 16)$$