MATHEMATICS

Chapter 14: MATHEMATICAL REASONING

Important Questions

Multiple Choice questions-

Question 1. Which of the following statement is a conjunction?

(a) Ram and Shyam are friends

(b) Both Ram and Shyam are friends

(c) Both Ram and Shyam are enemies

(d) None of these

Question 2. Which of the following is true?

(a) A prime number is either even or odd

(b) √3 is irrational number.

(c) 24 is a multiple of 2, 4 and 8

(d) Everyone in India speaks Hindi.

Question 3. The contrapositive of the statement If a number is not divisible by 3, it is not divisible by 9 is:

(a) If a number is divisible by 3, it is not divisible by 9

(b) If a number is not divisible by 3, it is divisible by 9

(c) If a number is divisible by 3, it is divisible by 9

(d) If a number is not divisible by 3, it is not divisible by 9

Question 4. The negation of the statement The product of 3 and 4 is 9 is

(a) It is false that the product of 3 and 4 is 9

(b) The product of 3 and 4 is 12

(c) The product of 3 and 4 is not 12

(d) It is false that the product of 3 and 4 is not 9

Question 5. The connective in the statement Earth revolves round the Sun and Moon is a satellite of earth is

(a) or

(b) Earth

(c) Sun

(d) and

Question 6. If p is a statement then the negation of p is

(a) p

(b) -p

(c) ~p

(d) p~

Question 7. If (p or q) is false when

(a) p is true and q is false

(b) p is true and q is true

(c) p is false and q is false

(d) p is false and q is true

Question 8. If (p and q) is true then

(a) p is true and q is false

(b) p is false and q is false

(c) p is false and q is true

(d) p is true and q is true

Question 9. If (p and q) is false then

(a) p is true and q is false

(b) p is false and q is false

(c) p is false and q is true

(d) all of the above

Question 10. The compound statement with AND is true if all its component statements are

(a) true

(b) false

(c) either true or false

(d) None of these

Short Questions:

1. Give three examples of sentences which are not statements. Give reasons for the answers.
2. Write the negation of the following statements

   (i) Chennai is the capital of Tamil Nadu.

   (ii) Every natural number is an integer.
3. Find the component statements of the following compound statements and check whether they are true or false.

(i) The number 3 is prime or it is odd.

4. Check whether the following pair of statements are negations of each other Give reasons for your answer.

   (i) x + y = y + x is true for every real numbers x and y.

   (ii) There exists real numbers x and y for which x + y = y + x.

5. Write the contra-positive and converse of the following statements.

   (i) If x is a prime number, then x is odd.

   (ii) if the two lines are parallel, then they do not intersect in the same plane.

6. Show that the statement

   P : "If x is a real number such that x3 + 4x = 0 then x is 0" is true by

   (i) direct method, (ii) method of contradiction, (iii) method of contra-positive.

7. Given below are two statements

   P : 25 is a multiple of 5.

   q: 25 is a multiple of 8

   Write the compound statements connecting these two statements with "and" and "OR". In both cases check the validity of the compound statement.

8. Write the following statement in five different ways, conveying the same meaning.

   P : If a triangle is equiangular, then it is an obtuse angled triangle.

Answer Key:

MCQ:

1. (d) None of these
2. (d) Everyone in India speaks Hindi.
3. (c) If a number is divisible by 3, it is divisible by 9
4. (a) It is false that the product of 3 and 4 is 9
5. (d) and
6. (c) ~p
7. (a) p is true and q is false
8. (d) p is true and q is true
9. (d) all of the above
10. (a) true

Short Answer:

1. (i) The sentence "Rani is a beautiful girl" is not a statement. To some Rani may look beautiful and to other she may not look beautiful. We cannot say on logic whether or not this sentence is true.

(ii) The sentence 'shut the door' is not a statement. It is only an imperative sentence giving a direction to someone. There is no question of it being true or false.

(iii) The sentence 'yesterday was Friday' is not a statement. It is an ambiguous sentence which is true if spoken on Saturday and false if spoken on other days. Truth or false hood of the sentence depends on the time at which it is spoken and not on mathematical reasoning.

2. (i) Chennai is not the capital of Tamil Nadu.

(ii) Every natural number is not an integer.

3. The component statements of the given statement are

p: "The number 3 is prime"

q: "number 3 is odd"

There two have been connected by using the connective "or"

The given statement is true as both the statements are true.

4. The given statements are

P : " x + y = y + x is true for every real number x and y

q : "There exists real numbers x and y for which x + y = y + x".

These statements are not negations of each other as they can be true at the same time. Infect, negation of p is

$\sim p$ " There are real numbers x and y for which x + y ≠ y + x."

Note that p is always true whatever and may be and is always false.

5. If statement is p ⇒ q then its contra-positive is $\sim q \Rightarrow \sim p$ and its converse is q ⇒ p.

(i) Contra-positive : "If x is not odd, then is not a prime number."

Converse : "If x is odd, then x is a prime number."

(ii) Contra-positive : "If two lines intersect in the same plane, then they are not parallel."

Converse: "If two lines do not intersect in the same plane, then they are parallel."

6. Given statement is p: "If x is a real number such that $x^3 + 4x = 0$, then x = 0"

(i) Direct method: Let $x3 + 4x = 0$, $x \in R$

$\Rightarrow x(x^2+4)=0, x \in R \Rightarrow x=0 \left(\because \text{if } x \in R \text{ then } x^2+4 \geq 4\right)$

Note that if the product of two numbers is zero then atleast one of them is surely zero.

Thus, we find that p is a true statement.

(ii) Method of contradiction.

Let x be a nonzero real number

$\Rightarrow x^2 > 0$ ($\because$ Square of a non- zero real number is always positive)

$\Rightarrow x^2+4>4 \Rightarrow x^2+4\neq 0$

$\Rightarrow x(x^2+4)\neq 0 \ (\because x\neq 0 \text{ and } x^2+4\neq 0)$

$\Rightarrow x^3 + 4x \neq 0$, which is a contradiction.

Hence, x = 0

(iii) Method of contra-positive:

Let $q:"x\in R \text{ and } x^3+4x=0"$

$r:"x=0"$

$\therefore$ Given statement p is $q \Rightarrow r$

Its contra-positive is $\sim r \Rightarrow \sim q$

i.e. "if x is a non- zero real number then $x^3 + 4$ is also nonzero"

Now $x\neq 0, x\in R \Rightarrow x^2>0 \Rightarrow x^2+4>4 \Rightarrow x^2+4\neq 0$

$\Rightarrow x(x^2+4)\neq 0 \Rightarrow x^3+4x\neq 0$ *i.e.* $\sim r \Rightarrow \sim q.$

Thus the statement is $\sim r \Rightarrow \sim q.$ always true

Hence, q ⇒ r is always true

Note: Infact, 'Method of contradiction' is another form of 'contra-positive method' while proving an implication.

7. Case I. Using the connective 'and', we obtain the compound statement "p and q".

i.e., "25 is a multiple of 5 and 8".

It is false statement as q is always false. ($\because$ 25 is not multiple of 8)

Case II. Using the connective 'or', we obtain the compound statement "p or q"

i.e., "25 is a multiple of 5 or 8".

It is a true statement as p always true. ($\because$ 25 is a multiple of 5)

8. Given statement is

"If a triangle is equiangular, then it is an obtuse angled triangle". Its five equivalents are as follows:

(i) "A triangle is equiangular only if it is an obtuse angled triangle".

(ii) "If a triangle is not obtuse angled triangle then it is not an equiangular triangle."

(iii) "equiangularity is a sufficient condition for triangle to be obtuse angled."

(iv) "A triangle being obtuse angled, is necessary condition for it to be equiangular".

(v) A triangle is obtuse is obtuse angled if it is equiangular.