MATHEMATICS

Chapter 13: STATISTICS

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Important Questions

Multiple Choice questions-

Question 1. If the varience of the data is 121 then the standard deviation of the data is

(a) 121

(b) 11

(c) 12

(d) 21

Question 2. The mean deviation from the mean for the following data: 4, 7, 8, 9, 10, 12, 13 and 17 is

(a) 2

(b) 3

(c) 4

(d) 5

Question 3. The mean of 1, 3, 4, 5, 7, 4 is m the numbers 3, 2, 2, 4, 3, 3, p have mean m – 1 and median q. Then, p + q =

(a) 4

(b) 5

(c) 6

(d) 7

Question 4. If the difference of mode and median of a data is 24, then the difference of median and mean is

(a) 12

(b) 24

(c) 8

(d) 36

Question 5. The coefficient of variation is computed by

(a) S.D/.Mean × 100

(b) S.D./Mean

(c) Mean./S.D × 100

(d) Mean/S.D.

Question 6. The geometric mean of series having mean = 25 and harmonic mean = 16 is

(a) 16

(b) 20

(c) 25

(d) 30

Question 7. When tested the lives (in hours) of 5 bulbs were noted as follows: 1357, 1090, 1666, 1494, 1623. The mean of the lives of 5 bulbs is

(a) 1445

(b) 1446

(c) 1447

(d) 1448

Question 8. Mean of the first n terms of the A.P. a + (a + d) + (a + 2d) + ……… is

(a) a + nd/2

(b) a + (n – 1)d

(c) a + (n – 1)d/2

(d) a + nd

Question 9. The mean of a group of 100 observations was found to be 20. Later on, it was found that three observations were incorrect, which was recorded as 21, 21 and 18. Then the mean if the incorrect observations are omitted is

(a) 18

(b) 20

(c) 22

(d) 24

Question 10. If covariance between two variables is 0, then the correlation coefficient between them is

(a) nothing can be said

(b) 0

(c) positive

(d) negative

Very Short Questions:

1. In a test with a maximum marks 25, eleven students scored 3, 9, 5, 3, 12, 10, 17, 4, 7, 19, 21 marks respectively. Calculate the range.
2. Coefficient of variation of two distributions is 70 and 75, and their standard deviations are

28 and 27 respectively what are their arithmetic mean?

3. Write the formula for mean deviation.
4. Write the formula for variance.
5. Find the median for the following data.

   x_i 579101215

   f_i 862226
6. Write the formula of mean deviation about the median
7. Write the formula of mean deviation about the median.
8. Find the mean of the following data 3,6,11,12,18.
9. Express in the form of a + ib (3i-7) + (7-4i) – (6+3i) + i^{23}
10. Find the conjugate of $\sqrt{-3} + 4i^2$

Short Questions:

1. The mean of 2,7,4,6,8 and p is 7. Find the mean deviation about the median of these observations.
2. Find the mean deviation about the mean for the following data!

   x_i 1030507090

   f_i 42428168
3. Find the mean, standard deviation and variance of the first natural n numbers.
4. Find the mean variance and standard deviation for following data.
5. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Long Questions:

1. Calculate the mean, variance and standard deviation of the following data:

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

2. The mean and the standard deviation of 100 observations were calculated as 40 and 5.1 respectively by a student who mistook one observation as 50 instead of 40. What are the correct mean and standard deviation?
3. 200 candidates the mean and standard deviation was found to be 10 and 15 respectively. After that if was found that the scale 43 was misread as 34. Find the correct mean and correct S.D

4. Find the mean deviation from the mean 6,7,10,12,13,4,8,20
5. Find two numbers such that their sum is 6 and the product is 14.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : In order to find the dispersion of values of x from mean $\bar{x}$, we take absolute measure of dispersion.

   Reason (R) : Sum of the deviations from mean ($\bar{x}$) is zero.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3.

   Reason (R) : The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (b) 11
2. (b) 3
3. (d) 7
4. (a) 12

5. (b) S.D./Mean
6. (b) 20
7. (b) 1446
8. (c) a + (n − 1)d/2
9. (b) 20
10. (b) 0

Very Short Answer:

1. The marks can be arranged in ascending order as 3,3,4,5,7,9,10,12,17,19,21.

   Range = maximum value – minimum value

   =21-3

   = 18

2. Given C.V (first distribution) = 70

   Standard deviation $= \sigma_1 = 28$

   $$C.V \ \frac{\sigma 1}{\bar{x}1}\times 100$$

   $$= 70 = \frac{28}{\bar{x}1}\times 100$$

   $$\bar{x} = \frac{28}{70}\times 100$$

   $$\bar{x} = 40$$

   Similarly for second distribution

   $$C.V = \frac{\sigma_2}{x_2}\times 100$$

   $$75 = \frac{27}{\bar{x}_2}\times 100$$

   $$\bar{x}_2 = \frac{27}{75}\times 100$$

   $$\bar{x}_2 = 36$$

3.

$$\text{MD}\left(\bar{x}\right) = \frac{\Sigma f_i\,|x_i - \bar{x}|}{\Sigma f_i} = \frac{1}{x}\Sigma f_i\,|x_i - \bar{x}|$$

4.

Variance $\sigma^2 = \frac{1}{n}\Sigma f_i\left(x_i - \bar{x}\right)^2$

5.

x_i	**5**	**7**	**9**	**10**	**12**	**15**
f_i	8	6	2	2	2	6
$c.f$	8	14	16	18	20	26

n = 26Median is the average of 13th and 14th item, both of which lie in the c.f 14

$\therefore x_i = 7$

$\therefore \text{ median} = \frac{13\text{observation} + 14\text{th observation}}{2}$

$= \frac{7+7}{2} = 7$

6.

$MD.(M) = \frac{\Sigma f_i \mid x_i M \mid}{\Sigma f_i} = \frac{1}{n}\Sigma f_i \mid x_i - M \mid$

7. Range = maximum value – minimum value

= 113-4

= 9

8.

$\text{Mean} = \frac{\text{sun of observation}}{\text{Total no of observation}}$

$= \frac{50}{5} = 10$

9. Let

$Z = \cancel{3i} - \cancel{7} + \cancel{7} - 4i - 6 - \cancel{3i} + \left(i^4\right)^5 . i^3$

$= -4i - 6 - i \left[\begin{array}{l} \because i^4 = 1 \\ i^3 = -i \end{array}\right.$

$= -5i - 6$

$= -6 + (-5i)$

10.

$\text{Let } z = \sqrt{-3} + 4i^2$

$=\sqrt{3}\,i - 4$

$\bar{z} = -\sqrt{3}\,i - 4$

Short Answer:

1. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers n = 6

The near of these observations is 7

$\frac{2+7+4+6+8+p}{6} = 7$

$= 27 + p = 42$

$= p = 15$

Arrange the observations in ascending order 2,4,6,7,8,15

$\therefore \text{Medias (M)} = \frac{\frac{n}{2}\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$

$= \frac{\text{3rd observation} + \text{4th observation}}{2}$

$= \frac{6+7}{2} = \frac{13}{2}$

$= 6.5$

Calculation of mean deviation about Median.

xi	xi-M	|xi-M|
2	-4.5	4.5
4	-2.5	2.5
6	-0.5	0.5
7	0.5	0.5
8	1.5	1.5
15	8.5	8.5
Total		18

$\therefore \text{Media's deviation about median} = \frac{\overset{3}{\cancel{18}}}{\cancel{6}} = 3$

2. To calculate mean, we require $f_i x_i$ values then for mean deviation, we require $|x_i - \bar{x}|$ values and $f_i|x_i - \bar{x}|$ values.

xi	f_i	$f_i xi$	$\|xi - \bar{x}\|$	$fi\|xi - \bar{x}\|$

10	4	4	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$n = \sum f_i = 80 \quad \sigma d \sum f_i xi = 4000$

$\bar{x} = \frac{\sum f_i xi}{n} = \frac{4000}{80} = 50$

Mean deviation about the mean

$\text{MD}\left(\bar{x}\right) = \frac{\sum f_i \,|\, xi - \bar{x} \,|}{n} = \frac{1280}{80} = 16$

3. The given numbers are 1, 2, 3, ……, n

Mean

$\bar{x} = \frac{\sum n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$

Variance

$\sigma 2 = \frac{\sum xi^2}{n} - \bar{x}$

$= \frac{\sum n^2}{n} - \left(\frac{n+1}{2}\right)^2$

$= \frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4}$

$= (n+1)\left[\frac{2n+1}{6} - \frac{n+1}{4}\right]$

$= (n+1)\left(\frac{n-1}{12}\right) = \frac{n^2-1}{12}$

$\therefore$ Standard deviation $\sigma = \frac{\sqrt{n^2-1}}{12}$

4.

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**

f_i	3	5	9	5	4	3	1

Note: - 4th, 5th and 6th columns are filled in after calculating the mean.

xi	f_i	$f_i x_i$	$xi - \bar{x}$	$(xi - \bar{x})^2$	$f_i x_i (xi - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	402			1374

Here $n = \sum f_i = 30, \quad \sum f_i x_i = 42$

$\therefore$ Mean $\bar{x} = \dfrac{\sum f_i x_i}{n} = \dfrac{420}{30} = 14$

$\therefore$ Variance $\sigma^2 = \dfrac{1}{n} \sum f_i (x_i - \bar{x})^2$

$= \dfrac{1}{30} \times 1374$

$= 45.8$

$\therefore$ Standard deviation $\sigma = \sqrt{45.8}$

$= 6.77$

5. Let $x_i, x_2 \ldots\ldots x_6$ be the six given observations

Then $\bar{x} = 8$ and $\sigma = 4$

$\bar{x} = \dfrac{\sum x_i}{n} = 8 = \dfrac{x_1 + x_2 + \ldots\ldots + x_6}{6}$

$x_1 + x_2 + \ldots\ldots x_6 = 48$

Also $\sigma^2 \frac{\sum x_1^2}{n} - \left(\bar{x}\right)^2$

$= 4^2 = \frac{x_1^2 + x_2^2 ......+ x_6^2}{6} - (8)^2$

$= x_1^2 + x_2^2 + ...x_6^2$

$= 6 \times (16 + 64) = 480$

As each observation is multiplied by 3, new observations are

$3x_1, 3x_2, ...3x_6$

New near $\overline{X} = \frac{3x_1 + 3x_2 + ...3x_6}{6}$

$= \frac{3(x_1 + x_2 + ...x_6)}{6}$

$= \frac{3 \times 48}{6}$

$= 24$

Let σ_1 be the new standard deviation, then

$\sigma_1^2 = \frac{(3x_1)^2 + (3x_2)^2 + ...+ (3x_6)^2}{6} - \left(\overline{X}\right)^2$

$= \frac{9(x_1^2 + x_2^2 + ...x_6^2)}{6} - (24)^2$

$= \frac{9 \times 480}{6} - 576$

$= 720 - 576$

$= 144$

$\sigma_1 = 12$

Long Answer:

1.

Classes	Frequency	Mid Point	$f_i xi$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135

70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
Total	50		3100		10050

Here $n=\sum f_i=50, \sum f_i x_i=3100$

$\therefore$ Mean $\bar{x}=\frac{\sum f_i x_i}{n}=\frac{3100}{50}=62$

Variance $\sigma^2=\frac{1}{n}\sum f_i\left(xi-\bar{x}\right)^2$

$=\frac{1}{50}\times 10050$

$=201$

Standard deviation $\sigma=\sqrt{201}=14.18$

2. Given that n = 100

Incorrect mean $\bar{x}=40$

Incorrect S.D $(\sigma)=5.1$

As $\bar{x}=\frac{\sum x_i}{n}$

$40=\frac{\sum x_i}{100}=\sum x_i=4000$

= incorrect sum of observation =4000

= correct sum of observations = 4000 – 50 + 40

= 3990

So correct mean = $\frac{3990}{100}=39.9$

Also $\sigma=\sqrt{\frac{1}{n}\sum x_i^2-\left(\bar{x}\right)^2}$

Using incorrect values,

$5.1=\sqrt{\frac{1}{100}\sum x_i^2-(40)^2}$

$=26.01=\left[\frac{1}{100}\sum x_i^2-1600\right]$

$=\sum x_i^2=2601+160000$

= 162601

$\sum x_i^2=162601$

= incorrect

$= \text{correct } \sum x_i^2 = 162601 - (50)^2 + (40)^2$

= 162601 - 2500 + 1600 = 161701

$\therefore \text{Correct } \sigma = \sqrt{\frac{1}{100}\text{correct}\sum x_i^2 - (\text{correct } \bar{x})^2}$

$= \sqrt{\frac{1}{100}(161701) - (39.9)^2} = \sqrt{1617.01 - 1592.01}$

$= \sqrt{25} = 5$

Hence, correct mean is 39.9 and correct standard deviation is 5.

3.

$n = 200, \overline{X} = 40, \sigma = \overline{15}$

$\overline{X} = \frac{1}{n}\sum x_i = \sum x_i = n\overline{X} = 200 \times 40 = 8000$

Corrected $\sum x_i$ = Incorrect $\sum x_i$ – (sum of incorrect +sum of correct value)

= 8000-34+43 = 8009

$\therefore$ Corrected mean = $\frac{\text{corrected} \sum x_i}{n} = \frac{8009}{200} = 40.045$

$\sigma = 15$

$15^2 = \frac{1}{200}\left(\sum x_i^2\right) - \left(\frac{1}{200}\sum x_i\right)^2$

$225 = \frac{1}{200}\left(\sum x_i^2\right) - \left(\frac{8000}{200}\right)^2$

$225 = \frac{1}{200} \times 1825 = 365000$

Incorrect $\sum x_i^2 = 365000$

Corrected $\sum x_i^2$ = (incorrect $\sum x_i^2$) – (sum of squares of incorrect values) + (sum of square of correct values)

$= 365000 - (34)^2 + (43)^2 = 365693$

Corrected $\sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\frac{1}{n}\sum x_i\right)^2} = \sqrt{\frac{365693}{200} - \left(\frac{8009}{200}\right)^2}$

$\sqrt{1828.465 - 1603.602} = 14.995$

4. Let $\overline{X}$ be the mean

$$\overline{X} = \frac{6+7+10+12+13+4+8+20}{8} = 10$$

x_i	$\lvert d_i \rvert = \lvert x_i - \overline{X} \rvert = \lvert x_i - 10 \rvert$
6	4
7	3
10	0
12	2
13	3
4	6
8	2
20	10
Total	$\sum d_i = 30$

$\sum d_i = 30$ and n = 8

$$\therefore MD = \frac{1}{n}\sum \lvert d_i \rvert = \frac{30}{8} = 3.75$$

$$\therefore MD = 3.75$$

5. Let x and y be the no.

x + y = 6

xy = 14

$$x^2 - 6x + 14 = 0$$

$$D = -20$$

$$x = \frac{-(-6) \pm \sqrt{-20}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{5}\,i}{2}$$

$$= 3 \pm \sqrt{5}\,i$$

$$x = 3 + \sqrt{5}\,i$$

$$y = 6 - \left(3 + \sqrt{5}\,i\right)$$

$= 3 - \sqrt{5}\, i$

when $x = 3 - \sqrt{5}\, i$

$y = 6 - \left(3 - \sqrt{5}\, i\right)$

$= 3 + \sqrt{5}\, i$

Assertion Reason Answer:

1. (i) Both assertion and reason are true and reason is the correct explanation of assertion.
2. (iii) Assertion is true but reason is false.