MATHEMATICS

Chapter 16: PROBABILITY

Important Questions

Multiple Choice questions-

Question 1. Events A and B are independent if

(a) $P(A \cap B) = P(A/B)\, P(B)$

(b) $P(A \cap B) = P(B/A)\, P(A)$

(c) $P(A \cap B) = P(A) + P(B)$

(d) $P(A \cap B) = P(A) \times P(B)$

Question 2. A single letter is selected at random from the word PROBABILITY. The probability that it is a vowel is

(a) 2/11

(b) 3/11

(c) 4/11

(d) 5/11

Question 3. A die is rolled, find the probability that an even prime number is obtained

(a) 1/2

(b) 1/3

(c) 1/4

(d) 1/6

Question 4. When a coin is tossed 8 times getting a head is a success. Then the probability that at least 2 heads will occur is

(a) 247/265

(b) 73/256

(c) 247/256

(d) 27/256

Question 5. The probability that the leap year will have 53 sundays and 53 monday is

(a) 2/3

(b) 1/2

(c) 2/7

(d) 1/7

Question 6. Let A and B are two mutually exclusive events and if P(A) = 0.5 and $P(\bar{B})$ =0.6

then P(AUB) is

(a) 0

(b) 1

(c) 0.6

(d) 0.9

Question 7. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) 1/2

(b) 7/15

(c) 2/15

(d) 1/3

Question 8. The events A, B, C are mutually exclusive events such that P (A) = (3x + 1)/3, P (B) = (x – 1)/4 and P (C) = (1 – 2x)/4. The set of possible values of x are in the interval

(a) [1/3, 1/2]

(b) [1/3, 2/3]

(c) [1/3, 13/3]

(d) [0, 1]

Question 9. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. The probability that none of the balls drawn is blue is

(a) 10/21

(b) 11/21

(c) 2/7

(d) 5/7

Question 10. If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5 and 7, then the probability of forming a number divisible by 5 when the digits are repeated is

(a) 1/5

(b) 2/5

(c) 3/5

(d) 4/5

Very Short Questions:

1. Three coins are tossed simultaneously list the sample space for the event.

2. Two dice are thrown simultaneously. Find the prob. of getting doublet.
3. 20 cards are numbered from 1 to 20. One card is then drawn at random. What is the prob. of a prime no.
4. If $\frac{3}{10}$ is the prob. that an event will happen, what is the prob. that it will not happen?
5. If A and B are two mutually exclusive events such that

   P (A) $= \frac{1}{2}$ and

   P (B) $= \frac{1}{3}$ find P (A or B)
6. If E and F are events such that P(E) $= \frac{1}{4}$ P(F) $= \frac{1}{2}$ and P(E and F) $= \frac{1}{8}$ find P(not E and not F)
7. A letter is chosen at random from the word 'ASSASSINATION'. Find the prob. that letter is a consonant.
8. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that at it is a woman?
9. 4 cards are drawn from a well snuffled deck of 52 cards what is the prob. of obtaining 3 diamonds and one spade.
10. Describe the sample space. A coin is tossed and a die is thrown.

Short Questions:

1. A coin is tossed three times consider the following event A : No head appears, B : Exactly one head appears and C : At least two heads appears do they form a set of mutually exclusive and exhaustive events.
2. A and B are events such that P (A) = 0.42, P (B) = 0.48, and P (A and B) = 0.16 determine (i) P (not A) (ii) P (not B) (iii) P (A or B)
3. Find the prob. that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains (i) all king (ii) 3 kings (iii) at least 3 kings
4. From a group of 2 boys and 3 girls, two children are selected at random. Describes the sample space associated with

   (i) E_1 : both the selected children are boys

   (ii) E_2 : at least one selected child is a boy

   (iii) E_3 : one boy and one girl is selected

   (iv) E_4 : both the selected children are girls
5. A book contains 100 pages. A page is chosen at random. What is the chance that the sum of the digit on the page is equal to 9.

Long Questions:

1. Three letters are dictated to three persons and an envelope is addressed to each of them, those letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the prob. that at least one letter is in its proper envelope.
2. If 4 digit no. greater than 5,000 are randomly formed the digits 0,1,3,5 and 7 what is the probability of forming a no. divisible by 5 when

   (i) The digits are repeated (ii) The repetition of digits is not allowed.
3. 20 cards are numbered from 1 to 20. One card is drawn at random what is the prob. that the no. on the card drawn is

   (i) A prime no. (ii) An odd no. (iii) A multiple of 5 (iv) Not divisible by 3.
4. In a single throw of three dice, find the prob. of getting

   (i) A total of 5 (ii) A total of at most 5.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The probability of a sure event is 1.

   Reason (R) : Let E be an event. Then, $0 \leq P(E) \leq 1$.

   (i) Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

   (ii) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

   (iii) Assertion is true and Reason is false.

   (iv) Assertion is false and Reason is true.
2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : If the probability of winning a game is $\frac{8}{18}$, then the probability of losing the game is $\frac{7}{15}$.

   Reason (R) For any event E, we have P(E) + P(not E) = 1.

   (i) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

   (ii) Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

   (iii) Assertion (A) is true and Reason (R) is false.

   (iv) Assertion (A) is false and Reason (R) is true.

Answer Key:

MCQ:

1. (d) P (A ∩ B) = P (A) × P (B)

2. (b) 3/11

3. (d) 1/6

4. (c) 247/256

5. (d) 1/7

6. (d) 0.9

7. (b) 7/15

8. (a) [1/3, 1/2]

9. (a) 10/21

10.(b) 2/5

Very Short Answer:

1. S = HHH, HHT, HTH, THH, HTT, TTH, THT, TTT.

2. n = (s) = 36 S be the sample space

let E be the event of getting doublet

$$P(E)=\frac{6}{36}\left[\because E=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}\right]$$

$$=\frac{1}{6}$$

3. Let S be the sample space and E be the event of prime no.

$$n(s)=\{1,2,3,.....20\}$$

$$n(E)=\{2,3,5,7,11,13,17,19\}$$

$$P(E)=\frac{n(E)}{n(S)}=\frac{8}{20}=\frac{2}{5}$$

4. Let E be the event

$$P(E)=\frac{3}{10}$$

$$P(E)=1-P(E)$$

$$=1-\frac{3}{10}$$

$$=\frac{7}{10}$$

5.

$$P(A\ or\ B)=P(A)+P(B)-p(A\cap B)$$

$$=\frac{1}{2}+\frac{1}{3}-\phi\ [P(A\cap B)=\phi]$$

$$=\frac{5}{6}$$

6.

$$P(E'\cap F')=P(E\cup F)'$$

$$=1-P(E\cup F)\left[\because P(E\cup F)=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}\right]$$

$$=1-\frac{5}{8}$$

$$=\frac{3}{8}$$

7. P (consonant) = $\frac{7}{13}$

8. P (a woman member is selected) = $\frac{6}{10}=\frac{3}{5}$

9.

$$\frac{{}^{13}C_3\times{}^{13}C_1}{{}^{52}C_4}=\frac{286}{20825}\left[\begin{array}{l}\because\ 3\ \text{Spades out of } 13\\ \text{and one ace out of } 13\end{array}\right]$$

10.{H1,H2,H3,H4,H5,H6,T1,T2,T3,T4,T5,T6}

Short Answer:

1. Observations are 2, 7, 4, 6, 8 and p which are 6 in numbers n = 6

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A = {TTT}, B = {HTT, THT, TTH}, C = {HHT, HTH, THH, HHH}

$A \cup B \cup C = S$

Therefore A, B and C are exhaustive events.

Also $A \cap B = \phi, A \cap C = \phi, C \cap C = \phi,$ disjoint i.e. they are mutually exclusive.

2.

$P(notA) = 1 - p(A) = 1 - 0.42 = 0.58$

$P(notB) = 1 - p(B) = 1 - 0.48 = 0.52$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= 0.42 + 0.48 - 0.16$

$= 0.74$

3.

$$\text{P (all king)} = \frac{{}^{4}C_{4} \times {}^{48}C_{3}}{{}^{52}C_{7}} = \frac{1}{7735}$$

$$\text{P (3 king)} = \frac{{}^{4}C_{3} \times {}^{48}C_{4}}{{}^{52}C_{7}} = \frac{9}{1547}$$

P (at least 3 king) = p (3 king) + p (4 king)

$$= \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$$

4.

$S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$

$E_1 = \{B_1B_2\}$

$E_2 = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

$E_3 = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

$E_4 = \{G_1G_2, G_1G_3, G_2G_3\}$

5.

$E = \{9, 18, 27, 36, 45, 54, 63, 72, 81, 90\}$

$S = 100$

$P(E) = \frac{10}{100}$

$= \frac{1}{10}$

Long Answer:

1. Let the tree letters be denoted by A_1 A_2 and A_3 and three envelopes by E_1 E_2 and E_3.

Total No. of ways to putting the letter into three envelopes is $3P_3 = 6$

No. of ways in which none of the letters is put into proper envelope = 2

Req. prob.

P (at least one letters is put into proper envelope) =1- P (none letters is put into proper envelopes)

$$= 1 - \frac{2}{6}$$

$$= \frac{2}{3}$$

2. (i)

Thousand	H	T	U
5,7			

For a digit greatest then 5000 Thousand Place filled in 2 ways and remaining three place be filled in 5 ways

No. 40. can be formed = 2 × 5 × 5 × 5 - 1 = 249

ATQ

Thousand	H	T	U
5,7			0,5

If no. is divisible by 5

Unit place filled in 2 ways and thousand place also by 2 ways (5, 7)

No. formed =2 × 5 × 5 × 2 - 1 = 99

Req. prob. $\frac{99}{249}$

(ii) Digit not repeated

Thousand	H	T	U
5,7			

Thousand place filled in 2 ways

4 digit no. greater than 5 thousand = 2 × 4 × 3 × 2 = 48

Thousand	H	T	U
5			0
7			5,0

Favorable case = 1 × 3 × 2 × 2 + 1 × 3 × 2 × 1

7 at thousand place 5 at thousand places

= 12 + 6 = 18

Req. prob. $= \frac{18}{48} = \frac{3}{8}$

3. Let S be the sample space

S = {1,2,3,4,5,……20}

Let E_1, E_2 and E_3, E_4 are the event of getting prime no., an odd no, multiple of 5 and not divisible by 3 respectively

$$P(E_1) = \frac{8}{20} = \frac{2}{5}, \quad E_1 = \{2,3,5,7,11,13,17,19\}$$

$$P(E_2) = \frac{10}{20} = \frac{1}{2}, \quad E_2 = \{1,3,5,7,9,11,13,15,17,19\}$$

$$P(E_3) = \frac{4}{20} = \frac{1}{5}, \quad E_3 = \{5,10,15,20\}$$

$$P(E_4) = \frac{14}{20} = \frac{7}{10}, \quad E_4 = \{1,2,4,5,7,8,10,11,13,14,16,17,19,20\}$$

4. Let S be the sample space E_1 be the event of total of 5.

(i) $E_1 = \{(1,1,3),(1,3,1),(3,1,1),(1,2,2),(2,1,2),(2,2,1)\}$

$S = 6 \times 6 \times 6 = 216$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{216} = \frac{1}{26}$$

(ii) $E_2 = \{(1,1,1),(1,1,2),(1,2,1),(2,1,1),(1,1,3),(1,3,1),(3,1,1),(1,2,2),(2,1,2),(2,2,1)\}$

$$P(E_2) = \frac{10}{216} = \frac{5}{108}$$

Assertion Reason Answer:

1. (ii) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

2. (i) Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).