MATHEMATICS

Important Questions

Multiple Choice questions-

1. HCF of 8, 9, 25 is

(a) 8

(b) 9

(c) 25

(d) 1

2. Which of the following is not irrational?

(a) $(2 - \sqrt{3})^2$

(b) $(\sqrt{2} + \sqrt{3})^2$

(c) $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$

(d) $\frac{2\sqrt{7}}{7}$

3. The product of a rational and irrational number is

(a) rational

(b) irrational

(c) both of above

(d) none of above

4. The sum of a rational and irrational number is

(a) rational

(b) irrational

(c) both of above

(d) none of above

5. The product of two different irrational numbers is always

(a) rational

(b) irrational

(c) both of above

(d) none of above

6. The sum of two irrational numbers is always

(a) irrational

(b) rational

(c) rational or irrational

(d) one

7. If b = 3, then any integer can be expressed as a =

(a) 3q, 3q+ 1, 3q + 2

(b) 3q

(c) none of the above

(d) 3q+ 1

8. The product of three consecutive positive integers is divisible by

(a) 4

(b) 6

(c) no common factor

(d) only 1

9. The set A = {0,1, 2, 3, 4, …} represents the set of

(a) whole numbers

(b) integers

(c) natural numbers

(d) even numbers

10. Which number is divisible by 11?

(a) 1516

(b) 1452

(c) 1011

(d) 1121

Very Short Questions:

1. What is the HCF of the smallest composite number and the smallest prime number?

2. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

3. If HCF of a and b is 12 and product of these numbers is 1800. Then what is LCM of these numbers?

4. What is the HCF of $3^3 \times 5$ and $3^2 \times 5^2$?

5. if a is an odd number, b is not divisible by 3 and LCM of a and b is P, what is the LCM of 3a and 2b?

6. If P is prime number then, what is the LCM of P, P^2, P^3?

7. Two positive integers p and q can be expressed as $p = ab^2$ and $q = a^2b$, a and b are prime numbers. What is the LCM of p and q?

8. A number N when divided by 14 gives the remainder 5. What is the remainder when the same number is divided by 7?

9. Examine whether $\frac{17}{30}$ is a terminating decimal or not.

10. What are the possible values of remainder r, when a positive integer a is divided by 3?

11. A rational number in its decimal expansion is 1.7351. What can you say about the prime factors of q when this number is expressed in the form $\frac{p}{q}$? Give reason.

12. Without actually performing the long division, find $\frac{987}{10500}$ will have terminating or non. terminating repeating decimal expansion. Give reason for your answer.

Short Questions :

1. Can the number 4^n, n be a natural number, end with the digit 0? Give reason.

2. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

3. Can two numbers have 18 as their HCF and 380 as their LCM? Give reason.

4. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

5. Find the LCM and HCF of 12, 15 and 21 by applying the prime factorisation method.

6. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(1) 26 and 91 (ii) 198 and 144

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

8. Write down the decimal expansiwns of the following numbers:

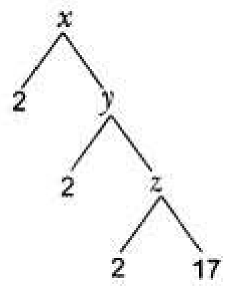
(i) $\frac{35}{50}$ (ii) $\frac{15}{1600}$

9. Express the number $\overline{0.3178}$ in the form of rational number $\frac{a}{b}$.

10. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.

11. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

12. Find the value of x, y and z in the given factor tree. Can the value of ‘x’ be found without finding the value of ‘y’ and ‘z’? If yes, explain.



13. Show that any positive odd integer is of the form 6q + 1 or 6q + S or 6q + 5 where q is some integer.

14. The decimal expansions of some real numbers are given below. In each case, decide whether they are rational or not. If they are rational, write it in the form $\frac{p}{q}$. What can you say about the prime factors of q?

(i) 0.140140014000140000... (ii) $\overline{0.16}$

Long Questions :

1. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

2. Show that one and only one out of n, n + 2, n + 4 is divisible by 3, where n is any positive integer.

3. Use Euclid's division algorithm to find the HCF of:

(i) 960 and 432

(ii) 4052 and 12576.

4. Using prime factorisation method, find the HCF and LCM of 30, 72 and 432. Also show that HCF × LCM ≠ Product of the three numbers.

5. Prove that √7 is an irrational number.

6. Show that 5 – √3 is an irrational number.

7. Using Euclid's division algorithm, find whether the pair of numbers 847,2160 are co-prime or not.

8. Check whether 6^n can end with the digit O for any natural number n.

9. Show that there is iw positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

10. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Case Study Questions:

1. Srikanth has made a project on real numbers, where he finely explained the applicability of exponential laws and divisibility conditions on real numbers. He also included some assessment questions at the end of his project as listed below. Answer them.

i. For what value of n, 4n ends in 0?

a. 10

b. When n is even.

c. When n is odd.

d. No value of n.

ii. If a is a positive rational number and n is a positive integer greater than I, then for what value of n, 4n is a rational number?

a. When n is any even integer.

b. When n is any odd integer.

c. For all n > 1.

d. Only when n = 0.

iii. If x and y are two odd positive integers, then which of the following is true?

a. $x^2 + y^2$ is even.

b. $x^2 + y^2$ is not divisible by 4.

c. $x^2 + y^2$ is odd.

d. Both (a) and (b).

iv. The statement 'One of every three consecutive positive integers is divisible by 3' is:

a. Always true.

b. Always false.

c. Sometimes true.

d. None of these.

v. If n is any odd integer, then n2 - 1 is divisible by:

a. 22

b. 55

c. 88

d. 8

2. Real numbers are extremely useful in everyday life. That is probably one of the main reasons we all learn how to count and add and subtract from a very young age. Real numbers help us to count and to measure out quantities of different items in various fields like retail, buying, catering, publishing etc. Every normal person uses real numbers in his daily life. After knowing the importance of real numbers, try and improve your knowledge about them by answering the following questions on real life based situations.

i. Three people go for a morning walk together from the same place. Their steps measure 80cm, 85cm and 90cm respectively. What is the minimum distance travelled when they meet at first time after starting the walk assuming that their walking speed is same?

a. 6120cm
b. 12240cm
c. 4080cm
d. None of these

ii. In a school Independence Day parade, a group of 594 students need to march behind a band of 189 members. The two groups have to march in the same number of columns. What is the maximum number of columns in which they can march?

a. 9
b. 6
c. 27
d. 29

iii. Two tankers contain 768 litres and 420 litres of fuel respectively. Find the maximum capacity of the container which can measure the fuel of either tanker exactly.

a. 4 litres
b. 7 litres
c. 12 litres

d. 18 litres

iv. The dimensions of a room are 8m, 25cm, 6m, 75cm and 4m, 50cm. Find the length of the largest measuring rod which can measure the dimensions of room exactly.

a. 1m, 25cm
b. 75cm
c. 90cm
d. 1m, 35cm

v. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

a. 3 and 2
b. 2 and 5
c. 3 and 4
d. 4 and 5

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false but R is true.

Assertion: 11 × 4 × 3 × 2 + 4 is a composite number.

Reason: Every composite number can be expressed as product of primes.

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
   (a) Both A and R are true and R is the correct explanation of A.
   (b) Both A and R are true and R is not the correct explanation of A.
   (c) A is true but R is false.
   (d) A is false but R is true.

Assertion: If LCM = 350, product of two numbers is 25 × 70, then their HCF = 5

Reason: LCM × product of numbers = HCF

Answer Key-

Multiple Choice questions-

1. (d) 1
2. (c) $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$
3. (b) irrational
4. (b) irrational
5. (b) irrational
6. (a) irrational
7. (a) 3q, 3q+ 1, 3q + 2
8. (b) 6
9. (a) whole numbers
10. (b) 1452

Very Short Answer :

1. Smallest composite number = 4

Smallest prime number = 2

So, HCF (4, 2) = 2

2.

$$\frac{6}{1250} = \frac{3}{625} = \frac{3}{5^4} \times \frac{2^4}{2^4} = \frac{48}{(5 \times 2)^4} = \frac{48}{10^4} = 0.0048$$

This representation will terminate after 4 decimal places.

3. Product of two numbers = Product of their LCM and HCF

⇒ 1800 = 12 × LCM

⇒ LCM $= \frac{1800}{12} = 150.$

4. HCF of $3^3 \times 5$ and $3^2 \times 5^2 = 3^2 \times 5 = 45$

5. 6P

6. P^3

7. a^2h^2

8. 5, because 14 is multiple of 7.

Therefore, remainder in both cases are same.

9.

$$\frac{17}{30} = \frac{17}{2\times3\times5}$$

Since the denominator has 3 as its factor.

$\therefore \frac{17}{30}$ is a non4ermznatlng decimal.

10. According to Euclid's division lemma

a = 3q + r, where O r < 3 and r is an integer.

Therefore, the values of r can be 0, 1 or 2.

11. As 1.7351 is a terminating decimal number, so q must be of the form $2^m 5^n$, where in, n are natural numbers.

12. $\frac{987}{10500} = \frac{47}{500}$ and 500 = 22 × 53, so it has terminating decimal expansion.

Short Answer :

1. if 4^n ends with 0, then it must have 5 as a factor. But, $(4)^n = (2^2)^n = 2^{2n}$ i.e., the only prime factor.

   of 4^n is 2. Also, we know from the fundamental theorem of arithmetic that the prime factorization of each number is unique.

   $\therefore 4^n$ can never end with 0.

2. No, because any positive integer can be written as 3q, 3q + 1, 3q + 2, therefore, square will be

   $9q^2 = 3m$, $9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$,

$9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3m + 1$.

3. No, because here HCF (18) does not divide LCM (380).

4. For the maximum number of columns, we have to find the HCF of 616 and 32.

Now, since 616 > 32, we apply division lemma to 616 and 32.

We have, 616 = 32 × 19 + 8

Here, remainder 8 ≠ O. So, we again apply division lemma to 32 and 8.

We have, 32 = 8 × 4 + O

Here, remainder is zero. So, HCF (616, 32) = 8

Hence, maximum number of columns is 8.

5. The prime factors of 12, 15 and 21 are

$12\ 2^2 \times 3$, $15 = 3 \times 5$ and $21 = 3 \times 7$

Therefore, the HCF of these integers is 3.

2^2, 3^1, 5^1 and 7^1 and are the greatest powers involved in the prime factors of 12, 15 and 21.

So, LCM (12, 15, 21) = $2^2 \times 3^1 \times 5^1 \times 7^1 = 420$.

6. (i) We have, 26 = 2 × 13 and 91 = 7 × 13

Thus, LCM (26, 91) = 2 × 7 × 13 = 182

HCF (26, 91) = 13

Now, LCM (26, 91) × HCF (26, 91) = 182 × 13 = 2366

and Product of the two numbers = 26 × 91 = 2366

Hence, LCM × HCF = Product of two numbers.

(ii) 144 = 24 × 32 and 198 = 2 × 32 × 11

∴ LCM(198,144)24 × 32 × 11 = 1584

HCF(198, 144) = 2 × 32 = 18

Now, LCM (198, 144) × HCF (198, 144) = 1584 × 18 = 28512 and product of 198 and 144 = 28512

Thus, product of LCM (198, 144) and HCF (198, 144)

= Product of 198 and 144.

7. To find the time after which they meet again at the starting point, we have to find LCM of 18 and 12 minutes. We have

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

Therefore, LCM of 18 and 12 = $2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.

8. (i)

We have, $\frac{35}{50} = \frac{35}{5^2 \times 2} = \frac{35 \times 2}{5^2 \times 2 \times 2} = \frac{70}{5^2 \times 2^2}$

$= \frac{70}{10^2} = \frac{70}{100} = 0.70$

(ii)

We have, $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^4 \times 2^2 \times 5^2 \times 5^4} = \frac{15 \times 625}{2^6 \times 5^6}$

$= \frac{9375}{10^6} = \frac{9375}{1000000} = 0.009375$

9. Let x = $\overline{0.3178}$

then x = 0.3178178178 … … (i)

10x = 3.178178178 … …(ii)

10000x = 3178.178178… …(iii)

On subtracting (ii) from (iii), we get

$$9990x = 3175 \Rightarrow x = \frac{3175}{9990} = \frac{635}{1998}$$

$$\therefore 0.3\overline{178} = \frac{635}{1998}$$

10. We know that an odd positive integer n is of the form (4q + 1) or (4 + 3) for some integer q.

Case – I When n = (4q + 1)

In this case $n^2 - 1 = (4q + 1)^2 1 = 16q^2 + 8q = 8q(2q + 1)$

which is clearly divisible by 8.

Case – II When n = (4q + 3)

In this case, we have

$n2^2 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

which is clearly divisible by 8.

Hence $(n^2 - 1)$ is divisible by 8.

11. Let HCF of the numbers hex then according to question LCM of the number will be 14x

And x + 14 = 600 ⇒ 15x = 600 ⇒ x = 40

Then HCF = 40 and LCM = 14 × 40 = 560

∵ LCM × HCF = Product of the numbers

560 × 40 = 280 × Second number Second number $= \frac{560 \times 40}{280} = 80$

Then other number is 80.

12. z = 2 × 17 = 34; y = 34 × 2 = 68 and x = 2 × 68 = 136

Yes, value of x can be found without finding value of y or z as

x = 2 × 2 × 2 × 17 which arc prime 1tctors of x.

13. Let a he any positive odd integer and h = 6. Then, by Euclid's algorithm, a = 6q + r, for some

integer q ≥ O and O ≤ r < 6.

i.e., the possible remainders are 0, 1, 2, 3, 4, 5.

1'hus,a canbeoftheform6q,or6q + l,or6q + 2,orßq + 3,ör6q + 4,

or 6q + 5, where q is some quotient.

Since a. is odd integer, so a cannot be of the form , or + 2, or 6q + 4, (since they are even).

Thus, a is of the form 6q + 1, 6q + 3, or 6q + 5, where q is some integer.

Hence, any odd positive integer is of the form 6q + 1 or 6q -1- 3 or 6q + 5, where q is sorne integer.

14. (i) We have, 0.140140014000140000... a non-terminating and non-repeating decimal expansion. So it is irrational. It cannot be written in the form of $\frac{p}{q}$

(ii) We have, $\overline{0.16}$ a non-terminating but repeating decimal expansion. So it is rational.

Let x = $\overline{0.16}$

Then, x = 0.1616... (i)

100x16.1616... ..(ii)

On subtracting (i) from (ii), we get

100x – x = 16.1616 – 0.1616

⇒ 99x = 16 ⇒ x = $\frac{16}{99} = \frac{p}{q}$

The denominator (q) has factors other than 2 or 5.

Long Answer :

1. Let a be an arbitrary positive integer.

Then by Euclid's division algorithm, corresponding to the positive integers a and 3 there exist

non-negative integers q and r such that

a = 3q + r where 0 ≤ r <3

$a^2 = 9q^2 + 6qr + r^2$ ....(i) 0 ≤ r <3

Case – I: When r = 0 [putting in (i)]

$a^2 = 9q^2 = 3(3q^2) = 3m$ where $m = 3q^2$

Case – II: r = 1

$a^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q)+ 1 = 3m + 1$ where $m = 3q^2 + 2q$

Case – III: r = 2

$a^2 = 9q^2 + 12 + 4 = 3(3q^2 + 4q + 1) + 1 = 3m + 1$ where $m = (3q^2 + 4q + 1)$

Hence, square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

2. Let q be the quotient and r be the remainder when n is divided by 3.

Therefore, n = 3q + r, where r = 0, 1, 2

n = 3q or n = 3q + 1 or n = 3q + 2

Case (i) if n = 3q, then n is divisible by 3, n + 2 and n +. 4 arc not divisible by 3.

Case (ii) if 71 = 3q + 1 then n + 2 = 3q + 3 = 3(q + 1), which is divisible by 3 and

n + 4 = 3q + 5, which is not divisible by 3.

So, only (n + 2) is divisible by 3.

Case (iii) If n = 3q + 2, then n + 2 = 3q + 4, which is not divisible by 3 and

(n + 4) = 3q + 6 = 3(q + 2), which is divisible by 3.

So, only (n + 4) is divisible by 3.

Hence one and only one out of n, (n + 2), (n + 4), is divisible by 3.

3. (j) Since 960 > 432, we apply the division lemma to 960 and 432.

We have, 960 = 432 × 2 + 96

Since the remainder 96 ≠ 0, so we apply the division lemma to 432 and 96.

We have, 432 = 96 × 4 + 48

Again remainder 48 ≠ 0 so we again apply division lemma to 96 and 48.

We have, 96 = 48 × 2 + O

The remainder has now become zero. So our procedure stops.

Since the divisor at this stage is 48.

Hence, HOE of 960 and 432 is 48.

i.e., HCF (960, 432) = 4H

(ii) Since 12576 > 4052, we apply the division lemma to 12576 and 4052, to get

12576 = 4052 × 3 + 420

Since the remainder 420 ≠ 0, we apply the division lemma to 4052 and 420, to get

4052 = 420 × 9 + 272

We consider the new divisor 420 and the new remainder 272, and apply the division lemma to get

420 = 272 × 1 + 148

We consider the new divisor 272 and the new remainder 148, and apply the division lemma to get

272 = 148 × 1 + 124

We consider the new divisor 148 and the new remainder 124, and apply the division lemma to get

148 = 124 × 1 + 24

We consider the new divisor 124 and the new remainder 24, and apply the division lemma to get

124 = 24 × 5 + 4

We consider the new divisor 24 and the new remainder 4, and apply the division lemma to get

24 = 4 × 6 + O

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 4, the HCF of 12576 and 4052 is 4.

4. Given members = 30, 72, 432 .

30 = 2 × 3 × 5; 72 = 23 × 32 and 432 = 24 × 33

Here, 2’ and 31 are the smallest powers of the common factors 2 and 3 respectively.

So, HCF (30, 72, 432) = 21 × 31 = 2× 3 = 6

Again, 2, 33 and 51 arc the greatest powers of the prime factors 2, 3 and 5 respectively.

So, LCM (30, 72, 432) = 24 × 33 × 51 = 2160

HCF × LCM = 6 × 2160 = 12960

Product of numbers = 30 × 72 × 432 = 933120 .

Therefore, HCF × LCM ≠ Product of the numbers.

5. Let us assume, to the contrary, that √7 is a rational number.

Then, there exist co-prime positive integers and such that

$\sqrt{7} = \frac{a}{b}$, b ≠ 0

So, a = √7 b

Squaring both sides, we have

$a^2 = 7b^2$ …… (i)

⇒ 7 divides a^2 ⇒ 7 divides a

So, we can write

a = 7c (where c is an integer)

Putting the value of a = 7c in (i), we have

$49c^2 = 7b^2$ $7^2 = b^2$

It means 7 divides b^2 and so 7 divides b.

So, 7 is a common factor of both a and b which is a contradiction.

So, our assumption that √7 is rational is wrong.

Hence, we conclude that √7 is an irrational number.

6. Let us assume that 5 – √3 is rational.

So, 5 – √3 may be written as

$5 - \sqrt{3} = \frac{p}{q}$, where p and q are integers, having no common factor except 1 and q ≠ 0.

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \Rightarrow \sqrt{3} = \frac{5q-p}{q}$

Since $\frac{5q-p}{q}$ is a rational number as p and q are integers.

∴ √3 is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, 5 – √3 is an irrational number.

7. Since 2160 > 847 we apply the division lemma to 2160 and 847

we have, 2160 847 × 2 + 466

Since remainder 466 ≠ 0. So, we apply the division lemma to 847 and 466

847 = 466 × 1+ 381

Again remainder 381 ≠ 0. So we again apply the division lemma to 466 and 381.

466 = 381 × 1 + 85

Again remainder 85 ≠0. So, we again apply the division lemma to 381 and 85

381 = 85 × 4 + 41

Again remainder 41 ≠ 0. So, we again apply the division lemma to 85 and 41.

85 = 41 × 2 + 3

Again remainder 3 ≠ 0. So, we again apply the division lemma to 4 1 and 3.

41 = 3 × 13 + 2

Again remainder 2 ≠ 0. So, we again apply the division lemma to 3 and 2.

3 = 2 × 1 + 1

Again remainder 1 ≠ 0. So, we apply division lemma to 2 and 1

2 = 1 × 2 + 0

The remainder now becomes O. So, our procedure stops.

Since the divisor at this stage is 1.

Hence, HCF of 847 and 2160 is 1 and numbers are co-prime.

8. If the number 6^n, for any n, were to end with the digit zero, then h would bc divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. But $6^n = (2 \times 3)^n = 2^n \times 3n$ So the primes in factorisation of 6n are 2 and 3. So the uniqueness of the Fundamental Theorem of Arithmetic guarantees that (here are no other primes except 2 and 3 in the factorisation of 6n. So there is no natural number n for which 6" ends with digit zero.

9. Let there be a positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ be rational number.

$$\sqrt{n-1} + \sqrt{n+1} = \frac{p}{q}\text{; where } p, q \text{ are integers and } q \neq 0 \qquad ...(i)$$

$$\Rightarrow \frac{1}{\sqrt{n-1}+\sqrt{n+1}} = \frac{q}{p} \qquad \Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{\left(\sqrt{n-1}+\sqrt{n+1}\right)\times\left(\sqrt{n-1}-\sqrt{n+1}\right)} = \frac{q}{p}$$

$$\Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)} = \frac{q}{p} \qquad \Rightarrow \frac{\sqrt{n-1}-\sqrt{n+1}}{n-1-n-1} = \frac{q}{p}$$

$$\Rightarrow \frac{\sqrt{n+1}-\sqrt{n-1}}{2} = \frac{q}{p} \qquad \Rightarrow \sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \qquad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$\sqrt{n-1}+\sqrt{n+1}+\sqrt{n+1}-\sqrt{n-1} = \frac{p}{q}+\frac{2q}{p}$$

$$\Rightarrow 2\sqrt{n+1} = \frac{p^2+2q^2}{pq} \qquad \Rightarrow \sqrt{n+1} = \frac{p^2+2q^2}{2pq}$$

$\Rightarrow$ $\sqrt{n+1}$ is rational number as $\frac{p^2+2q^2}{2pq}$ is rational

$\Rightarrow$ $\sqrt{n+1}$ is perfect square of positive integer ...(A)

Again subtracting (*ii*) from (*i*), we get

$$\sqrt{n-1}+\sqrt{n+1}-\sqrt{n+1}+\sqrt{n-1} = \frac{p}{q}-\frac{2q}{p} \Rightarrow 2\sqrt{n-1} = \frac{p^2-2q^2}{pq}$$

$\Rightarrow$ $\sqrt{n-1}$ is rational number as $\frac{p^2-2q^2}{2pq}$ is rational.

$\Rightarrow \sqrt{n-1}$is also perfect. square of positive integer From (A) and (B)

$\sqrt{n+1}$ and $\sqrt{n-1}$ are perfect squares of positive integer. It contradict the fact that two perfect squares differ at least by 3.

Hence, there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

10. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is exactly divisible by the required timber In other words, required number is a factor of 391.

Similarly, required positive integer is a Íctor of 436 – 11 = 425 and 542 – 15 = 527

Clearly, the required number is the HCF of 391, 425 and 527.

Using the factor tree, we get the prime factorisations of 391, 425 and 527 as follows:

391 = 17 × 23, 425 52 × 17 and 527 17 × 31

∴ HCF of 391, 425, and 527 is 17.

Hence, the required number = 17.

Case Study Answers:

1. Answer :

i. (d) No value of n.

Solution:

For a number to end in zero it must be divisible by 5, but $4^n = 2^{2n}$ is never divisible by 5. So, 4^n never ends in zero for any value of n.

ii. (c) For all n > 1.

Solution:

We know that product of two rational numbers is also a rational number.

So, $a^2 = a \times a$ = rational number.

$a^3 = a^2 \times a$ = rational number.

$a^4 = a^3 \times a$ = rational number.

....

....

$a^n = a^{n-1} \times a$ = rational number.

iii. (d) Both (a) and (b).

Solution:

Let x = 2m + 1 and y = 2k + 1

Then $x^2 + y^2 = (2m + 1)^2 + (2k + 1)^2$

$= 4m^2 + 4m + 1 + 4k^2 + 4k + 1$

$= 4(m^2 + k^2 + m + k) + 2$

So, it is even but not divisible by 4.

iv. (a) Always true.

Solution:

Let three consecutive positive integers be n, n + 1 and n + 2.

We know that when a number is divided by 3, the remainder obtained is either 0 or 1 or 2.

So, n = 3p or 3p + 1 or 3p + 2, where pis some integer.

If n = 3p, then 2 is divisible by 3.

If n = 3p + 1, then n + 2 = 3p + 1 + 2 = 3p + 3 = 3(p + 1) is divisible by 3.

If n = 3p + 2, then n + 1 = 3p + 2 + 1 = 3p + 3 = 3(p + 1) is divisible by 3.

So, we can say that one of the numbers among n, n + 1 and n + 2 is always divisible by 3.

v. (d) 8

Solution:

Any odd number is of the form of (2k + 1), where k is any integer.

So, $n^2 - 1 = (2k + 1)^2 - 1 = 4k^2 + 4k$

For k = 1, $4k^2 + 4k = 8$, which is divisible by 8.

Similarly, for k = 2, $4k^2 + 4k = 24$, which is divisible by 8.

And fork= 3, $4k^2 + 4k = 48$, which is also divisible by 8.

So, $4k^2 + 4k$ is divisible by 8 for all integers k, i.e., $n^2 - 1$ is divisible by 8 for all odd values of n.

2. Answer :

i. (b) 12240cm

Solution:

Here 80 = 24 × 5, 85 = 17 × 5

and 90 = 2 × 32 × 5

L.C.M of 80, 85 and 90 = 24 × 3 × 3 × 5 × 17 = 12240

Hence, the minimum distance each should walk when they at first time is 12240cm.

ii. (c) 27

Solution:

Here 594 = 2 × 33 × 11 and 189 = 33 × 7

HCF of 594 and 189 = $3^3 = 27$

Hence, the maximum number of columns in which they can march is 27.

iii. (c) 12 litres

Solution:

Here 768 = 28 × 3 and 420 = 22 × 3 × 5 × 7

HCF of 768 and 420 = 22 × 3 = 12

So, the container which can measure fuel of either tanker exactly must be of 12 litres.

iv. (b) 75cm

Solution:

Here, Length = 825cm, Breadth = 675cm and Height = 450cm

Also, 825 = 5 × 5 × 3 × 11, 675 = 5 × 5 × 3 × 3 × 3 and 450 = 2 × 3 × 3 × 5 × 5

HCF = 5 × 5 × 3 = 75

Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75cm.

v. (a) 3 and 2

Solution:

LCM of 8 and 12 is 24.

$\therefore$ The least number of pack of pens $= \frac{24}{8} = 3$

$\therefore$ The least number of pack of note pads $= \frac{24}{12} = 2$

Assertion Reason Answer-

1. (a) Both A and R are true and R is the correct explanation of A.
2. (c) A is true but R is false.