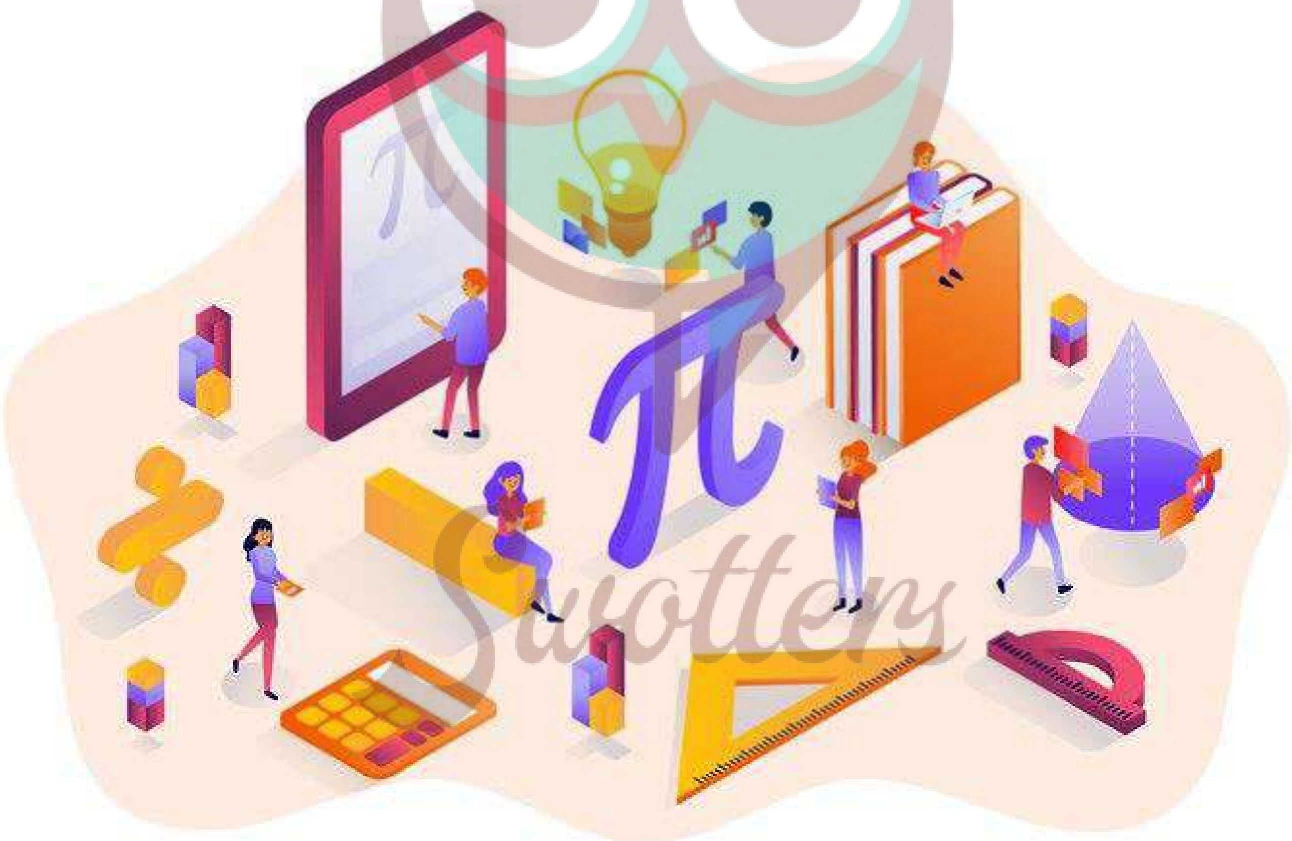
MATHEMATICS



Important Questions

Multiple Choice questions-

1. Let R be the relation in the set (1, 2, 3, 4}, given by:

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}.

Then:

(a) R is reflexive and symmetric but not transitive

(b) R is reflexive and transitive but not symmetric

(c) R is symmetric and transitive but not reflexive

(d) R is an equivalence relation.

2. Let R be the relation in the set N given by: R = {(a, b): a = b – 2, b > 6}. Then:

(a) (2, 4) ∈ R

(b) (3, 8) ∈ R

(c) (6, 8) ∈ R

(d) (8, 7) ∈ R.

3. Let A = {1, 2, 3}. Then number of relations containing {1, 2} and {1, 3}, which are reflexive and symmetric but not transitive is:

(a) 1

(b) 2

(c) 3

(d) 4.

4. Let A = (1, 2, 3). Then the number of equivalence relations containing (1, 2) is

(a) 1

(b) 2

(c) 3

(d) 4.

5. Let f: R → R be defined as $f(x) = x^4$. Then

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto.

6. Let f: R → R be defined as f(x) = 3x. Then

(a) f is one-one onto

(b) f is many-one onto

(c) f is one-one but not onto

(d) f is neither one-one nor onto.

7. If f: R → R be given by $f(x) = (3 – x^3)^{1/3}$, then $f_o f$ (x) is

(a) $x^{1/3}$

(b) x^3

(c) x

(d) $3 - x^3$.

8. Let f: R – {$-\frac{4}{3}$} → R be a function defined as: $f(x) = \frac{4x}{3x + 4}$, $x \neq -\frac{4}{3}$. The inverse of f is map g: Range f → R -{$-\frac{4}{3}$} given by

(a) $g(y) = \frac{3y}{3 - 4y}$

(b) $g(y) = \frac{4y}{4 - 3y}$

(c) $g(y) = \frac{4y}{3 - 4y}$

(d) $g(y) = \frac{3y}{4 - 3y}$

9. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is

(a) Reflexive and symmetric

(b) Transitive and symmetric

(c) Equivalence

(d) Reflexive, transitive but not symmetric.

10. Set A has 3 elements, and the set B has 4 elements. Then the number of injective mappings that can be defined from A to B is:

(a) 144

(b) 12

(c) 24

(d) 64

Very Short Questions:

1. If R = {(x, y) : x + 2y = 8} is a relation in N, write the range of R.
2. Show that a one-one function:

   f {1, 2, 3} → {1, 2, 3} must be onto. (N.C.E.R.T.)
3. What is the range of the function $f(x) = \frac{|x-1|}{x-1}$? (C.B.S.E. 2010)
4. Show that the function f : N → N given by f(x) = 2x is one-one but not onto. (N.C.E.R.T.)
5. If f : R → R is defined by f(x) = 3x + 2 find f(f(x)). C.B.S.E. 2011 (F))
6. If $f(x) = \frac{x}{x-1}$, $x \neq 1$ then find fof. (N.C.E.R.T)
7. If f: R → R is defined by $f(x) = (3 - x^3)^{1/3}$, find fof (x)
8. Are f and q both necessarily onto, if gof is onto? (N.C.E.R.T.)

Short Questions:

1. Let A be the set of all students of a Boys' school. Show that the relation R in A given by:

   R = {(a, b): a is sister of b} is an empty relation and the relation R' given by :

   R' = {(a, b) : the difference between heights of a and b is less than 3 metres} is an universal relation. (N.C.E.R.T.)

2. Let f : X → Y be a function. Define a relation R in X given by :

   R = {(a,b):f(a) = f(b)}.

   Examine, if R is an equivalence relation. (N.C.E.R.T.)

3. Let R be the relation in the set Z of integers given by:

   R = {(a, b): 2 divides a – b}.

   Show that the relation R is transitive. Write the equivalence class [0]. (C.B.S.E. Sample Paper 2019-20)

4. Show that the function:

   f : N → N

   given by f(1) = f(2) = 1 and f(x) = x -1, for every x > 2 is onto but not one-one. (N.C.E.R.T.)

5. Find gof and fog, if:

   f : R → R and g : R → R are given by f (x) = cos x and g (x) = $3x^2$. Show that gof ≠ fog. (N. C.E.R. T.)

6. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ find fof(x)

7. Let A = N x N be the set of ail ordered pairs of natural numbers and R be the relation on the set A defined by (a, b) R (c, d) iff ad = bc. Show that R is an equivalence relation.

8. Let f: R → R be the Signum function defined as:

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

   and g : R → R be the Greatest Integer Function given by g (x) = [x], where [x] is greatest integer less than or equal to x. Then does fog and gof coincide in (0,1]?

Long Questions:

1. Show that the relation R on R defined as R = {(a, b):a ≤ b}, is reflexive and transitive but not symmetric.

2. Prove that function f : N → N, defined by f(x) = $x^2 + x + 1$ is one-one but not onto. Find inverse of f : N → S, where S is range of f.

3. Let A = (x ∈Z : 0 ≤ x ≤ 12}.

   Show that R = {(a, b) : a, b ∈ A; |a – b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. (C.B.S.E 2018)

4. Prove that the function f: [0, ∞) → R given by f(x) = $9x^2 + 6x - 5$ is not invertible. Modify the co-domain of the function f to make it invertible, and hence find f-1. (C.B.S.E. Sample Paper 2018-19

Assertion and Reason Questions-

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is also false.

Assertion(A): Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L1, L2): L1 is perpendicular to L2}.R is not equivalence realtion.

Reason (R): R is symmetric but neither reflexive nor transitive

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is also false.

Assertion (A): = {(T1, T2): T1 is congruent to T2}. Then R is an equivalence relation.

Reason(R): Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive.

Case Study Questions-

1. Consider the mapping f: A → B is defined by f(x) = x - 1 such that f is a bijection.

Based on the above information, answer the following questions.

(i) Domain of f is:

a) R - {2}
b) R
c) R - {1, 2}
d) R - {0}

(ii) Range of f is:

a) R
b) R - {2}
c) R - {0}
d) R - {1, 2}

(iii) If g: R - {2} → R - {1} is defined by g(x) = 2f(x) - 1, then g(x) in terms of x is:

a. $\frac{x+2}{x}$

b. $\frac{x+1}{x-2}$

c. $\frac{x-2}{x}$

d. $\frac{x}{x-2}$

(iv) The function g defined above, is:

a) One-one
b) Many-one
c) into
d) None of these

(v) A function f(x) is said to be one-one if.

a. $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$
b. $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$
c. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
d. None of these

2. A relation R on a set A is said to be an equivalence relation on A iff it is:

I. Reflexive i.e., (a, a) ∈ R ∀ a ∈ A.
II. Symmetric i.e., (a, b) ∈ R ⇒ (b, a) ∈ R ∀ a, b ∈ A.
III. Transitive i.e., (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R ∀ a, b, c ∈ A.

Based on the above information, answer the following questions.

(i) If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then R is:

a) Reflexive
b) Symmetric
c) Transitive
d) Equivalence

(ii) If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then R is:

a) Reflexive
b) Symmetric
c) Transitive
d) Equivalence

(iii) If the relation R on the set N of all natural numbers defined as R = {(x, y): y = x + 5 and x < 4}, then R is:

a) Reflexive
b) Symmetric
c) Transitive
d) Equivalence

(iv) If the relation R on the set A = {1, 2, 3, ..., 13, 14} defined as R = {(x, y): 3x - y = O}, then R is:

a) Reflexive
b) Symmetric
c) Transitive
d) Equivalence

(v) If the relation R on the set A = {I, 2, 3} defined as R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}, then R is:

a) Reflexive only
b) Symmetric only
c) Transitive only
d) Equivalence

Answer Key-

Multiple Choice questions-

(b) R is reflexive and transitive but not symmetric

(c) (6, 8) ∈ R

(a) 1

(b) 2

(d) f is neither one-one nor onto.

(a) f is one-one onto

(c) x

(b) $g(y) = \frac{4y}{4-3y}$

(b) Transitive and symmetric

(c) 24

Very Short Answer:

1. Solution: Range of R = {1, 2, 3}.

   [∵ When x = 2, then y = 3, when x = 4, then y = 2, when x = 6, then y = 1]

2. Solution: Since 'f' is one-one,

   ∴ under 'f', all the three elements of {1, 2, 3} should correspond to three different elements of the co-domain {1, 2, 3}.

   Hence, 'f' is onto.

3. Solution: When x > 1,

   than $f(x) = \frac{x-1}{x-1} = 1$.

   When x < 1,

   than $f(x) = \frac{-(x-1)}{x-1} = -1$

   Hence, Rf = {-1, 1}.

4. Solution:

   Let $x_1, x_2 \in N$.

   Now, $f(x_1) = f(x_2)$

⇒ $2x_1 = 2x_2$

⇒ $x_1 = x_2$

⇒ f is one-one.

Now, f is not onto.

∵ For 1 ∈ N, there does not exist any x ∈ N such that f(x) = 2x = 1.

Hence, f is ono-one but not onto.

5. Solution:

f(f(x)) = 3 f(x) + 2

= 3(3x + 2) + 2 = 9x + 8.

6. Solution:

$$fof(x) = f(f(x)) = \frac{f(x)}{f(x)-1}$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-x+1}$$

$$= \frac{x}{1} = x.$$

7. Solution:

$f_o f(x) = f(f(x)) = (3-(f(x))^3)^{1/3}$

$= (3 - ((3 - x^3)^{1/3})^3)^{1/3}$

$= (3 - (3 - x^3))^{1/3} = (x^3)^{1/3} = x.$

8. Solution:

Consider f: {1, 2, 3, 4} → {1, 2, 3, 4}

and g: {1, 2, 3,4} → {1,2.3} defined by:

f(1) = 1, f(2) = 2, f(3) = f(4) = 3

g (1) = 1, g (2) = 2, g (3) = g (4) = 3.

∴ gof = g (f(x)) {1, 2,3}, which is onto

But f is not onto.

[∵ 4 is not the image of any element]

Short Answer:

1. Solution:

(i) Here R = {(a, b): a is sister of b}.

Since the school is a Boys' school,

∴ no student of the school can be the sister of any student of the school.

Thus R = Φ Hence, R is an empty relation.

(ii) Here R' = {(a,b): the difference between heights of a and b is less than 3 metres}.

Since the difference between heights of any two students of the school is to be less than 3 metres,

∴ R' = A x A. Hence, R' is a universal relation.

2. Solution:

For each a ∈ X, (a, a) ∈ R.

Thus R is reflexive. [∵ f (a) = f(a)]

Now (a, b) ∈ R

⇒ f(a) = f(b)

⇒ f(b) = f (a)

⇒ (b, a) ∈ R.

Thus R is symmetric.

And (a, b) ∈ R

and (b, c) ∈ R

⇒ f(a) = f(b)

and f(b) = f(c)

⇒ f(a)= f(c)

⇒ (a, c) ∈ R.

Thus R is transitive.

Hence, R is an equivalence relation.

3. Solution:

Let 2 divide (a – b) and 2 divide (b – c), where a,b,c ∈ Z

⇒ 2 divides [(a – b) + (b – c)]

⇒ 2 divides (a – c).

Hence, R is transitive.

And [0] = {0, ± 2, ± 4, ± 6,…].

4. Solution:

Since f(1) = f(2) = 1,

∴ f(1) = f(2), where 1 ≠ 2.

∴ ‘f’ is not one-one.

Let y ∈ N, y ≠ 1,

we can choose x as y + 1 such that f(x) = x – 1

= y + 1 – 1 = y.

Also 1 ∈ N, f(1) = 1.

Thus ‘f ’ is onto.

Hence, ‘f ’ is onto but not one-one.

5. Solution:

We have:

f(x) = cos x and g(x) = $3x^2$.

∴ gof (x) = g (f(x)) = g (cos x)

$= 3 (\cos x)^2 = 3 \cos^2 x$

and fog (x) = f(g (x)) = $f(3x^2) = \cos 3x^2$.

Hence, gof ≠ fog.

6. Solution:

We have: $\frac{4x+3}{6x-4}$ ...(1)

∴ fof(x) - f (f (x))

$$= \frac{4f(x)+3}{6f(x)-4}$$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} \quad [Using\ (1)]$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16}$$

$$= \frac{34x}{34} = x.$$

7. Solution:

Given: (a, b) R (c, d) if and only if ad = bc.

(I) (a, b) R (a, b) iff ab – ba, which is true.

[∵ ab = ba ∀ a, b ∈ N]

Thus, R is reflexive.

(II) (a, b) R (c,d) ⇒ ad = bc

(c, d) R (a, b) ⇒ cb = da.

But cb = be and da = ad in N.

∴ (a, b) R (c, d) ⇒ (c, d) R (a, b).

Thus, R is symmetric.

(III) (a,b) R (c, d)

⇒ ad = bc ...(1)

(c, d) R (e,f)

⇒ cf = de ... (2)

Multiplying (1) and (2), (ad). (cf) – (be), (de)

⇒ af = be

⇒ (a,b) = R(e,f).

Thus, R is transitive.

Thus, R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

8. Solution:

For x ∈ (0,1].

$$(fog)\ (x) = f(g(x)) = f([x])$$
$$= \begin{cases} f(0); \text{if } 0 < x < 1 \\ f(1); \text{if } x = 1 \end{cases}$$

$$\Rightarrow \quad f(g(x)) = \begin{cases} 0; \text{if } 0 < x < 1 \\ 1; \text{if } x = 1 \end{cases} \quad ...(1)$$

And (gof) (x) = g(f(x)) = g(1)

[∵ f(x) = 1 ∀ x > 0]

= [1] = 1

⇒ (gof) (x) = 1 ∀ x ∈ (0, 1] ...(2)

From (1) and (2), (fog) and (gof) do not coincide in (0, 1].

Long Answer:

1. Solution:

We have: R = {(a, b)} = a ≤ b}.

Since, a ≤ a ∀ a ∈ R,

∴ (a, a) ∈ R,

Thus, R reflexive.

Now, (a, b) ∈ R and (b, c) ∈ R

⇒ a ≤ b and b ≤ c

⇒ a ≤ c

⇒ (a, c) ∈ R.

Thus, R is transitive.

But R is not symmetric

[∵ (3, 5) ∈ R but (5, 3) ∉ R as 3 ≤ 5 but 5 > 3]

Solution:

Let $x_1, x_2 \in N$.

Now, $f(x_1) = f(x_2)$

$$\Rightarrow x^2_1 + x_1 + 1 = x^2_2 + x_2 + 1$$
$$\Rightarrow x^2_1 + x_1 = x^2_2 + x_2$$
$$\Rightarrow (x^2_1 - x^2_2) + (x_1 - x_2) = 0$$
$$\Rightarrow (x_1 - x_2) + (x_1 + x_2 + 1) = 0$$
$$\Rightarrow x_1 - x_2 = 0 \quad [\because x_1 + x_2 + 1 \neq 0]$$
$$\Rightarrow x_1 = x_2.$$

Thus, f is one-one.

Let y ∈ N, then for any x,

$f(x) = y$ if $y = x^2 + x + 1$

$\Rightarrow \quad y = \left(x^2 + x + \frac{1}{4}\right) + \frac{3}{4}$

$\Rightarrow \quad y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

$\Rightarrow \quad x + \frac{1}{2} = \pm\sqrt{y - \frac{3}{4}}$

$\Rightarrow \quad x = \pm\frac{\sqrt{4y-3}}{2} - \frac{1}{2}$

$\Rightarrow \quad x = \frac{\pm\sqrt{4y-3} - 1}{2}$

$\Rightarrow \quad x = \frac{\sqrt{4y-3} - 1}{2}$

$\left[\frac{-\sqrt{4y-3} - 1}{2} \notin N \text{ for any value of } y\right]$

Now, for $y = \frac{3}{4}$, $x = -\frac{1}{2} \notin N$.

Thus, f is not onto.

⇒ f(x) is not invertible.

Since, x > 0, therefore, $\frac{\sqrt{4y-3}-1}{2} > 0$

$\Rightarrow \sqrt{4y-3} > 1$

⇒ 4y − 3 > 1

⇒ 4y > 4

⇒ y > 1.

Redefining, f : (0, ∞) → (1, ∞) makes

$f(x) = x^2 + x + 1$ on onto function.

Thus, f (x) is bijection, hence f is invertible and f^{-1} : (1, ∞) → (1,0)

$f^{-1}(y) = \frac{\sqrt{4y-3}-1}{2}$

2. Solution:

We have:

R = {(a, b): a, b ∈ A; |a – b| is divisible by 4}.

(1) Reflexive: For any a ∈ A,

∴ (a, b) ∈ R.

$|a - a| = 0$, which is divisible by 4.

Thus, R is reflexive.

Symmetric:

Let (a, b) ∈ R

⇒ $|a - b|$ is divisible by 4

⇒ $|b - a|$ is divisible by 4

Thus, R is symmetric.

Transitive: Let (a, b) ∈ R and (b, c) ∈ R

⇒ $|a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4

⇒ $|a - b| = 4\lambda$

⇒ $a - b = \pm 4\lambda$ ………….(1)

and $|b - c| = 4\mu$

⇒ $b - c = \pm 4\mu$ ………….(2)

Adding (1) and (2),

$(a-b) + (b-c) = \pm 4(\lambda + \mu)$

⇒ $a - c = \pm 4(\lambda + \mu)$

⇒ (a, c) ∈ R.

Thus, R is transitive.

Now, R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

(ii) Let 'x' be an element of A such that (x, 1) ∈ R

⇒ $|x - 1|$ is divisible by 4

$\Rightarrow x - 1 = 0, 4, 8, 12, \ldots$

$\Rightarrow x = 1, 5, 9, 13, \ldots$

Hence, the set of all elements of A which are related to 1 is {1, 5, 9}.

(iii) Let $(x, 2) \in R$.

Thus $|x - 2| = 4k$, where $k \leq 3$.

$\therefore x = 2, 6, 10$.

Hence, equivalence class [2] = {2, 6, 10}.

3. Solution:

Let $y \in R$.

For any x, $f(x) = y$ if $y = 9x^2 + 6x - 5$

$\Rightarrow y = (9x^2 + 6x + 1) - 6$

$= (3x + 1)^2 - 6$

$$\Rightarrow \quad 3x + 1 = \pm\sqrt{y+6}$$

$$\Rightarrow \quad x = \frac{\pm\sqrt{y+6} - 1}{3}$$

$$\Rightarrow \quad x = \frac{\sqrt{y+6} - 1}{3}$$

$$\left[\because \frac{-\sqrt{y+6} - 1}{3} \notin [0, \infty) \text{ for any value of } y\right]$$

For $y = -6 \in R$, $x = -\frac{1}{3} \notin [0, \infty)$.

Thus, f(x) is not onto.

Hence, f(x) is not invertible.

Since, $x \geq 0$, $\therefore \frac{\sqrt{y+6}-1}{3} \geq 0$

$\Rightarrow \quad \sqrt{y+6}-1 \geq 0$

$\Rightarrow \quad \sqrt{y+6} \geq 1$

$\Rightarrow \quad y+6 \geq 1$

$\Rightarrow \quad y \geq -5.$

We redefine,

f: $[0, \infty) \rightarrow [-5, \infty)$,

which makes $f(x) = 9x^2 + 6x - 5$ an onto function.

Now, $x_1, x_2 \in [0, \infty)$ such that $f(x_1) = f(x_2)$

$\Rightarrow (3x_1 + 1)^2 = (3x_2 + 1)^2$

$\Rightarrow [(3x_1 + 1) + (3x_2 + 1)][(3x_1 + 1) - (3x_2 + 1)]$

$\Rightarrow [3(x_1 + x_2) + 2][3(x_1 - x_2)] = 0$

$\Rightarrow x_1 = x_2$

$[\because 3(x_1 + x_2) + 2 > 0]$

Thus, f(x) is one-one.

$\therefore$ f(x) is bijective, hence f is invertible

and $f^{-1}: [-5, \infty) \rightarrow [0, \infty)$

$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

Assertion and Reason Answers-

1. (a) Both A and R are true and R is the correct explanation of A.

2. (a) Both A and R are true and R is the correct explanation of A.

Case Study Answers-

1. Answer :

(i) (a) R - {2}

Solution:

For f(x) to be defined x - 2; ≠ 0 i.e., x; ≠ 2.

$\therefore$ Domain of f = R - {2}

(ii) (b) R - {2}

Solution:

Let y = f(x), then $y = \frac{x-1}{x-2}$

⇒ xy - 2y = x - 1 ⇒ xy - x = 2y -

$\Rightarrow x = \frac{2y-1}{y-1}$

Since, x ∈∈ R - {2}, therefore y ≠ 1

Hence, range of f = R - {1}

(iii) (d) $\frac{x}{x-2}$

Solution:

We have, g(x) = 2f(x) - 1

$= 2\left(\frac{x-1}{x-2}\right) - 1 = \frac{2x-2-x+2}{x-2} = \frac{x}{x-2}$

(iv) (a) One-one

Solution:

We have, $g(x) = \frac{x}{x-2}$

Let $g(x_1) = g(x_2) \Rightarrow \frac{x_1}{x_1-2} = \frac{x_2}{x_2-2}$

$\Rightarrow x_1x_2 - 2x_1 = x_1x_2 - 2x_2 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

Thus, $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$

Hence, g(x) is one-one.

(v) (c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

2. Answer :

(i) (a) Reflexive

Solution:

Clearly, (1, 1), (2, 2), (3, 3), ∈ R. So, R is reflexive on A.

Since, (1, 2) ∈ R but (2, 1) ∉ R. So, R is not symmetric on A.

Since, (2, 3), ∈ R and (3, 1) ∈ R but (2, 1) ∉ R. So, R is not transitive on A.

(ii) (b) Symmetric

Solution:

Since, (1, 1), (2, 2) and (3, 3) are not in R. So, R is not reflexive on A.

Now, (1, 2) ∈ R ⇒ (2, 1) ∈ R and (1, 3) ∈ R ⇒ (3, 1) ∈ R. So, R is symmetric,

Clearly, (1, 2) ∈ R and (2, 1) ∈ R but (1, 1) ∉ R. So, R is not transitive on A.

(iii) (c) Transitive

Solution:

We have, R = {(x, y): y = x + 5 and x < 4}, where x, y ∈ N.

∴ R = {(1, 6), (2, 7), (3, 8)}

Clearly, (1, 1), (2, 2) etc. are not in R. So, R is not reflexive.

Since, (1, 6) ∈ R but (6, 1) ∉ R. So, R is not symmetric.

Since, (1, 6) ∈ R and there is no order pair in R which has 6 as the first element.

Same is the case for (2, 7) and (3, 8). So, R is transitive.

(iv) (d) Equivalence

Solution:

We have, R = {(x, y): 3x - y = 0}, where x, y ∈ A = {1,2, ..., 14}.

∴ R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Clearly, (1, 1) ∉ R. So, R is not reflexive on A.

Since, (1, 3) ∈ R but (3, 1) ∉ R. So, R is not symmetric on A.

Since, (1, 3) ∈ Rand (3, 9) ∈ R but (1, 9) ∉ R. So, R is not transitive on A.

(v) (d) Equi0076alence

Solution:

Clearly, (1, 1), (2, 2), (3, 3) ∈ R. So, R is reflexive on A.

We find that the ordered pairs obtained by interchanging the components of ordered pairs in R are also in R. So, R is symmetric on A. For 1, 2, 3 ∈ A such that (1, 2) and (2, 3) are in R implies that (1, 3) is also, in R. So, R is transitive on A. Thus, R is an equivalence relation.