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Test / Exam Name: Maths - Relations And Functions **Standard: 12th Science** **Subject: Mathematics**

Student Name: ..... **Section:** ..... **Roll No.:** .....

Questions: 28 | Time: 01:30 hh:mm | Marks: 50

Instructions

1. New section on new page
2. Rough work at the last page should be in proper manner too
3. Make sure to write in the point formation. You handwriting should be neat and clean
4. Honesty is the best policy.

SECTION-A

Q1. If f and g are two functions from R to R defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, then fog (x) for x<0 is **1 Mark**

A 4x **B** 2x **C** 0 **D** -4x

Q2. The two lines x = ay + b, z = cy + d; and x = a'y + b', z = c'y + d' are perpendicular to each other, if: **1 Mark**

A $\frac{a}{a'} + \frac{c}{c'} = 1$ **B** $\frac{a}{a'} + \frac{c}{c'} = -1$

C $aa' + cc' = 1$ **D** $aa' + cc' = -1$

Q3. The function f : R→[– 1, 1] defined by $f(x) = \cos x$ is **1 Mark**

A Both one-one and onto. **B** Not one-one, but onto.

C One-one, but not onto. **D** Neither one-one, nor onto.

Q4. A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by f(x) = 4x + 3 is: **1 Mark**

A one-one but not onto **B** onto but not one-one

C both one-one and onto **D** neither one-one nor onto

Q5. Let A = {1, 3, 5}. Then the number of equivalence relations in A containing (1, 3) is: **1 Mark**

A 1 **B** 2 **C** 3 **D** 4

Q6. If f: R → R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then fof(x) is: **1 Mark**

A $x^{\frac{1}{3}}$ **B** x^3

C x **D** (3 - x^3).

Q7. If N be the set of all-natural numbers, consider f : N → N such that f(x) = 2x, ∀ x ∈ N, then f is: **1 Mark**

A One-one onto. **B** One-one into. **C** Many-one onto. **D** None of these.

Q8. The function $f : \left[\frac{-1}{2}, \frac{1}{2}, \frac{1}{2}\right] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, defined by $f(x) = \sin^{-1}(3x - 4x^3)$, is: **1 Mark**

A Bijection. **B** Injection but not a surjection.

C Surjection but not an injection. **D** Neither an injection nor a surjection.

Q9. Let A = {2, 3, 4, 5, ..., 17, 18}. Let $' \simeq '$ be the equivalence relation on A × A, cartesian product of A with itself, defined by $(a, b) \simeq (c, d)$ if ad = bc. Then, the number of ordered pairs of the equivalence class of (3, 2) is: **1 Mark**

A 4 **B** 5 **C** 6 **D** 7

Q10. If $F : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals: **1 Mark**

A $\frac{x+\sqrt{x^2-4}}{2}$ **B** $\frac{x}{1+x^2}$

C $\frac{x-\sqrt{x^2-4}}{2}$ **D** $1 + \sqrt{x^2 - 4}$

Q11. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not. **1 Mark**

Q12. Let * be the binary operation on N given by a * b = L.C.M. of a and b. Find: **1 Mark**
5 * 7, 20 * 16

Q13. Let * be a 'binary' operation on N given by a * b = LCM (a, b) for all a, b ∈ N. Find 5 * 7. **1 Mark**

Q14. State with reason whether following functions have inverse **1 Mark**
f: {1, 2, 3, 4} → {10} with
f = {(1, 10), (2, 10), (3, 10), (4, 10)}

Q15. Find which of the binary operations are commutative and which are associative. **1 Mark**
State whether the following statements are true or false. Justify
For an arbitrary binary operation * on a set N, $a * a = a \ \forall a \in N$.

Q16. Let * be the binary operation on N given by a * b = L.C.M. of a and b. Find: **1 Mark**
Is * commutative?

SECTION-B

Q17. If an operation * on the set of integers Z is defined by a * b = $2a^2$ + b, then find (i) whether it is a binary or not, and (ii) if a binary, then is it commutative or not. **2 Marks**

Q18. Let * be a binary operation on R - {-1} defined by **2 Marks**
$a * b = \frac{a}{b+1}$, for all $a, \ b \in R - \{-1\}$
Show that * is neither commutative nor associative in R - {-1}.

Q19. If * is defined on the set R of all real numbers by $* : a * b = \sqrt{a^2 + b^2}$, find the identity element, if it exists in with respect to *. **2 Marks**

Q20. If'a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$. **2 Marks**

Q21. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, then show that (fof) (x) = x, for all $x \neq \frac{2}{3}$. Also, write inverse of f. **2 Marks**

Q22. A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive. **3 Marks**

Q23. Given a non empty set X, consider P(X) which is the set of all subsets of X. **3 Marks**
Define the relation R in P(X) as follows:
For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify your answer.

Q24. Show that the Signum Function f: R → R, given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is neither one-one nor onto. **3 Marks**

Q25. Show that f: [–1, 1] →R, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function f:[-1, 1] → Range f. **3 Marks**
(Hint: For $y \in \text{Range } f, y = f(x) = \frac{x}{x+2}$, for some x in [-1, 1], i.e., $x = \frac{2y}{(1-y)}$)

SECTION-C

Q26. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$. **4 Marks**

Q27. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}. **4 Marks**

Q28. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, \ b \in Z, \ |a - b| \text{ is divisible by } 3\}$ is an equivalence relation. **4 Marks**