MATHEMATICS

Important Questions

Multiple Choice questions-

1. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10

(b) -10

(c) 5

(d) -5

2. If the zeroes of the quadratic polynomial $x^2 + (a + 1) x + b$ are 2 and -3, then

(a) a = -7, b = -1

(b) a = 5, b = -1

(c) a = 2, b = -6

(d) a – 0, b = -6

3. The number of polynomials having zeroes as -2 and 5 is

(a) 1

(b) 2

(c) 3

(d) more than 3

4. If one of the zeroes of the cubic polynomial $x3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) b – a + 1

(b) b – a – 1

(c) a – b + 1

(d) a – b – 1

5. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal

5. The zeroes of the quadratic polynomial $x^2 + kx + k$, k? 0,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal

6. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, c # 0 are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have the same sign

(d) c and b have the same sign

7. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

8. The number of polynomials having zeroes as 4 and 7 is

(a) 2

(b) 3

(c) 4

(d) more than 4

9. A quadratic polynomial, whose zeores are -4 and -5, is

(a) $x^2-9x + 20$

(b) $x^2 + 9x + 20$

(c) $x^2-9x- 20$

(d) $x^2 + 9x- 20$

10. The zeroes of the quadratic polynomial $x^2 + 1750x + 175000$ are

(a) both negative

(b) one positive and one negative

(c) both positive

(d) both equal

Very Short Questions:

1. What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^2 + qx^2 + rx + 5$, $p \neq 0$?

2. If on division of a polynomial p(x) by a polynomial g(x), the quotient is zero, what is the relation between the degrees of p(x) and g(x)?

3. Can $x – 2$ be the remainder on division of a polynomial p(x) by $x + 3$?

4. Find the quadratic polynomial whose zeros are -3 and 4.

5. If one zero of the quadratic polynomial $x^2 – 5x – 6$ is 6 then find the other zero.

6. If both the zeros of the quadratic polynomial $ax^2 + bx + c$ are equal and opposite in sign, then find the value of b.

7. What number should be added to the polynomial $x^2 – 5x + 4$, so that 3 is the zero of the polynomial?

8. Can a quadratic polynomial $x^2 + kx + k$ have equal zeros for some odd integer $k > 1$?

9. If the zeros of a quadratic polynomial $ax^2 + bx + c$ are both negative, then can we say a, b and c all have the same sign? Justify your answer.

10. If the graph of a polynomial intersects the x-axis at only one point, can it be a quadratic polynomial?

11. If the graph of a polynomial intersects the x-axis at exactly two points, is it necessarily a quadratic polynomial?

Short Questions :

1. If one of the zeros of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is equal in magnitude but opposite in sign of the other, find the value of k.

2. If one of the zeros of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3 then find the value of k.

3. If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

4. If α and β are zeros of polynomial $p(x) = x^2 - 5x + 6$, then find the value of α + B – 3aß.

5. Find the zeros of the polynomial $p(x) = 4x^2 - 12x + 9$.

6. What must be subtracted from $p(x) = 8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $g(x) = 4x^2 + 3x - 2$?

7. What must be added to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$?

8. Obtain the zeros of quadratic polynomial $3x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeros and coefficients.

9. If α and β are the zeros of the polynomial 6y2 – 7y + 2, find a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

10. If one zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Long Questions :

1. Verify that the numbers given alongside the cubic polynomial below are their zeros. Also verify the relationship between the zeros and the coefficients.

$x^3 - 4x^2 + 5x - 2$; 2,1,1

2. Find a cubic polynomial with the sum of the zeros, sum of the products of its zeros taken two at a time, and the product of its zeros as 2, -7, -14 respectively.

3. Find the zeros of the polynomial f(x) = x3 – $5x^2$ – 2x + 24, if it is given that the product of its two zeros is 12.

4. If the remainder on division of $x^3 - kx^2 + 13x - 21$ by 2x – 1 is -21, find the quotient and the value of k. Hence, find the zeros of the cubic polynomial $x^3 - kx^2 + 13x$.

5. Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

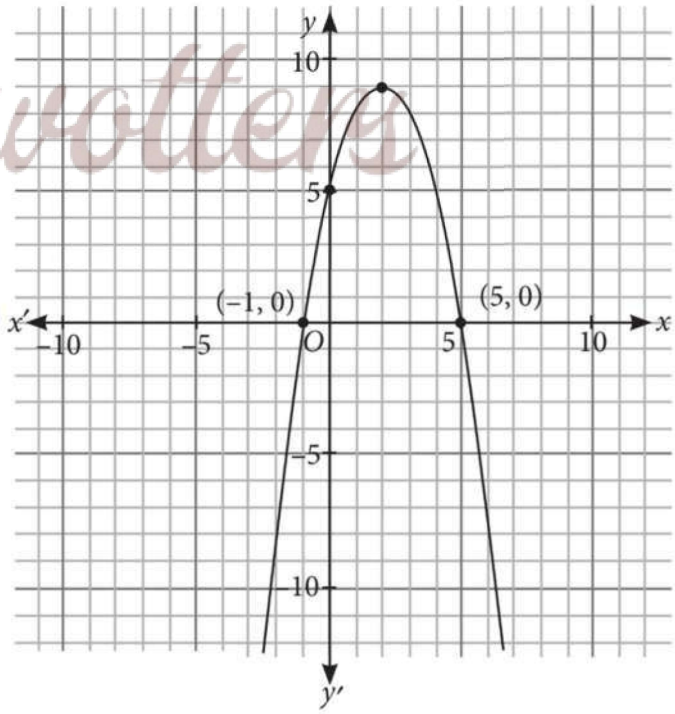
6. Given that √2 is a zero of the cubic polynomial $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other zeros.

7. If α, β, γ bezerosofpo1ynomial $6x^3 + 3x^2 - 5x + 1$, then find die value of α-1 + β-1 + γ-1.

8. Find the zeros of the polynomial f(x) = – $12x^2$ + 39x – 28, if it is given that the zeros are in AP.

Case Study Questions:

1. ABC construction company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents overspeeding, minimise accidents and gives a chance for pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.



Based on the above information, answer the following questions.

i. The polynomial represented by the graph can be _____ polynomial.

a. Linear
b. Quadratic
c. Cubic
d. Zero

ii. The zeroes of the polynomial represented by the graph are:

a. 1, 5
b. 1, -5
c. -1, 5
d. -1, -5

iii. Sum of zeroes of the polynomial represented by the graph are:

a. 4
b. 5
c. 6
d. 7

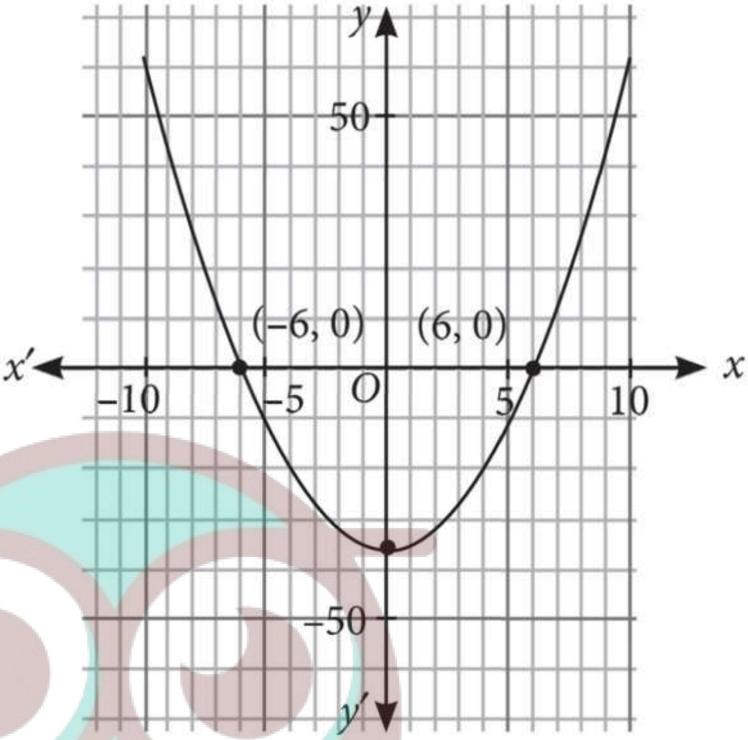
iv. If α and β are the zeroes of the polynomial represented by the graph such that β > α, β > α, then |8 α + β| = |8 α + β| =

a. 1
b. 2
c. 3
d. 4

v. The expression of the polynomial represented by the graph is:

a. $x^2 - 4x - 5$
b. $x^2 + 4x + 5$
c. $x^2 + 4x - 5$
d. $-x^2 + 4x + 5$

2. While playing in garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph .



Based on the above information, answer the following questions.

i. Graph of a quadratic polynomial is _____ in shape.

a. Straight line.
b. Parabolic.
c. Circular.
d. None of these.

ii. The expression of the polynomial represented by the graph is:

a. $x^2 - 49$
b. $x^2 - 64$
c. $x^2 - 36$
d. $x^2 - 81$

iii. Find the value of the polynomial represented by the graph when x = 6.

a. -2
b. -1
c. 2
d. 1

iv. The sum of zeroes of the polynomial $x^2 + 2x - 3$ is:

a. -1
b. -2
c. 2
d. 1

v. If the sum of zeroes of polynomial $at^2 + 5t + 3a$ is equal to their product, then find the value of a.

a. -5
b. -3
c. 5353
d. −53−53

Assertion reason questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Assertion: $x^2 + 7x + 12$ has no real zeroes.

Reason: A quadratic polynomial can have at the most two zeroes.

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Assertion: If the sum of the zeroes of the quadratic polynomial x^2-2kx+8 is 2 then value of k is 1.
Reason: Sum of zeroes of a quadratic polynomial ax^2+bx+c is -b/a

Answer Key-

Multiple Choice questions-

1. (b) -10
2. (d) a – 0, b = -6
3. (d) more than 3
4. (a) b – a + 1
5. (b) both negative
6. (a) cannot both be positive
7. (c) c and a have the same sign
8. (a) has no linear term and the constant term is negative.
9. (d) more than 4
10. (b) $x^2 + 9x + 20$
11. (a) both negative

Very Short Answer :

1. 0, $ax^2 + bx + C$.
2. Since the quotient is zero, therefore

   deg p(x) < deg g(x)
3. No, as degree (x – 2) = degree (x + 3)
4. Sum of zeros = -3 + 4 = 1,

   Product of zeros = – 3 x 4 = -12

   ∴ Required polynomial = $x^2 - x - 12$
5. Let α,6 be the zeros of given polynomial.

   Then α + 6 = 5 3 ⇒ α = -1

6. Let α and -α be the roots of given polynomial.

Then α + (-α) = 0 ⇒ $-\frac{b}{a} = 0 \Rightarrow b = 0$.

7. Let $f(x) = x^2 - 5x + 4$

Then $f(3) = 3^2 - 5 \times 3 + 4 = -2$

For f(3) = 0, 2 must be added to f(x).

8. No, for equal zeros, k = 0, 4 ⇒ k should be even.

9. Yes, because $-\frac{b}{a}$ = sum of zeros < 0, so that $\frac{b}{a} = 0 > 0$. Also the product of the zeros $= \frac{c}{a} = 0 > 0$.

10. Yes, because every quadratic polynomial has at the most two zeros.

11. No, $x^4 - 1$ is a polynomial intersecting the x-axis at exactly two points.

Short Answer :

1. Let one root of the given polynomial be α.

Then the other root = -α

Sum of the roots = (-α) + α = 0

$$\Rightarrow -\frac{b}{a} = 0 \text{ or } -\frac{8k}{4} = 0 \text{ or } k = 0$$

2. Since – 3 is a zero of the given polynomial

$\therefore (k-1)(-3)^2 + k(-3) + 1 = 0$:

$\Rightarrow 9k - 9 - 3k + 1 = 0 \Rightarrow k = 4/3$.

3. Put x = 1 in p(x)

$\therefore p(1) = a(1)^2 - 3(a-1) \times 1 - 1 = 0$

$\Rightarrow a - 3a + 3 - 1 = 0 \Rightarrow 2a = -2 \Rightarrow a = 1$

4. Here, α + β = 5, αβ = 6

$= \alpha + \beta - 3\alpha\beta = 5 - 3 \times 6 = -13$

5. $p(x) = 4x^2 - 12x + 9 = (2x - 3)+$

For zeros, $p(x) = 0$

$\Rightarrow (2x - 3)(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$

6. Let y be subtracted from polynomial p(x)

: $8x^4 + 14x^3 - 2x^2 + 7x - 8 - y$ is exactly divisible by g(x)

Now,

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
\hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 - y \\
 & \underline{8x^4 \pm 6x^3 \mp 4x^2} \\
 & \quad 8x^3 + 2x^2 + 7x - 8 - y \\
 & \quad \underline{8x^3 \pm 6x^2 \mp 4x} \\
 & \qquad -4x^2 + 11x - 8 - y \\
 & \qquad \underline{\mp 4x^2 \mp 3x \pm 2} \\
 & \qquad\qquad 14x - 10 - y
\end{array}
$$

$\because$ Remainder should be 0.

$\therefore 14x - 10 - y = 0$ or $14x - 10 = y$ or $y = 14x - 10$

$\therefore (14x - 10)$ should be subtracted from p(x) so that it will be exactly divisible by g(x)

7. By division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$= f(x) - r(x) = g(x) \times q(x) \Rightarrow f(x) + \{-r(x)\} = g(x) \times q(x)$

Clearly, RHS is divisible by g(x). Therefore, LHS is also divisible by g(x). Thus, if we add –r(x) to f(x), then the resulting polynomial is divisible by g(x). Let us now find the remainder when f(x) is divided by g(x).

$$\begin{array}{r} 4x^2 - 6x + 22 \\ x^2 + 2x - 3 \overline{\smash{\big)} 4x^4 + 2x^3 - 2x^2 + x - 1} \\ \underline{4x^4 \pm 8x^3 \mp 12x^2} \\ -6x^3 + 10x^2 + x - 1 \\ \underline{\mp 6x^3 \mp 12x^2 \pm 18x} \\ 22x^2 - 17x - 1 \\ \underline{22x^2 \pm 44x \mp 66} \\ -61x + 65 \end{array}$$

∴ r(x) = -61x + 65 or -r(x) = 61x – 65

Hence, we should add –r(x) = 61x – 65 to f(x) so that the resulting polynomial is divisible by g(x).

8. We have,

$$\alpha + \beta = -\left(\frac{-7}{6}\right) = \frac{7}{6}; \quad \alpha\beta = \frac{2}{6} = \frac{1}{3}$$

9. Let $p(y) = 6y^2 - 7y + 2$

$$\text{Now,} \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7}{6 \times \frac{1}{3}} = \frac{7}{2}$$

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{1}{3}} = 3$$

$$\text{The required polynomial} = y^2 - \frac{7}{2}y + 3 = \frac{1}{2}(2y^2 - 7y + 6)$$

10. Let α and β be the zeros of the polynomial. Then as per question β = 7α

Now sum of zeros = $\alpha + \beta = \alpha + 7\alpha = -\left(\frac{-8}{3}\right)$

$\Rightarrow \quad 8\alpha = \frac{8}{3} \quad$ or $\quad \alpha = \frac{1}{3}$

and $\quad \alpha \times \beta = \alpha \times 7\alpha = \frac{2k+1}{3}$

$\Rightarrow \quad 7\alpha^2 = \frac{2k+1}{3} \quad \Rightarrow \quad 7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3} \qquad \left(\because \alpha = \frac{1}{3}\right)$

$\Rightarrow \quad \frac{7}{9} = \frac{2k+1}{3} \quad \Rightarrow \quad \frac{7}{3} = 2k+1$

$\Rightarrow \quad \frac{7}{3} - 1 = 2k \quad \Rightarrow \quad k = \frac{2}{3}$

Long Answer :

1. Let $p(x) = x^3 - 4x^2 + 5x - 2$

On comparing with general polynomial px) $ax^3 + bx^2 + cx + d$, we get a = 1, b = -4, c = 5 and d = -2

Given zeros 2, 1, 1.

$\therefore p(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$

and $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0$

Hence, 2, 1 and I are the zeros of the given cubic polynomial.

Again, consider $\alpha = 2, \beta = 1, \gamma = 1$

$\therefore \alpha + 13 + y = 2 + 1 + 1 = 4$

and $\quad \alpha + \beta + \gamma = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-b}{a} = \frac{-(-4)}{1} = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2) = 2 + 1 + 2 = 5$

and $\quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a} = \frac{5}{1} = 5$

$\alpha\beta\gamma = (2)(1)(1) = 2$

and $\quad \alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Coefficient of } x^3} = \frac{-d}{a} = \frac{-(-2)}{1} = 2$

2. Let the cubic polynomial be $p(x) = ax^3 + bx^2 + cx + d$. Then

Sum of zeros = $\frac{-b}{a} = 2$

Sum of the products of zeros taken two at a time = $\frac{c}{a} = -7$

and product of the zeros = $\frac{-d}{a} = -14$

$\Rightarrow \quad \frac{b}{a} = -2, \quad \frac{c}{a} = -7, \quad -\frac{d}{a} = -14 \quad$ or $\quad \frac{d}{a} = 14$

$\therefore \quad p(x) = ax^3 + bx^2 + cx + d \quad \Rightarrow \quad p(x) = a\left[x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a}\right]$

$p(x) = a[x^3 + (-2)x^2 + (-7)x + 14] \Rightarrow p(x) = a[x^3 - 2x^2 - 7x + 14]$

For real value of a = 1, $p(x) = x^3 - 2x^2 - 7x + 14$

3. Let α, β and γ be the zeros of polynomial (fx) such that αβ = 12.

We have, $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-2}{1} = -2$ and $\alpha\beta\gamma = \frac{-d}{a} = \frac{-24}{1} = -24$

Putting αβ = 12 in αβγ = –24, we get

$12\gamma = -24 \quad \Rightarrow \quad \gamma = -\frac{24}{12} = -2$

Now, α + β + γ = 5 α + β – 2 = 5

= α + β = 7 a = 7 – β

= (7 – β) β =12 ⇒ 7β – β^2 – 12

= β^2 + 7β + 12 = 0 ⇒ β^2 – 3β – 4β + 12 = O

= β = 4 or β = 3

β = 4 or β = 3

∴ α = 3 or α = 4

4. Let $f(x) = x^3 - kx^2 + 13x - 21$

Then, $f\left(\frac{1}{2}\right)=-21 \Rightarrow \left(\frac{1}{2}\right)^3-k\left(\frac{1}{2}\right)^2+13\left(\frac{1}{2}\right)-21=-21$

or $\frac{1}{8}-\frac{1}{4}k+\frac{13}{2}-21+21=0$ or $\frac{k}{4}=\frac{53}{8} \Rightarrow k=\frac{53}{2}$

$\therefore \quad f(x)=x^3-\frac{53}{2}x^2+13x-21$

Now, $f(x)=q(x)(2x-1)-21$

$\Rightarrow \quad x^3-\frac{53}{2}x^2+13x-21=q(x)(2x-1)-21$

$\Rightarrow \quad \left(x^3-\frac{53}{2}x^2+13x\right)\div(2x-1)=q(x)$

$$\begin{array}{r|l} & \frac{1}{2}x^2-13x \\ \hline 2x-1 & x^3-\frac{53}{2}x^2+13x \\ & \underline{x^3 \mp \frac{1}{2}x^2} \\ & -26x^2+13x \\ & \underline{\mp 26x^2 \pm 13x} \\ & 0 \end{array}$$

i.e., $x^3-\frac{53}{2}x^2+13x=(2x-1)\left(\frac{1}{2}x^2-13x\right)=\frac{1}{2}x(2x-1)(x-26)$

For zeros, $x^3-\frac{53}{2}x^2+13x=0 \Rightarrow \frac{1}{2}x(2x-1)(x-26)=0 \Rightarrow x=0, \frac{1}{2}, 26$

5.

Since two zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$, so $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=x^2-\frac{5}{3}$ is a factor of the given polynomial.

Now, we divide the given polynomial by $\left(x^2-\frac{5}{3}\right)$ to obtain other zeros.

$$\begin{array}{r|l} & 3x^2+6x+3 \\ \hline x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & \underline{\mp 3x^4 \quad \mp 5x^2} \\ & 6x^3+3x^2-10x \\ & \underline{\mp 6x^3 \quad \mp 10x} \\ & 3x^2-5 \\ & \underline{\mp 3x^2 \mp 5} \\ & 0 \end{array}$$

So, $\quad 3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)(3x^2+6x+3)$

Now, $\quad 3x^2+6x+3=3(x^2+2x+1)=3(x+1)^2=3(x+1)(x+1)$

So its zeros are –1, –1.

Thus, all the zeros of given polynomial are $\sqrt{5/3}, -\sqrt{5/3}$, –1 and –1.

6. The given polynomial is f(x) = $(6x^3+\sqrt{2}x^2-10x-4\sqrt{2})$. Since $\sqrt{2}$ is the zero of f(x), it follows that $(x-\sqrt{2})$ is a factor of f(x).

On dividing f(x) by $(x-\sqrt{2})$, we get

$$\begin{array}{r|l} & 6x^2+7\sqrt{2}x+4 \\ \hline x-\sqrt{2} & 6x^3+\sqrt{2}x^2-10x-4\sqrt{2} \\ & \underline{\mp 6x^3 \pm 6\sqrt{2}x^2} \\ & 7\sqrt{2}x^2-10x \\ & \underline{\mp 7\sqrt{2}x^2 \pm 14x} \\ & 4x-4\sqrt{2} \\ & \underline{\mp 4x \pm 4\sqrt{2}} \\ & 0 \end{array}$$

$\therefore \quad f(x)=0 \Rightarrow (x-\sqrt{2})(6x^2+7\sqrt{2}x+4)=0 \Rightarrow (x-\sqrt{2})(3\sqrt{2}x+4)(\sqrt{2}x+1)=0$

$x-\sqrt{2}=0,\quad 3\sqrt{2}x+4=0,\quad \sqrt{2}x+1=0$

Hence, $x=\sqrt{2}, x=-\frac{2\sqrt{2}}{3}, x=\frac{-\sqrt{2}}{2}$ and All zeros of $f(x)$ are $\sqrt{2}, \frac{-2\sqrt{2}}{3}, \frac{-\sqrt{2}}{2}$.

7. ∵ p(x) = 6x3 + 3x2 – 5x + 1 so a = 6, b = 3, c = -5, d = 1

∴ α, β and γ are zeros of the polynomial p(x).

$$\therefore \quad \alpha+\beta+\gamma=\frac{-b}{a}=\frac{-3}{6}=\frac{-1}{2}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}=\frac{-5}{6} \quad \text{and} \quad \alpha\beta\gamma=\frac{-d}{a}=\frac{-1}{6}$$

Now $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}=\frac{-5/6}{-1/6}=5$

8. If α, β, γ are in AP., then,

$\beta-\alpha=\gamma+\beta \Rightarrow 2\beta=\alpha+\gamma$

$\alpha+\beta+\gamma=-\frac{b}{a}=\frac{-(-12)}{1}=12 \Rightarrow \alpha+\gamma=12-\beta$ .... (i)

From (i) and (ii)

$2\beta=12-\beta$ or $3\beta=12$ or $\beta=4$

Putting the value of β in (i), we have

$8=a+\gamma$

$\alpha\beta\gamma=-\frac{d}{a}=\frac{-(-28)}{1}=28$ .... (iii)

$(\alpha\gamma)\,4=28$ or $\alpha\gamma=7$ or $\gamma=7\alpha$ ..... (iv)

Putting the value of $\gamma=7\alpha$ in (iii), we get

$\Rightarrow 8=\alpha+\frac{7}{\alpha} \Rightarrow 8\alpha=\alpha^2+7$

$\Rightarrow \alpha^2-8\alpha+7=0 \Rightarrow \alpha^2-7\alpha-1\alpha+7=0$

$\Rightarrow \alpha(\alpha-7)-1(\alpha-7)=0 \Rightarrow (\alpha-1)(\alpha-7)=0$

$\Rightarrow \alpha=1$ or $\alpha=7$

Putting $\alpha=1$ in (*iv*), we get	Putting $\alpha=7$ in (*iv*), we get
$\gamma=\frac{7}{1}$	$\gamma=\frac{7}{7}$
or $\gamma=7$	or $\gamma=1$
and $\beta=4$	and $\beta=4$
∴ zeros are 1, 7, 4.	∴ zeros are 7, 4, 1.

Case Study Answers:

1. **Answer :**

i. (b) Quadratic

Solution:

Since, the given graph is parabolic is shape, therefore it will represent a quadratic polynomial.

[∵ Graph of quadratic polynomial is parabolic in shape)

ii. (c) -1, 5

Solution:

Since, the graph cuts the x-axis at -1, 5. So the polynomial has 2 zeroes i.e., -1 and 5.

iii. (a) 4

Solution:

Sum of zeroes = -1 + 5 = 4

iv. (c) 3

Solution:

Since α and β are zeroes of the given polynomial and $\beta > \alpha$, $\beta > \alpha$,

$\therefore \alpha = -1 \therefore \alpha = -1$ and $\beta = 5\beta = 5$

$\therefore |8\alpha + \beta| = |8(-1) + 5| = |-8 + 5| = |-3| = 3.$

$\therefore |8\alpha + \beta| = |8(-1)+5| = |-8+5| = |-3| = 3.$

v. (d) $-x^2 + 4x + 5$

Solution:

Since the zeroes of the given polynomial are -1 and 5.

$\therefore$ Required polynomial p(x)

$= k^2 \{x^2 - (-1 + 5)x + (-1)(5)\} = k(x^2 - 4x - 5)$

For k = -1, we get,

p(x) = $-x^2 + 4x + 5$, which is the required polynomial.

2. Answer :

i. (b) Parabolic.

Solution:

Graph of a quadratic polynomial is a parabolic in shape.

ii. (c) $x^2 - 36$

Solution:

Since the graph of the polynomial cuts the x-axis at (-6, 0) and (6, 0). So, the zeroes of polynomial are -6 and 6.

∴ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

iii. (c) 2

Solution:

We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

iv. (b) -2

Solution:

Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = -\frac{\text{coefficient of x}}{\text{coefficient of x}^2}$$

$$= -\frac{(2)}{1} = -2$$

v. (d) −53−53

Solution:

The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \frac{-5}{a} = \frac{3a}{a}$$

$$\Rightarrow a = \frac{-5}{3}$$

Assertion Reason Answer-

1. (d) Assertion (A) is false but reason (R) is true.
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).