MATHEMATICS

Important Questions

Multiple Choice questions-

Question. 1 x^2-2x +1 is a polynomial in:

a. One Variable

b. Two Variables

c. Three variable

d. None of the above

Question. 2 The coefficient of x^2 in $3x^3+2x^2-x+1$ is:

a. 1

b. 2

c. 3

d. -1

Question. 3 A binomial of degree 20 in the following is:

a. 20x + 1

b. x/20 + 1

c. x20 +1

d. x2+20

Question. 4 The degree of $4x^3-12x^2+3x+9$ is

a. 0

b. 1

c. 2

d. 3

Question. 5 $x^2 – x$ is ________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 6 $x – x^3$ is a ________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 7 1 + 3x is a __________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 8 The value of $f(x) = 5x-4x^2+3$ when x = -1, is:

a. 3

b. -12

c. -6

d. 6

Question. 9 The value of $p(t) = 2+t+2t^2-t^3$ when t=0 is

a. 2

b. 1

c. 4

d. 0

Question. 10 The zero of the polynomial f(x) = 2x+7 is

a. 2/7

b. -2/7

c. 7/2

d. -7/2

Very Short:

1. Factorise: $125x^3 - 64y^3$
2. Find the value of $(x + y)^2 + (x - y)^2$.
3. If $p(x)= x^2 - 2\sqrt{2}x+1$, then find the value of $p(2\sqrt{2})$
4. Find the value of m, if x + 4 is a factor of the polynomial $x^2 + 3x + m$.
5. Find the remainder when $x^3 + x^2 + x + 1$ is divided by $x - \frac{1}{2}$ using remainder theorem.
6. Find the common factor in the quadratic polynomials $x^2 + 8x + 15$ and $x^2 + 3x - 10$.

Short Questions:

1. Expand:

   (i) $(y - \sqrt{3})^2$

   (ii) $(x - 2y - 3z)^2$

2. If, $x + = \frac{1}{x} . = 7$

3. then find the value of $x^3 + \frac{1}{x^3}$

4. Show that $p - 1$ is a factor of $p^{10} + p^8 + p^6 - p^4 - p^2 - 1$.

5. If $3x + 2y = 12$ and $xy = 6$, find the value of $27x^3 + 8y^3$

6. Factorise: $4x^2 + 9y^2 + 16z^22 + 12xy - 24\ yz - 16xz$.

7. Factorise: $1 - 2ab - (a^2 + b^2)$.

8. Factories:

$$27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}.$$

Long Questions:

1. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b)(b + c)(c + a)$.
2. Factorise: $(m + 2n)^2 x^2 - 22x(m + 2n) + 72$.
3. If $x - 3$ is a factor of $x^2 - 6x + 12$, then find the value of k. Also, find the other factor of the – polynomial for this value of k.
4. Find a and b so that the polynomial $x^3 - 10x^2 + ax + b$ is exactly divisible by the polynomials $(x - 1)$ and $(x - 2)$.
5. Factorise: $x^2 - 6x^2 + 11x - 6$.

Assertion and Reason Questions:

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: If $f(x) = 3x^7 - 4x^6 + x + 9$ is a polynomial, then its degree is 7.

Reason: Aromatic aldehydes are almost as reactive as formaldehyde.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The expression $3x^4 – 4x^{3/2} + x^2 = 2$ is not a polynomial because the term $– 4x^{3/2}$ contains a rational power of x.

Reason: The highest exponent in various terms of an algebraic expression in one variable is called its degree.

Answer Key:

MCQ:

1. (a) One Variable
2. (b) 2
3. (c) x^{20} +1
4. (d) 3
5. (b) Quadratic
6. (c) Cubic
7. (a) Linear
8. (c) -6
9. (a) 2
10. (d) -7/2

Very Short Answer:

1. $125x^3– 6443 = (5x)^3 – (4y)^3$

   By using $a3 – b3 = (a – b) (a^2 + a^b + b^2)$, we obtain

   $125x^3– 64y3 = (5x – 4y) (25x2 + 20xy + 16y^2)$

2. $(x + y)^2 + (x – y)^2 = x^2 + y^2 + 2xy + x^2 + y^2 – 2xy$

$= 2x^2 + 2y^2 = 21x^2 + y^2)$

3. Put x = 2√2 in p(x), we obtain

$p(2\sqrt{2}) = (2\sqrt{2})^2 - 2\sqrt{2}(2\sqrt{2}) + 1 = (2\sqrt{2})^2 - (2\sqrt{2})^2 + 1 = 1$

4. Let $p(x) = x^2 + 3x + m$

Since (x + 4) or (x – (-4)} is a factor of p(x).

∴ p (-4) = 0

$\Rightarrow (-4)^2 + 3(-4) + m = 0$

$\Rightarrow 16 - 12 + m = 0$

$\Rightarrow m = -4$

5. Let p(x) = x3+ x2 + x + 1 and q(x) = $x - \frac{1}{2}$

Here, p(x) is divided by q(x)

∴ By using remainder theorem, we have

$$\text{Remainder} = p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1$$

$$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+2+4+8}{8} = \frac{15}{8}.$$

6. $x^2 + 8x + 15 = x^2 + 5x + 3x + 15 = (x + 3)(x + 5)$

$x^2 + 3x - 10 = x^2 + 5x - 2x - 10 = (x - 2)(x + 5)$

Clearly, the common factor is x + 5.

Short Answer:

Ans: 1. $(y - \sqrt{3})^2$ = y2 -2 × y × √3 + $(\sqrt{3})^2 = y^2 - 2\sqrt{3}\,y + 3$ $(x - 2y - 3z)^2$

$= x^2 + 1 - 2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

Ans: 2. We have $x + \frac{1}{x} = 7$

Cubing both sides, we have

$$\left(x+\frac{1}{x}\right)^3 = 7^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 7 = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21 = 322$$

Ans: 3. Let $f(p) = p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

Put p = 1, we obtain

$f(1) = 1^{10} + 1^8 + 1^6 - 1^4 - 1^2 - 1$

$= 1 + 1 + 1 - 1 - 1 - 1 = 0$

Hence, $p - 1$ is a factor of $p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

Ans: 4. We have $3x + 2y = 12$

On cubing both sides, we have

$\Rightarrow (3x + 2y)^3 = 12^3$

$\Rightarrow (3x)^3 + (2y)^3 + 3 \times 3x \times 2y(3x + 2y) = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 18xy(3x + 2y) = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 18 \times 6 \times 12 = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 1296 = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 = \sqrt{728} - 1296$

$\Rightarrow 27x^3 + 8y^3 = 432$

Ans: 5. $4x^2 + 9y^2 + 16z22 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(= 42) + 2(- 42)(2x)$

By using $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b + c)^2$, we obtain

$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

Ans: 6. $1 - 2ab - (a^2 + b^2) = 1 - (a^2 + b^2 + 2ab)$

$= 1^2 - (a + b)^2$

$= (1 + a + b)(1 - a - b)$

$[\because x^2 - y^2 = (x + y)(x - y)]$

Ans: 7.

$$27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2} = (3a)^3 + \frac{1}{(4b)^3} + 3.(3a).\left(\frac{1}{4b}\right)\left(3a + \frac{1}{4b}\right)$$

By using $x^3 + y^3 + 3xy(x + y) = (x + y)^3$, we have

$$= \left(3a + \frac{1}{4b}\right)^3.$$

Long Answer:

Ans: 1. L.H.S. $= (a + b + c)^3 - a^3 - b^3 - c^3$

$= \{(a + b + c)^3 - 3\} - \{b^3 + c^3\}$

$= (a + b + c - a)\{(a + b + c)^2 + a^2 + a(a + b + c)\} - (b + c)(b^2 + c^2 - bc)$

$= (b + c)\{a^2 + b^2 + 2 + 2ab + 2bc + 2ca + a^2 + a^2 + ab + ac - b^2 - a^2 + bc)$

$= (b + c)(3a2 + 3ab + 3bc + 3ca\}$

$= 3(b + c)\{a^2 + ab + bc + ca\}$

$= 31b + c)\{\{a^2 + ca) + (ab + bc)\}$

$= 3(b + c)\{a(a + c) + b(a + c)\}$

$= 3(b + c)(a + c)(a + b)$

$= 3(a + b)(b + c)(c + a)$ = R.H.S.

Ans: 2. Let $m + 2n = a$

$\therefore (m + 2n)^2 x^2 - 22x(m + 2n) + 72 = a^2x^2 - 22ax + 72$

$= a^2x^2 - 18ax - 4ax + 72$

$= ax(ax - 18) - 4(ax - 18)$

$= (ax - 4)(ax - 18)$

$= \{(m + 2n)x - 4)\}\{(m + 2n)x - 18)\}$

$= (mx + 2nx - 4)(mx + 2nx - 18)$.

Ans: 3. Here, $x - 3$ is a factor of $x^2 - kx + 12$

∴ By factor theorem, putting $x = 3$, we have remainder 0.

$\Rightarrow (3)^2 - k(3) + 12 = 0$

$\Rightarrow 9 - 3k + 12 = 0$

$\Rightarrow 3k = 21$

$\Rightarrow k = 7$

Now, $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= x(x - 3) - 4(x - 3)$

$= (x - 3)(x - 4)$

Hence, the value of k is 7 and other factor is $x - 4$.

Ans: 4. Let $p(x) = x^3 - 10x^2 + ax + b$

Since p(x) is exactly divisible by the polynomials $(x - 1)$ and $(x - 2)$.

∴ By putting $x = 1$, we obtain

$(1)^3 - 10(1)^2 + a(1) + b = 0$

$\Rightarrow a + b = 9$

And by putting x = 2, we obtain

$(2)^3 - 10(2)^2 + a(2) + b = 0$

$8 - 40 + 2a + b = 0$

$\Rightarrow 2a + b = 32$

Subtracting (i) from (ii), we have

a = 23

From (i), we have 23 + b = 9 = b = -14

Hence, the values of a and b are a = 23 and b = -14

Ans: 5. Let $p(x) = x^2 - 6x^2 + 11x - 6$

Here, constant term of p(x) is -6 and factors of -6 are ± 1, ± 2, ± 3 and ± 6

By putting x = 1, we have

$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

∴ (x – 1) is a factor of p(x)

By putting x = 2, we have

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

∴ (x – 2) is a factor of p(x)

By putting x = 3, we have

$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

∴ (x – 3) is a factor of p(x) Since p(x) is a polynomial of degree 3, so it cannot have more than three linear factors.

∴ $x^3 - 6x^2 + 11x - 6 = k(x-1)(x-2)(x-3)$

By putting x = 0, we obtain

$0 - 0 + 0 - 6 = k(-1)(-2)(3)$

$-6 = -6k$

k = 1

Hence, $x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$.

Assertion and Reason Answers:

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.