MATHEMATICS

Chapter 2: RELATIONS AND FUNCTIONS

Important Questions

Multiple Choice questions-

Question 1. The domain of the function ${}^{7-x}P_{x-3}$ is

(a) {1, 2, 3}

(b) {3, 4, 5, 6}

(c) {3, 4, 5}

(d) {1, 2, 3, 4, 5}

Question 2. The domain of $\tan^{-1}(2x + 1)$ is

(a) R

(b) R – {1/2}

(c) R – {-1/2}

(d) None of these

Question 3. Two functions f and g are said to be equal if f

(a) The domain of f = the domain of g

(b) The co-domain of f = the co-domain of g

(c) f(x) = g(x) for all x

(d) all of above

Question 4. If the function f : R → R be given by $f(x) = x^2 + 2$ and g : R → R is given by g(x) = x/(x – 1). The value of g of (x) is

(a) $(x^2 + 2)/(x^2 + 1)$

(b) $x^2/(x^2 + 1)$

(c) $x^2/(x^2 + 2)$

(d) None of these

Question 5. Given g(1) = 1 and g(2) = 3. If g(x) is described by the formula g(x) = ax + b, then the value of a and b is

(a) 2, 1

(b) -2, 1

(c) 2, -1

(d) -2, -1

Question 6. Let f : R → R be a function given by $f(x) = x^2 + 1$ then the value of $f^{-1}(26)$ is

(a) 5

(b) -5

(c) ±5

(d) None of these

Question 7. The function f(x) = x – [x] has period of

(a) 0

(b) 1

(c) 2

(d) 3

Question 8. The function $f(x) = \sin(\pi x/2) + \cos(\pi x/2)$ is periodic with period

(a) 4

(b) 6

(c) 12

(d) 24

Question 9. The domain of the function $f(x) = x/(1 + x^2)$ is

(a) R – {1}

(b) R – {-1}

(c) R

(d) None of these

Question 10. If f: R → R is defined by $f(x) = x^2 - 3x + 2$, the f(f(y)) is

(a) $x^4 + 6x^3 + 10x^2 + 3x$

(b) $x^4 - 6x^3 + 10x^2 + 3x$

(c) $x^4 + 6x^3 + 10x^2 - 3x$

(d) $x^4 - 6x^3 + 10x^2 - 3x$

Very Short Questions:

1. Find a and b if (a – 1, b + 5) = (2, 3) If A = {1,3,5}, B = {2,3} find:
2. A × B
3. B × A

Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. A × (B ∩ C)
5. A × (B ∪ C)
6. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B
7. If A = {1,2,3,5} and B = {4,6,9}, R = {(x, y): |x – y| is odd, x ∈ A, y ∈ B} Write R in roster form Which of the following relations are functions? Give reason.
8. R = {(1,1), (2,2), (3,3), (4,4), (4,5)}
9. R = {(2,1), (2,2), (2,3), (2,4)}
10. R = {(1,2), (2,5), (3,8), (4,10), (5,12), (6,12)} Which of the following arrow diagrams represent a function? Why?

Short Questions:

1. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as, R = {(x, y) : x ϵ A, y B and y = x2}

   (a) Depict this relation using arrow diagram.

   (b) Find domain of R.

   (c) Find range of R.

   (d) Write co-domain of R.
2. Let R = { (x, y) : x, y ϵ N and y = 2x} be a relation on N. Find :

   (i) Domain

   (ii) Codomain

   iii) Range

   Is this relation a function from N to N
3. Find the domain and range of, f(x) = |2x – 3| – 3
4. Draw the graph of the Constant function, f : R ϵ R; f(x) = 2 ∀ x ϵ R. Also find its domain and range.
5. Let R = {(x, - y) : x, y ϵ W , 2x + y = 8) then

   (i) Find the domain and the range of R (ii) Write R as a set of ordered pairs.
6. Let R be a relation from Q to Q defined by R = {(a,b) : a, b ϵ Q and a -b ϵ z), Show that.

   $(i)(a,a) \in R$ for all $a \in Q$ $(ii)(a,b) \in R$ implies that $(b,a) \in R$

   $(iii)(a,b) \in R$ and $(b,c) \in R$ implies that $(a,c) \in R$
7. If $f(x) = \frac{x^2-3x+1}{x-1}$, find $f(-2) + f\left(\frac{1}{3}\right)+$
8. Find the domain and the range of the function f(x) = $3x^2 – 5$. Also find f (-3) and the

numbers which are associated with the number 43 m its range.

9. If f (x) = x^2 - 3x + 1, find x such that f (2x) = 2 f(x).

10. Find the domain and the range of the function $f(x) - \sqrt{x-1}$.

Long Questions:

1. Draw the graphs of the following real functions and hence find their range

$f(x) = \frac{1}{x}, x \in R, x \neq 0$

2. If f(x) = $x - \frac{1}{4}$, Prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

3. Draw the graphs of the following real functions and hence find their range

4. Let f be a function defined by $F: x \to 5x^2 + 2, x \in R$

(i) find the image of 3 under f.

(ii) find f(3) + f(2).

(iii) find x such that f(x) = 22

5. The function $f(x) = \frac{9x}{5} + 32$ is the formula to connect x˚c to Fahrenheit units find (i) f(0) (ii) f(-10) (iii) the value of x f(x) = 212 interpret the result is each case.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : If (x +1, y – 2) = (3, 1), then x = 2 and y = 3.

   Reason (R) : Two ordered pairs are equal if their corresponding elements are equal.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The cartesian product of two non-empty sets P and Q is denoted as P × Q and P × Q = {(p, q) : p ∈ P, q ∈ Q}.

   Reason (R) : If A = {red, blue} and B = {b, c, s}, then A × B = {(red, b), (red, c), (red), (blue, b)

(blue, c) (blue, s)}

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (c) {3, 4, 5}

2. (a) R

3. (d) all of above

4. (a) $(x^2 + 2)/(x^2 + 1)$

5. (c) 2, -1

6. (c) ±5

7. (b) 1

8. (a) 4

9. (c) R

10. (d) $x^4 - 6x^3 + 10x^2 - 3x$

Very Short Answer:

1. a = 3, b = –2

2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)

3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)

4. {(1,4), (2,4)

5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)

6. $2^6 = 6$

7. R = { (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)

8. Not a function because 4 has two images.

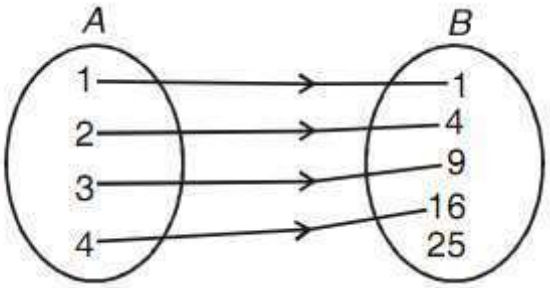
9. Not a function because 2 does not have a unique image.

10. Function

Short Answer:

1.

(a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

2. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

3. Domain is R

Range is [–3, ∞)

4. Domain = R

Range = {2}

5. (i) Given and $2x + y = 8$ and $x, y \in w$

Put

$x = 0, 2\times 0 + y = 8 \Rightarrow y = 8,$

$x = 1, 2\times 1 + y = 8 \Rightarrow y = 6,$

$x = 2, 2\times 2 + y = 8 \Rightarrow y = 4,$

$x = 3, 2\times 3 + y = 8 \Rightarrow y = 2,$

$x = 4, 2\times 4 + y = 8 \Rightarrow y = 0$

for all other values of $x, y \in w$ we do not get $y \in w$

$\therefore$ Domain of R = {0,1 2, 3, 4} and range of R = {8, 6, 4, 2, 0}

(ii) R as a set of ordered pairs can be written as

$$R = \{(0,8),(1,6),(2,4),(3,2),(4,0)\}$$

6.

$$R = [(a,b): a,b \in Q \text{ and } a-b \in z]$$

Swotters

(i) For all $a \in Q, a-a=0$ and $0 \in z$, it implies that $(a,a) \in R$.

(ii) Given $(a,b) \in R \Rightarrow a-b \in z \Rightarrow -(a-b) \in z$

$\Rightarrow b-a \in z \Rightarrow (b,a) \in R$.

(iii) Given $(a,b) \in R$ and $(b,c) \in R \Rightarrow a-b \in z$ and $b-c \in z \Rightarrow (a-b)+(b-c) \in z$

$\Rightarrow a-c \in z \Rightarrow (a-c) \in R$.

7.

Given $f(x)=\dfrac{x^2-3x+1}{x-1}, Df = R-\{1\}$

$\therefore f(-2)=\dfrac{(-2)^2-3(-2)+1}{-2-1}-\dfrac{4+6+1}{-3}=1\dfrac{1}{3}$ and

$f\left(\dfrac{1}{3}\right)=\dfrac{\left(\dfrac{1}{3}\right)^2-3\times\dfrac{1}{3}+1}{\dfrac{1}{3}-1}=\dfrac{\dfrac{1}{9-1+1}}{-\dfrac{2}{3}}=\dfrac{\dfrac{1}{9}}{-\dfrac{2}{3}}=\dfrac{1}{9}\times\left(-\dfrac{3}{2}\right)=-\dfrac{1}{6}$

$\therefore f(-2)+f\left(\dfrac{1}{3}\right)=-\dfrac{11}{3}-\dfrac{1}{6}=\dfrac{-22-1}{6}=\dfrac{-23}{6}=3\dfrac{5}{6}.$

8.

Given $f(x)=3x^2-5$

For Df, $f(x)$ must be real number

$\Rightarrow 3x^2-5$ must be a real number

Which is a real number for every $x \in R$

Swotters

$\Rightarrow Df = R.....(i)$

$for\ Rf$, let $y = f(x) = 3x^2 - 5$

We know that for all $x \in R, x^2 \geq 0 \Rightarrow 3x^2 \geq 0$

$\Rightarrow 3x^2 - 5 \geq -5 \Rightarrow y \geq -5 \Rightarrow Rf = [-5, \infty]$

Funthes, as $-3 \in Df, f(-3)$ exists is and $f(-3)$

$= 3(-3)^2 - 5 = 22.$

As $43 \in Rf$ on putting $y = 43$ is (i) weget

$3x^2$-5=43 $\Rightarrow 3x^2$=48 $\Rightarrow x^2$=16 $\Rightarrow x = -4, 4.$

There fore -4 and 4 are number

(is Df) which are associated with the number 43 in Rf

9.

Given $f(x) = x^2 - 3x + 1, Df = R$

$\therefore f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$

As $f(2x) = f(x)$(Given)

$\Rightarrow 4x^2 - 6x + 1 = x^2 - 3x + 1$

$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1.$

10.

Given $f(x) = \sqrt{x-1},$

for $Df, f(x)$ must be a real number

$\Rightarrow \sqrt{x-1}$must be a real number

$\Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1$

$\Rightarrow Df = [1, \infty]$

for Rf, let $y = f(x) = \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

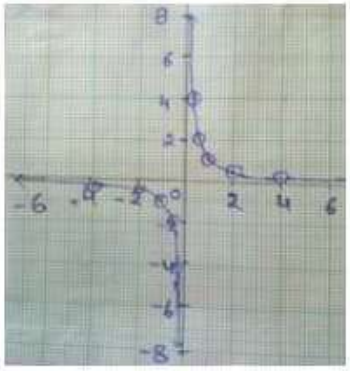
$\Rightarrow Rf = [0, \infty]$

Long Answer:

1.

Given $f(x)=\frac{1}{x}, x\in R, x\neq 0$

Let $y=f(x)=i\ell y=\frac{1}{x}, x\in R, x\neq 0$



(Fig for Answer 11)

x	−4	−2	−1	−0.5	−0.25	0.5	1	2	4
$y=\frac{1}{x}$	−0.25	−0.5	−1	−2	−4	2	1	0.5	0.25

Plot the points shown is the above table and join there points by a free hand drawing.

Portion of the graph are shown the right margin

From the graph, it is clear that Rf = R – [0]

This function is called reciprocal function.

2.

If $f(x)=x-\frac{1}{x}$, prove that $[f(x)]^3=f(x^3)+f\left(\frac{1}{x}\right)$

Given $f(x)=x-\frac{1}{x}, Df=R-[0]$

$\Rightarrow f(x^3)=x^3-\frac{1}{x^3}$ and $f\left(\frac{1}{x}\right)=\frac{1}{x}-\frac{1}{\frac{1}{x}}=\frac{1}{x}-x.....(i)$

$\therefore [f(x)]^3=\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3x.\frac{1}{x}\left(x-\frac{1}{x}\right)$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$$

$$= f\left(x^3\right) + 3f\left(\frac{1}{x}\right)\left[\text{using } (i)\right]$$

3.

(i)Given, f(x) i.e. y = x - 1 which is first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine straight lint uniquely

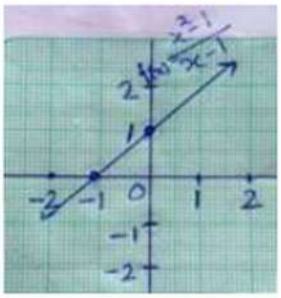


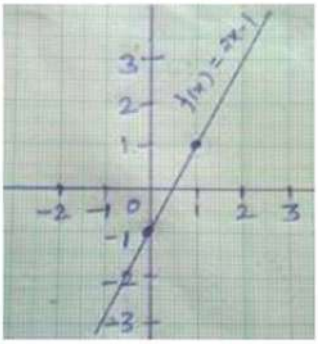
Table of values

x	0	1
y	-1	1

A portion of the graph is shown in the figure from the graph, it is clear that y takes all real values. It therefore that

$R_F = R$

(ii) Given $f(x) = \frac{x^2 - 1}{x - 1} \Rightarrow D_F = R - (1)$



Let $y = f(x) = \frac{x^2 - 1}{x - 1} = x + 1 (\because x \neq 1)$

i.e y = x + 1 which is a first degree equation is and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

Table of values

x	-1	0
y	0	1

A portion of the graph is shown is the figure from the graph it is clear that y takes all real values except 2. It fallows that $R_F = R - [2]$.

4.

Given $f(x) = 5x^2 + 2, x \in R$

(i) $f(3) = 5 \times 3^2 + 2 = 5 \times 9 + 2 = 47$

(ii) $f(2) = 5 \times 2^2 + 2 = 5 \times 4 + 2 = 22$

$\therefore f(3) \times f(2) = 47 \times 22 = 1034$

(iii) $f(x) = 22$

$\Rightarrow 5x^2 + 2 = 22$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2, \ -2$

5.

$f(x) = \frac{9x}{5} + 32 (given)$

$(i)\ f(0) = \left(\frac{9 \times 0}{5} + 32\right) = 32 \Rightarrow f(0) = 32 \Rightarrow 0^\circ c = 32^\circ F$

$(ii)\ f(-10) = \left(\frac{9 \times (-10)}{5} + 32\right) = 14 \Rightarrow f(-10) = 14^\circ \Rightarrow (-10)^\circ c = 14^\circ F$

$(iii)\ f(x) = 212 \Leftrightarrow \frac{9x}{5} + 32 = 212 \Leftrightarrow 9x = 5 \times (180)$

$\Leftrightarrow x = 100$

$\therefore 212^\circ f = 100^\circ c$

Assertion Reason Answer:

1. (i) Both assertion and reason are true and reason is the correct explanation of assertion.
2. (i) Both assertion and reason are true and reason is the correct explanation of assertion.