MATHEMATICS

Important Questions

Multiple Choice questions-

1. Graphically, the pair of equations $7x - y = 5$; $21x - 3y = 10$ represents two lines which are

(a) intersecting at one point

(b) parallel

(c) intersecting at two points

(d) coincident

2. The pair of equations $3x - 5y = 7$ and $-6x + 10y = 7$ have

(a) a unique solution

(b) infinitely many solutions

(c) no solution

(d) two solutions

3. If a pair of linear equations is consistent, then the lines will be

(a) always coincident

(b) parallel

(c) always intersecting

(d) intersecting or coincident

4. The pair of equations $x = 0$ and $x = 5$ has

(a) no solution

(b) unique/one solution

(c) two solutions

(d) infinitely many solutions

5. The pair of equation $x = -4$ and $y = -5$ graphically represents lines which are

(a) intersecting at (- 5, – 4)

(b) intersecting at (- 4, – 5)

(c) intersecting at (5, 4)

(d) intersecting at (4, 5)

6. One equation of a pair of dependent linear equations is $2x + 5y = 3$. The second equation will be

(a) $2x + 5y = 6$

(b) $3x + 5y = 3$

(c) $-10x – 25y + 15 = 0$

(d) $10x + 25y = 15$

7.If $x = a$, $y = b$ is the solution of the equations $x + y = 5$ and $2x – 3y = 4$, then the values of a and b are respectively

(a) 6, -1

(b) 2, 3

(c) 1, 4

(d) 19/5, 6/5

8. The graph of $x = -2$ is a line parallel to the

(a) x-axis

(b) y-axis

(c) both x- and y-axis

(d) none of these

9. The graph of $y = 4x$ is a line

(a) parallel to x-axis

(b) parallel to y-axis

(c) perpendicular to y-axis

(d) passing through the origin

10. The graph of y = 5 is a line parallel to the

(a) x-axis

(b) y-axis

(c) both axis

(d) none of these

Very Short Questions:

1. If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then find value of k.

2. Find the value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have infinitely many solutions.

3. Do the equations 4x + 3y – 1 = 5 and 12x + 9y = 15 represent a pair of coincident lines?

4. Find the co-ordinate where the line x – y = 8 will intersect y-axis.

5. Write the number of solutions of the following pair of linear equations:

x + 2y – 8 = 0, 2x + 4y = 16

6. Is the following pair of linear equations consistent? Justify your answer.

2ax + by = a, 4ax + 2by – 2a = 0; a, b≠ 0

7. For all real values of c, the pair of equations

x – 2y = 8, 5x + 10y = c

have a unique solution. Justify whether it is true or false.

8. Does the following pair of equations represent a pair of coincident lines? Justify your answer.

$$\frac{x}{2} + y + \frac{2}{5} = 0, 4x + 8y + \frac{5}{16} = 0.$$

9. If x = a, y = b is the solution of the pair of equation x – y = 2 and x + y = 4, then find the value of a and b.

10. $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

Short Questions :

1. Solve: $ax + by = a - b$ and $bx - ay = a + b$

2. Solve the following linear equations:

$152x - 378y = -74$ and $-378x + 152y = -604$

3. Solve for x and y

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; \quad x + y = 2ab$$

4. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

(ii) for which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

5. Find whether the following pair of linear equations has a unique solution. If yes, find the

$7x - 4y = 49$ and $5x - y = 57$

6. Solve for x and y.

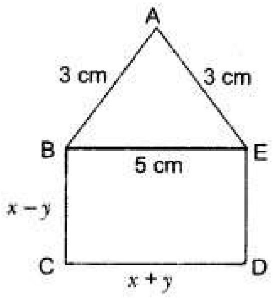
$$\frac{6}{x-1} - \frac{3}{y-2} = 1; \quad \frac{5}{x-1} + \frac{1}{y-2} = 2 \text{ where } x \neq 1, y \neq 2$$

7. Solve the following pair of equations for x and y.

$$\frac{a^2}{x} - \frac{b^2}{y} = 0; \quad \frac{a^2b}{x} + \frac{b^2a}{y} = a + b, \ x \neq 0, y \neq 0.$$

8. In ΔABC, LA = x, ∠B = 3x, and ∠C = y if 3y – 5x = 30°, show that triangle is right angled.

9. In Fig. 3.1, ABCDE is a pentagon with BE|CD and BC||DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm. Find the value of x and y.



10. Five years ago, A was thrice as old as B and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Long Questions :

1. Form the pair of linear equations in this problem and find its solution graphically: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

2. Show graphically the given system of equations

$2x + 4y = 10$ and $3x + 6y = 12$ has no solution.

3. Solve the following pairs of linear equations by the elimination method and the substitution method:

(i) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(ii) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

4. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay 31000 as hostel charges whereas a student B, who takes food for 26 days, pays 1180 as hostel charges. Find the fixed charges and the cost of food per day.

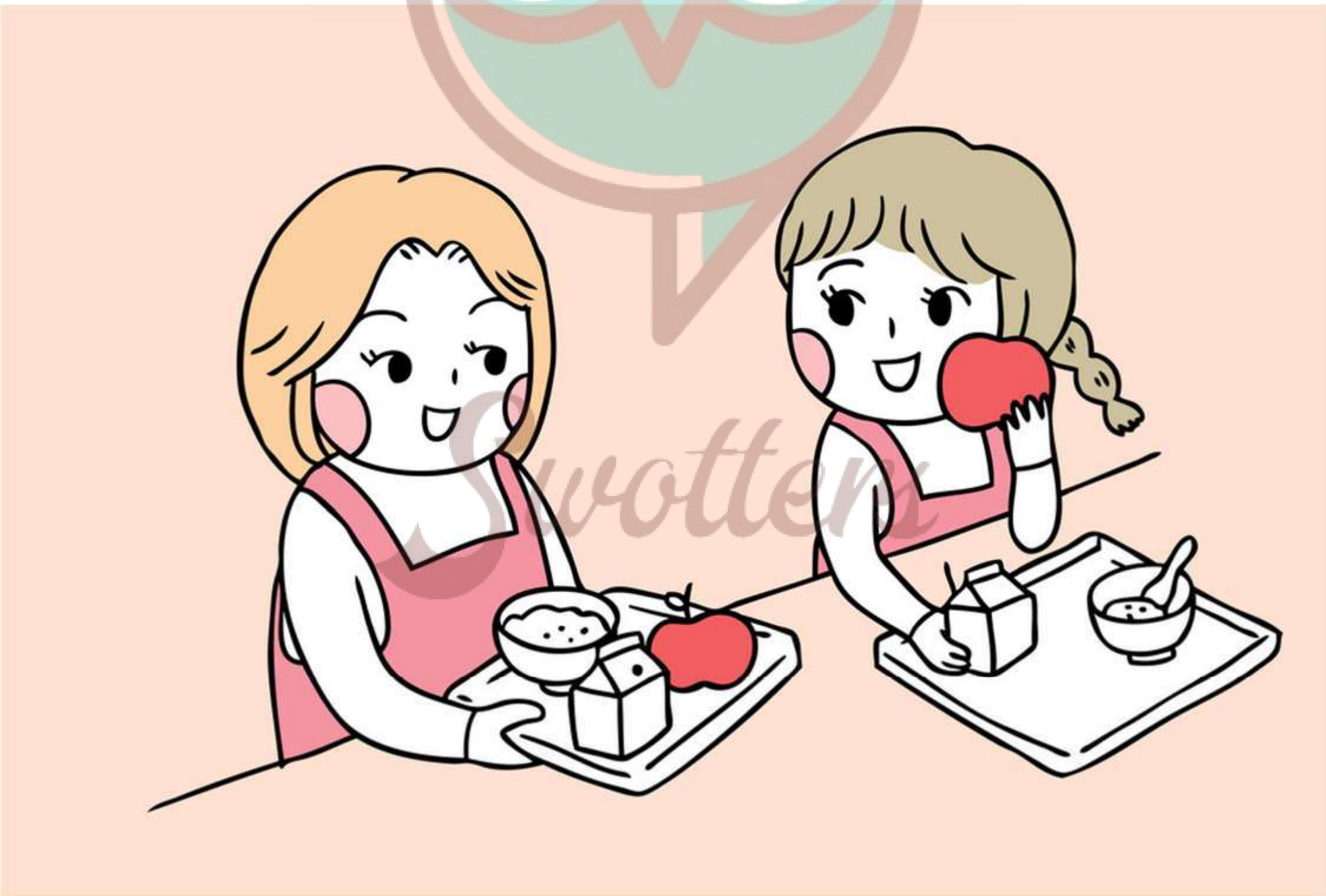
6. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deduced for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

7. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

8. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Case Study Questions:

1. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student Anu takes food for 25 days, she has to pay ₹ 4500 as hostel charges, whereas another student Bindu who takes food for 30 days, has to pay ₹ 5200 as hostel charges.



Considering the fixed charges per month by ₹ x and the cost of food per day by ₹ y, then answer the following questions.

i. Represent algebraically the situation faced by both Anu and Bindu.

a. $x + 25y = 4500$, $x + 30y = 5200$
b. $25x + y = 4500$, $30x + y = 5200$
c. $x - 25y = 4500$, $x - 30y = 5200$
d. $25x - y = 4500$, $30x - y = 5200$

ii. The system of linear equations, represented by above situations has.

a. No solution.
b. Unique solution.
c. Infinitely many solutions.
d. None of these.

iii. The cost of food per day is:

a. ₹ 120
b. ₹ 130
c. ₹ 140
d. ₹ 1300

iv. The fixed charges per month for the hostel is:

a. ₹ 1500
b. ₹ 1200
c. ₹ 1000
d. ₹ 1300

v. If Bindu takes food for 20 days, then what amount she has to pay?

a. ₹ 4000
b. ₹ 3500
c. ₹ 3600
d. ₹ 3800

2. From Bengaluru bus stand, if Riddhima buys 2 tickets to Malleswaram and 3 tickets to Yeswanthpur, then total cost is ₹ 46; but if she buys 3 tickets to Malleswaram and 5 tickets to Yeswanthpur, then total cost is ₹ 74.

Consider the fares from Bengaluru to Malleswaram and that to Yeswanthpur as ₹ x and ₹ y respectively and answer the following questions.

i. 1st situation can be represented algebraically as:

a. 3x - 5y = 74
b. 2x + 5y = 74
c. 2x - 3y = 46
d. 2x + 3y = 46

ii. 2nd situation can be represented algebraically as:

a. 5x + 3y = 74
b. 5x - 3y = 74
c. 3x + 5y = 74
d. 3x - 5y = 74

iii. Fare from Bengaluru to Malleswaram is:

a. ₹ 6
b. ₹ 8
c. ₹ 10
d. ₹ 2

iv. Fare from Bengaluru to Yeswanthpur is:

a. ₹ 10
b. ₹ 12
c. ₹ 14
d. ₹ 16

v. The system of linear equations represented by both situations has:

a. Infinitely many solutions.
b. No solution.
c. Unique solution.
d. None of these.

Assertion reason questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. (C) Assertion (A) is true but reason (R) is false.

d. (d) Assertion (A) is false but reason (R) is true.

Assertion: The graph of the linear equations 3x+2y=12 and 5x-2y=4 gives a pair of intersecting lines.

Reason: The graph of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$

2. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If the pair of lines are coincident, then we say that pair of lines is consistent and it has a unique solution.

Reason: If the pair of lines are parallel, then the pairs has no solution and is called inconsistent pair of equations.

Answer Key-

Multiple Choice questions-

1. (b) -10
2. (d) a – 0, b = -6
3. (d) more than 3
4. (a) b – a + 1
5. (b) both negative
6. (a) cannot both be positive
7. (c) c and a have the same sign
8. (a) has no linear term and the constant term is negative.
9. (d) more than 4
10. (b) $x^2 + 9x + 20$
11. (a) both negative

Very Short Answer :

1. Since the given lines are parallel

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1} \qquad i.e., \ k = \frac{15}{4}.$$

2. The given system of equations will have infinitely many solutions if $\frac{c}{6} = \frac{-1}{-2} = \frac{2}{3}$ which is not possible

∴ For no value of c, the given system of equations have infinitely many solutions.

3.

Here, $\frac{4}{12} = \frac{3}{9} \neq \frac{6}{15}$ i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Given equations do not represent a pair of coincident lines.

4. The given line will intersect y-axis when x = 0.

∴0 – y = 8 ⇒ y = -8

Required coordinate is (0, -8).

5.

Here, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴The given pair of linear equations has infinitely many solutions.

6. Yes,

Here, $\frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$

∵ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The given system of equations is consistent.

7.

Here, $\frac{a_1}{a_2} = \frac{1}{5}$, $\frac{b_1}{b_2} = \frac{-2}{+10} = \frac{-1}{5}$, $\frac{c_1}{c_2} = \frac{8}{c}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

8.

Here, $a_1 = \frac{1}{2}$, $b_1 = 1$, $c_1 = \frac{2}{5}$ and $a_2 = 4$, $b_2 = 8$, $c_2 = \frac{5}{16}$

$$\frac{a_1}{a_2} = \frac{\frac{1}{2}}{4} = \frac{1}{8}, \quad \frac{b_1}{b_2} = \frac{1}{8}, \quad \frac{c_1}{c_2} = \frac{\frac{2}{5}}{\frac{5}{16}} = \frac{32}{25}$$

$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ The given system does not represent a pair of coincident lines.

x – y = 2 … (i)

x + y = 4 … (ii)

9. On adding (i) and (ii), we get 2x = 6 or x = 3

From (i), 3 – y ⇒ 2 = y = 1

a = 3, b = 1.

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$, and, $\frac{c_1}{c_2}$ find out whether the following pair of linear equations consistent or inconsistent. is consistent or inconsistent.

10.

We have, $\frac{3}{2}x + \frac{5}{3}y = 7$ …(i)

$9x - 10y = 14$ …(ii)

Here, $a_1 = \frac{3}{2}$, $b_1 = \frac{5}{3}$, $c_1 = 7$

$a_2 = 9$, $b_2 = -10$, $c_2 = 14$

Thus, $\frac{a_1}{a_2} = \frac{3}{2 \times 9} = \frac{1}{6}$, $\frac{b_1}{b_2} = \frac{5}{3 \times (-10)} = -\frac{1}{6}$

Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. So, it has unique solution and it is consistent.

Short Answer :

1. The given system of equations may be written as

ax + by – (a – b) = 0

bx – ay – (a + b) = 0

By cross-multiplication, we have

$$\frac{x}{\begin{matrix} b & -(a-b) \\ -a & -(a+b) \end{matrix}} = \frac{-y}{\begin{matrix} a & -(a-b) \\ b & -(a+b) \end{matrix}} = \frac{1}{\begin{matrix} a & b \\ b & -a \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{b\times-(a+b)-(-a)\times-(a-b)} = \frac{-y}{a\times-(a+b)-b\times-(a-b)} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \quad \frac{x}{-b(a+b)-a(a-b)} = \frac{-y}{-a(a+b)+b(a-b)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-b^2-a^2} = \frac{-y}{-a^2-b^2} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad \frac{x}{-(a^2+b^2)} = \frac{y}{(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \quad x = -\frac{(a^2+b^2)}{-(a^2+b^2)} = 1 \quad \text{and} \quad y = \frac{(a^2+b^2)}{-(a^2+b^2)} = -1$$

Hence, the solution of the given system of equations is x = 1, y = -1

2. We have, $152x - 378y = -74$ …(i)

$-378x + 152y = -604$ ……(ii)

Adding equation (*i*) and (*ii*), we get

$$\begin{aligned} 152x - 378y &= -74 \\ -378x + 152y &= -604 \\ \hline -226x - 226y &= -678 \end{aligned}$$

$$\Rightarrow \quad -226(x+y) = -678$$

$$\Rightarrow \quad x + y = \frac{-678}{-226}$$

$$\Rightarrow \quad x + y = 3 \qquad \ldots(iii)$$

Subtracting equation (*ii*) from (*i*), we get

$$\begin{aligned} 152x - 378y &= -74 \\ -378x + 152y &= -604 \\ + \quad\quad - \quad\quad & \quad + \\ \hline 530x - 530y &= 530 \end{aligned}$$

$$\Rightarrow \quad x - y = 1 \qquad \ldots(iv)$$

Adding equations (*iii*) and (*iv*), we get

$$\begin{aligned} x + y &= 3 \\ x - y &= 1 \\ \hline 2x &= 4 \end{aligned} \quad \Rightarrow \quad x = 2$$

Putting the value of x in (iii), we get

$2 + y = 3 \Rightarrow y = 1$

Hence, the solution of given system of equations is x = 2, y = 1.

3.

We have, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$ ...(i)

$x + y = 2ab$ ...(ii)

Multiplying (ii) by b/a, we get

$\frac{b}{a}x + \frac{b}{a}y = 2b^2$ ...(iii)

Subtracting (iii) from (i), we get

$\left(\frac{a}{b} - \frac{b}{a}\right)y = a^2 + b^2 - 2b^2 \Rightarrow \left(\frac{a^2 - b^2}{ab}\right)y = (a^2 - b^2)$

$\Rightarrow y = (a^2 - b^2) \times \frac{ab}{(a^2 - b^2)} \Rightarrow y = ab$

4. (i) We have, 2x + 3y = 7

(a – b) x + (a + b) y = 3a + b – 2 ... (ii)

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = 7$ and

$a_2 = a - b$, $b_2 = a + b$, $c_2 = 3a + b - 2$

For infinite number of solutions, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a - b} = \frac{3}{a + b} = \frac{7}{3a + b - 2}$

Now, $\frac{2}{a - b} = \frac{3}{a + b}$

$\Rightarrow 2a + 2b = 3a - 3b \Rightarrow 2a - 3a = -3b - 2b$

$\Rightarrow -a = -5b$ ...(iii)

$\therefore a = 5b$

Again, we have

$\frac{3}{a + b} = \frac{7}{3a + b - 2} \Rightarrow 9a + 3b - 6 = 7a + 7b$

⇒ 9a – 7a + 3b – 75 -6 = 0 ⇒ 2a – 45 – 6 = 0 => 2a – 4b = 6

⇒ a – 2b = 3 …(iv)

Putting a = 5b in equation (iv), we get

56 – 2b = 3 or 3b = 3 i.e., b = $\frac{3}{3}$ =1

Putting the value of b in equation (ii), we get a = 5(1) = 5

Hence, the given system of equations will have an infinite number of solutions for a = 5 and b = 1.

(ii) We have, 3x + y = 1, 3x + y − 1 = 0 …(i)

(2k – 1) x + (k – 1) y = 2k + 1

⇒ (2k – 1) x + (k – 1) y – (2k + 1) = 0 ……(ii)

Here, a_1 = 3, b_1 = 1, C_1 = -1

a_2 = 2k – 1, b_2 = k – 1, C_2 = -(2k + 1)

For no solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

Now, $\frac{3}{2k-1} = \frac{1}{k-1} \quad \Rightarrow \quad 3k - 3 = 2k - 1$

⇒ 3k – 2k = 3 – 1 ⇒ k = 2

5. Hence, the given system of equations will have no solutions for k = 2.

We have, 7x – 4y = 49 ……..(i)

and 5x – 6y = 57 ……..(ii)

Here, $a_1 = 7,\ b_1 = -4,\ c_1 = 49$

$a_2 = 5,\ b_2 = -6,\ c_2 = 57$

So, $\frac{a_1}{a_2} = \frac{7}{5},\ \frac{b_1}{b_2} = \frac{-4}{-6} = \frac{2}{3}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, system has a unique solution.

Multiply equation (i) by 5 and equation (ii) by 7 and subtract

$$\begin{array}{r} 35x - 20y = 245 \\ \underset{-}{}35x \underset{+}{-} 42y = \underset{-}{}399 \\ \hline 22y = -154 \end{array} \quad \Rightarrow \quad y = -7$$

Put y = -7 in equation (ii)

5x – 6(-7)57 ⇒ 5x = 57 – 42 ⇒ x = 3

hence, x = 3 and y = -7.

6.

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$

The given equations become

$$6p - 3q = 1 \quad \text{...}(i)$$

$$5p + q = 2 \quad \text{...}(ii)$$

Multiply equation (*ii*) by 3 and add in equation (*i*)

$$\begin{array}{r} 15p + 3q = 6 \\ 6p - 3q = 1 \\ \hline 21p \quad = 7 \end{array} \quad \Rightarrow \quad p = \frac{7}{21} = \frac{1}{3}$$

Putting this value in equation (*i*) we get

$$6 \times \frac{1}{3} - 3q = 1 \quad \Rightarrow \quad 2 - 3q = 1 \quad \Rightarrow \quad 3q = 1, \quad \Rightarrow \quad q = \frac{1}{3}$$

Now, $\frac{1}{x-1} = p = \frac{1}{3} \quad \Rightarrow \quad x - 1 = 3 \quad \Rightarrow \quad x = 4$

$$\frac{1}{y-2} = q = \frac{1}{3} \quad \Rightarrow \quad y - 2 = 3 \quad \Rightarrow \quad y = 5$$

Hence, $x = 4$ and $y = 5$.

7.

$$\frac{a^2b}{x}+\frac{b^2a}{y}=a+b \quad ...(ii)$$

Multiply equation (i) by a and adding to equation (ii)

$$\frac{a^2a}{x}-\frac{b^2a}{y}+\frac{a^2b}{x}+\frac{b^2a}{y}=0+(a+b)$$

$$\Rightarrow \quad \frac{a^3}{x}+\frac{a^2b}{x}=a+b \quad \Rightarrow \quad \frac{a^2}{x}(a+b)=a+b \Rightarrow x=\frac{a^2(a+b)}{a+b}=a^2$$

Putting the value of x in equation (i), we get

$$\frac{a^2}{a^2}-\frac{b^2}{y}=0 \quad \Rightarrow \quad 1-\frac{b^2}{y}=0 \quad \Rightarrow \quad \frac{b^2}{y}=1 \quad \Rightarrow \quad y=b^2$$

Hence, $x=a^2, y=b^2$.

8. ∠A + 2B + ∠C = 180°

(Sum of interior angles of A ABC) x + 3x + y = 180°

4x + y = 180° …(i)

3y – 5x = 30° (Given) …(ii) Multiply equation (i) by 3 and subtracting from eq. (ii), we get

-17x = – 510 = x = 910 = 30°

17 then _A = x = 30° and 2B = 3x = 3 X 30o = 90°

∠C = y = 180° – (∠A + ∠B) = 180° – 120° = 60°

∠A = 30°, ∠B = 90°, ∠C = 60° Hence ΔABC is right triangle right angled at B.

9. Since BC||DE and BE||CD with BC||CD.

BCDE is a rectangle.

Opposite sides are equal BE = CD

∴ x + y = 5 …… (i)

DE = BC = x – y

Since perimeter of ABCDE is 21 cm.

AB + BC + CD + DE + EA = 21

$3 + x - y + x + y + x - y + 3 = 21 \Rightarrow 6 + 3x - y = 21$

$3x - y = 15$ ..... (iii)

Adding (i) and (ii), we get

$4x = 20 \Rightarrow x = 5$

On putting the value of x in (i), we get $y = 0$

Hence, $x = 5$ and $y = 0$.

10. Let the present ages of B and A be x years and y years respectively. Then

B's age 5 years ago = $(x - 5)$ years

and A's age 5 years ago = $(- 5)$ years

$(-5) = 3(x - 5) = 3x - y = 10$ ....(i)

B's age 10 years hence = $(x + 10)$ years

A's age 10 years hence = $(y + 10)$ years

$y + 10 = 2(x + 10) = 2x - y = -10$ .... (ii)

On subtracting (ii) from (i) we get $x = 20$

Putting $x = 20$ in (i) we get

$(3 \times 20) - y = 10 \Rightarrow y = 50$

$\therefore x = 20$ and $y = 50$

Hence, B's present age = 20 years and A's present age = 50 years.

Long Answer :

1. Let x be the number of girls and y be the number of boys.

According to question, we have

$x = y + 4$

$\Rightarrow x - y = 4$ ......(i)

Again, total number of students = 10

Therefore, x + y = 10 ...(ii)

Hence, we have following system of equations

x – y = 4

and x + y = 10

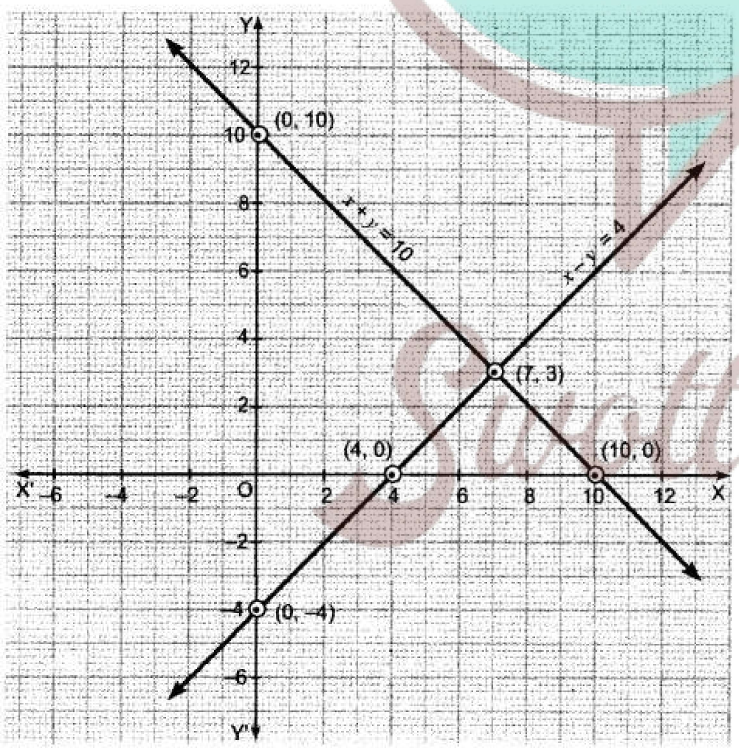
From equation (i), we have the following table:

x	0	4	7
y	– 4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have



Here, the two lines intersect at point (7,3) i.e., x = 7, y = 3.

So, the number of girls = 7

and number of boys = 3.

2. We have, 2x + 4y = 10

$\Rightarrow$ 4y = 10 – 2x $\Rightarrow y = \frac{5-x}{2}$

Thus, we have the following table:

x	1	3	5
y	2	1	0

Plot the points A (1, 2), B (3, 1) and C (5,0) on the graph paper. Join A, B and C and extend it on both sides to obtain the graph of the equation 2x + 4y = 10.

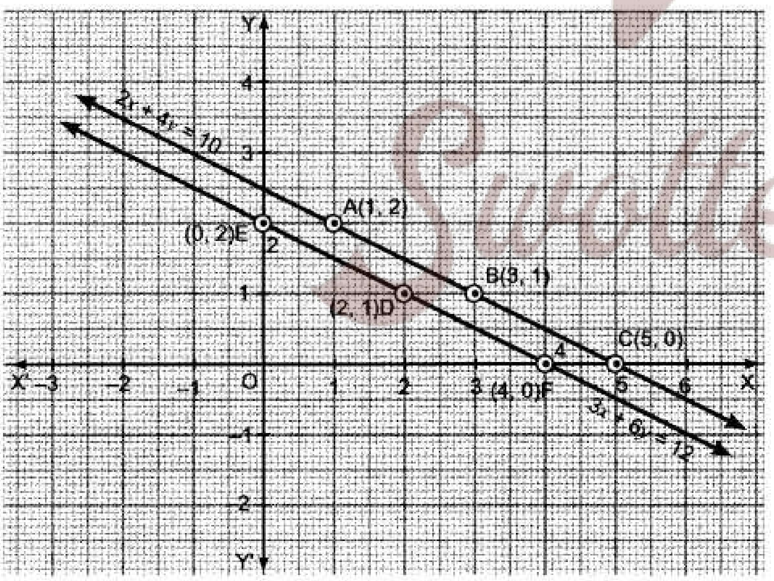
We have, 3x + 6y = 12

$\Rightarrow$ 6y = 12 – 3x $\Rightarrow y = \frac{4-x}{2}$

Thus, we have the following table :

x	2	0	4
y	1	2	0

Plot the points D (2, 1), E (0, 2) and F (4,0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation 3x + 6y = 12.



We find that the lines represented by equations 2x + 4y = 10 and 3x + y = 12 are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

3. (i) We have, $3x - 5y - 4 = 0$

$\Rightarrow 3x - 5y = 4$ ....(i)

Again, $9x = 2y + 7$

$9x - 2y = 7$ ...(ii)

By Elimination Method:

Multiplying equation (i) by 3, we get

$9x - 15y = 12$ ... (iii)

Subtracting (ii) from (iii), we get

$$\begin{array}{r} 9x - 15y = 12 \\ \underset{-}{9x} \underset{+}{-} 2y = \underset{-}{7} \\ \hline -13y = 5 \end{array}$$

$$\Rightarrow \quad y = -\frac{5}{13}$$

Putting the value of y in equation (*ii*), we have

$$9x - 2\left(-\frac{5}{13}\right) = 7 \quad \Rightarrow \quad 9x + \frac{10}{13} = 7 \quad \Rightarrow \quad 9x = 7 - \frac{10}{13}$$

$$\Rightarrow \quad 9x = \frac{91 - 10}{13} \quad \Rightarrow \quad 9x = \frac{81}{13} \quad \Rightarrow \quad x = \frac{9}{13}$$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$$x = \frac{4+5y}{3}$$

Substituting the value of x in equation (*ii*), we have

$$9 \times \left(\frac{4+5y}{3}\right) - 2y = 7$$

$\Rightarrow \quad 3 \times (4 + 5y) - 2y = 7$

$\Rightarrow \quad 12 + 15y - 2y = 7 \quad \Rightarrow \quad 13y = 7 - 12$

$\therefore \quad y = -\frac{5}{13}$

Putting the value of y in equation (*i*), we have

$$3x - 5 \times \left(-\frac{5}{13}\right) = 4 \quad \Rightarrow \quad 3x + \frac{25}{13} = 4$$

$\Rightarrow \quad 3x = 4 - \frac{25}{13} \quad \Rightarrow \quad 3x = \frac{27}{13}$

$\therefore \quad x = \frac{9}{13}$

Hence, the required solution is $x = \frac{9}{13}$, $y = -\frac{5}{13}$.

(*ii*) We have, $\frac{x}{2} + \frac{2y}{3} = -1 \quad \Rightarrow \quad \frac{3x+4y}{6} = -1$

$\therefore \quad 3x + 4y = -6 \quad$ ...(*i*)

and $\quad x - \frac{y}{3} = 3 \quad \Rightarrow \quad \frac{3x - y}{3} = 3$

$\therefore \quad 3x - y = 9 \quad$ ...(*ii*)

By Elimination Method:

Subtracting (ii) from (i), we have

5y = – 15 or y = – 55 = -3

Putting the value of y in equation (i), we have

3x + 4 × (-3) = -6 ⇒ 3x = – 6 + 12

∴ 3x – 12 = -6 ⇒ 3x = 6

∴ x = 63 = 2

Hence, solution is x = 2, y = -3.

By Substitution Method:

Expressing x in terms of y from equation (i), we have

$3 \times \left(\frac{-6-4y}{3}\right) - y = 9 \Rightarrow -6 - 4y - y = 9 \Rightarrow -6 - 5y = 9$

Substituting the value of x in equation (ii), we have

$\therefore -5y = 9 + 6 = 15$

$y = -\frac{15}{5} = -3$

Putting the value of y in equation (i), we have

$3x + 4 \times (-3) = -6 \Rightarrow 3x - 12 = -6$

$\therefore 3x = 12 - 6 = 6$

$\therefore x = \frac{6}{3} = 2$

Hence, the required solution is x = 2, y = – 3.

4. We have,' x – y + 1 = 0 and 3x + 2y – 12 = 0

Thus, x – y = -1 => x = y – 1 ...(i)

3x + 2y = 12 => $x = \frac{12-2y}{3}$ ... (ii)

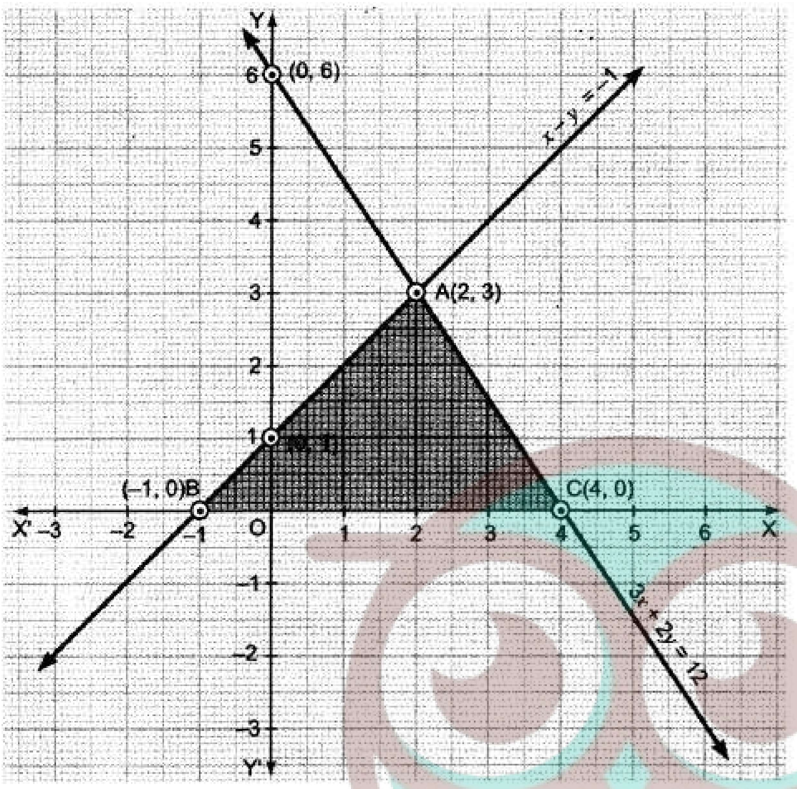
From equation (i), we have

x	– 1	0	2
y	0	1	3

From equation (ii), we have

x	0	4	2
y	6	0	3

Plotting this, we have



ABC is the required (shaded) region and point of intersection is (2, 3).

∴ The vertices of the triangle are (-1, 0), (4, 0), (2, 3).

5. Let the fixed charge be *x and the cost of food per day be by.

Therefore, according to question,

x + 20y = 1000 …(i)

x + 26y = 1180 …(ii)

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{r} x + 20y = 1000 \\ \underset{-}{} x \underset{-}{+} 26y = \underset{-}{} 1180 \\ \hline -6y = -180 \end{array}$$

$$\therefore \quad y = \frac{-180}{-6} = 30$$

Putting the value of y in equation (i), we have

x + 20 x 30 = 1000 ⇒ x + 600 = 1000 ⇒ x = 1000 – 600 = 400

Hence, fixed charge is ₹400 and cost of food per day is ₹30.

6. Let x be the number of questions of right answer and y be the number of questions of wrong answer.

According to question,

$3x - y = 40$ … (i)

and $4x - 2y = 50$

or $2x - y = 25$ …(ii)

Subtracting (ii) from (i), we have

$$\begin{array}{r} 3x - y = 40 \\ 2x - y = 25 \\ - \quad + \quad - \\ \hline x = 15 \end{array}$$

Putting the value of x in equation (i), we have

$3 \times 15 - y = 40 \Rightarrow 45 - y = 40$

$\therefore y = 45 - 40 = 5$

Hence, total number of questions is x + i.e.., 5 + 15 = 20.

7. Let one man alone can finish the work in x days and one boy alone can finish the work in y days

Then, One day work of one man $= \frac{1}{x}$, One day work of one boy $\frac{1}{y}$

$\therefore$ One day work of 8 men $= \frac{8}{x}$, One day work of 12 boys $= \frac{12}{y}$

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x}+\frac{12}{y}\right)=1 \quad \Rightarrow \quad \frac{80x}{x}+\frac{120}{y}=1 \qquad \ldots(i)$$

Again, 6 men and 8 boys can finish the work in 14 days

$$\therefore \quad 14\left(\frac{6}{x}+\frac{8}{y}\right)=1 \quad \Rightarrow \quad \frac{84}{x}+\frac{112}{y}=1 \qquad \ldots(ii)$$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equations (i) and (ii), we get

$$80u + 120v - 1 = 0 \quad \text{and} \quad 84u + 112v - 1 = 0$$

By using cross-multiplication, we have

$$\frac{u}{120\times -1-112\times -1}=\frac{-v}{80\times -1-84\times -1}=\frac{1}{80\times 112-84\times 120}$$

$$\Rightarrow \quad \frac{u}{-120+112}=\frac{-v}{-80+84}=\frac{1}{8960-10080}$$

$$\Rightarrow \quad \frac{u}{-8}=\frac{-v}{4}=\frac{1}{-1120}$$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

8. Let the speed of the boat in still water be x km/h and that of the stream be y km/h. Then,

Speed upstream (x – y) km/h

Speed downstream (x + y) km/h

Now, time taken to cover 25 km upstream $=\frac{25}{x-y}$ hours

Time taken to cover 44 km downstream $=\frac{44}{x+y}$ hours

The total time of journey is 9 hours

$$\frac{25}{x-y}+\frac{44}{x+y}=9 \qquad \ldots(i)$$

Time taken to cover 15 km upstream $= \frac{15}{x-y}$

Time taken to cover 22 km downstream $= \frac{22}{x+y}$

In this case, total time of journey is 5 hours.

$$\therefore \quad \frac{15}{x-y} + \frac{22}{x+y} = 5 \qquad \text{...(ii)}$$

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (*i*) and (*ii*), we get

25u + 44v = 9 ⇒ 25u + 44v – 9 = 0 ...(iii)

15u + 22v = 5 ⇒ 15u + 22v – 5 = 0 ...(iv)

By cross-multiplication, we have

$$\Rightarrow \quad u = \frac{22}{110} = \frac{1}{5} \text{ and } v = \frac{1}{11}$$

We have, $u = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5$ ...(*v*)

and $v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$ ...(*vi*)

$$\Rightarrow \quad u = \frac{22}{110} = \frac{1}{5} \text{ and } v = \frac{1}{11}$$

We have, $u = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5$ ...(*v*)

and $v = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$ ...(*vi*)

Solving equations (v) and (vi), we get x = 8 and y = 3.

Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Case Study Answers:

1. Answer :

i. (a) $x + 25y = 4500$, $x + 30y = 5200$

Solution:

For student Anu:

Fixed charge + cost of food for 25 days = ₹ 4500

i.e., $x + 25y = 4500$

For student Bindu:

Fixed charges + cost of food for 30 days = ₹ 5200

i.e., $x + 30y = 5200$

ii. (b) Unique solution.

Solution:

From above, we have $a_1 = 1$, $b_1 = 25$

$c_1 = -4500$ and $a_2 = 1$, $b_2 = 30$, $c_2 = -5200$

$$\therefore \frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = \frac{25}{30} = \frac{5}{6}, \frac{c_1}{c_2} = \frac{-4500}{-5200} = \frac{45}{52}$$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, system of linear equations has unique solution.

iii. (c) ₹ 140

Solution:

We have, $x + 25y = 4500$

and $x + 30y = 5200$

Subtracting (i) from (ii), we get

$5y = 700 \Rightarrow y = 140$

∴ Cost of food per day is ₹ 140

iv. (c) ₹ 1000

Solution:

We have, $x + 25y = 4500$

$\Rightarrow x = 4500 - 25 \times 140$

$\Rightarrow x = 4500 - 3500 = 1000$

∴ Fixed charges per month for the hostel is ₹ 100

v. (d) ₹ 3800

Solution:

We have, $x = 1000$, $y = 140$ and Bindu takes food for 20 days.

∴ Amount that Bindu has to pay = ₹ $(1000 + 20 \times 140)$ = ₹ 3800

2. Answer :

i. (d) $2x + 3y = 46$

Solution:

1^{st} situationcan berepresented algebraically as.

$2x + 3y = 46$

ii. (c) $3x + 5y = 74$

Solution:

2^{nd} situation can be represented algebraically as:

$3x + 5y = 74$

iii. (b) ₹ 8

Solution:

We have, $2x + 3y = 46$.........(i)

$3x + 5y = 74$.........(ii)

Multiplying (i) by 5 and (ii) by 3 and then subtracting, we get

$10x - 9x = 230 - 222 \Rightarrow x = 8$

∴ Fare from Bengaluru to Malleswaram is ₹ 8.

iv. (a) ₹ 10

Solution:

Putting the value of x in equation (i), we g

$3y = 46 - 2 \times 8 = 30 \Rightarrow y = 10$

∴ Fare from Bengaluru to Yeswanthpur is ₹ 10

v. (c) Unique solution.

Solution:

We have, $a_1 = 2$, $b_1 = 3$, $c_1 = -46$ and

$a_2 = 3$, $b_2 = 5$, $C_2 = -74$

$$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{3}{5}, \frac{c_1}{c_2} = \frac{-46}{-74} = \frac{23}{37}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus system of linear equations has unique solution.

Assertion reason Answer-

1. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
2. (d) Assertion (A) is true but reason (R) is false.