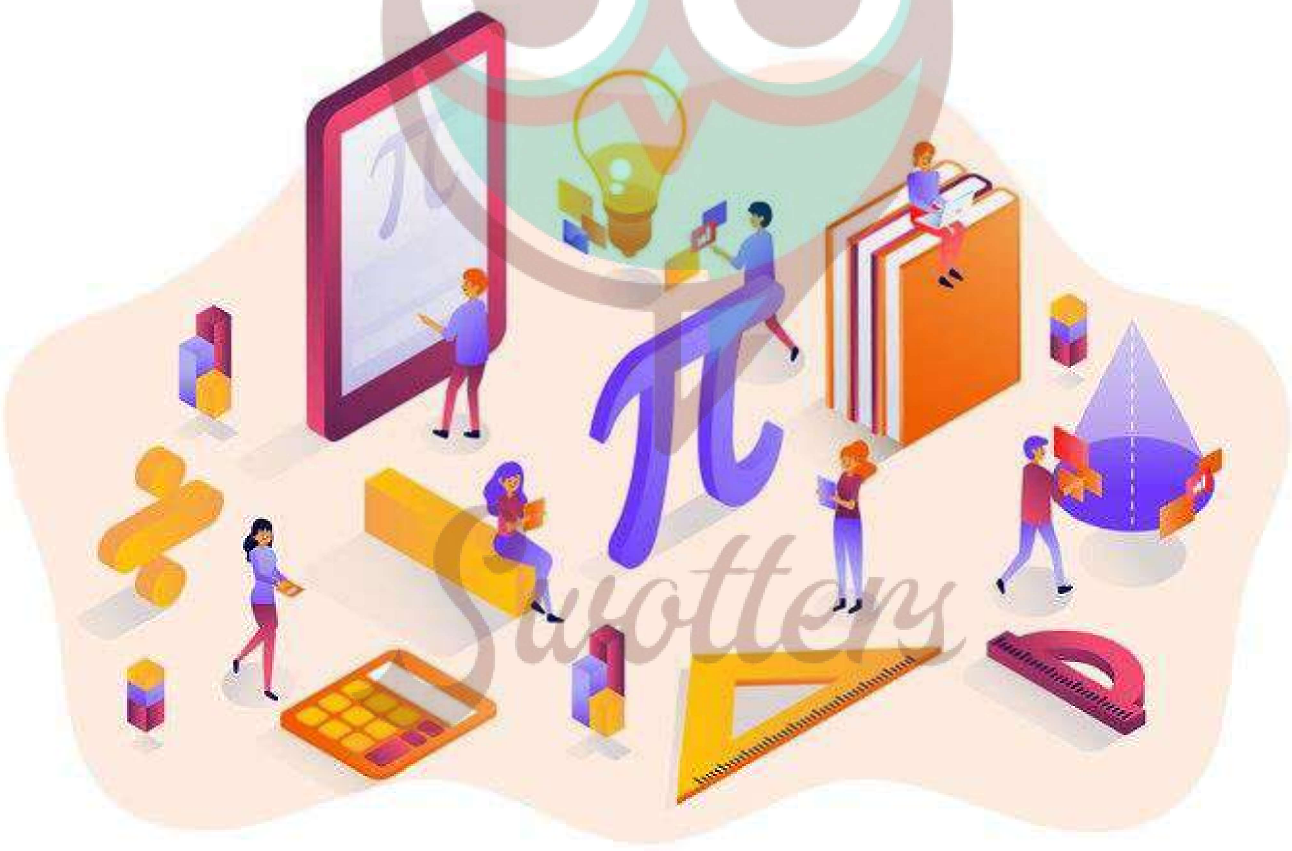
MATHEMATICS



Important Questions

Multiple Choice questions-

1. If A = $[a_{ij}]_{m \times n}$ is a square matrix, if:

(a) m < n

(b) m > n

(c) m = n

(d) None of these.

2. Which of the given values of x and y make the following pair of matrices equal:

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(a) $x = -\frac{1}{3}$, $y = 7$

(b) Not possible to find

(c) $y = 7$, $x = -\frac{2}{3}$

(d) $x = -\frac{1}{3}$, $y = -\frac{2}{3}$

3. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

(a) 27

(b) 18

(c) 81

(d) 512.

Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × 1, 2 × p, n × 3 and p × k respectively. Now answer the following (4-5):

4. The restrictions on n, k and p so that PY + WY will be defined are

(a) k = 3, p = n

(b) k is arbitrary, p = 2

(c) p is arbitrary

(d) k = 2, p = 3.

5. If n =p, then the order of the matrix 7X - 5Z is:

(a) $p \times 2$

(b) $2 \times n$

(c) $n \times 3$

(d) $p \times n$.

6. If A, B are symmetric matrices of same order, then AB – BA is a

(a) Skew-symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix.

7.

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then A + A' = I, the value of α is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{3\pi}{2}$

8. Matrices A and B will be inverse of each other only if:

(a) AB = BA

(b) AB – BA = O

(c) AB = O, BA = I

(d) AB = BA = I.

9.

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(a) $1 + \alpha^2 + \beta\gamma = 0$

(b) $1 - \alpha^2 + \beta\gamma = 0$

(c) $1 - \alpha^2 - \beta\gamma = 0$

(d) $1 + \alpha^2 - \beta\gamma = 0$

10. If a matrix is both symmetric and skew- symmetric matrix, then:

(a) A is a diagonal matrix

(b) A is a zero matrix

(c) A is a square matrix

(d) None of these.

Very Short Questions:

1. If a matrix has 8 elements, what are the possible orders it can have.
2. Identity matrix of orders n is denoted by.
3. Define square matrix
4. The no. of all possible metrics of order 3 × 3 with each entry 0 or 1 is
5. Write (1) a33, a12 (ii) what is its order

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

6. Two matrices $A = a_{ij}$ and $B = b_{ij}$ are said to be equal if
7. Define Diagonal matrix.
8. Every diagonal element of a skew symmetric matrix is

9. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, *then* $A + A' = I$ Find α

10. $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ *Find* $A + A'$

Short Questions:

1. Write the element a_{23} of a 3 x 3 matrix A = $[a_{ij}]$ whose elements atj are given by: $\frac{|i-j|}{2}$

2. For what value of x is

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0 \text{? (C.B.S.E. 2019(C))}$$

3. Find a matrix A such that 2A – 3B + 5C = 0,

Where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C = $\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

4.

If A = $\begin{pmatrix} \cos\alpha - \sin\alpha \\ \sin\alpha - \cos\alpha \end{pmatrix}$, then for what value of 'α' is A an identity matrix?

5. Find the values of x, y, z and t, if:

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

6.

If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$. (CBSE 2019)

7.

If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + b\,kI$

8. If A and B are symmetric matrices, such that AB and BA are both defined, then prove that

AB – BA is a skew symmetric matrix. (A.I.C.B.S.E. 2019)

Long Questions:

1. Find the values of a, b, c and d from the following equation:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$ (N.C.E.R.T)

2. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then find the matrix A. (C.B.S.E. 2013)

3. If A = = $\begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$ find the matrix C such that A + B + C is a zero matrix.

4. If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix 'X', of order 3 x 2, such that 2A + 3X = 5B. (N.C.E.R.T.)

Assertion and Reason Questions:

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion(A): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix.

Reason (R): A matrix A=[a_{ij}] is an identity matrix if $a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.

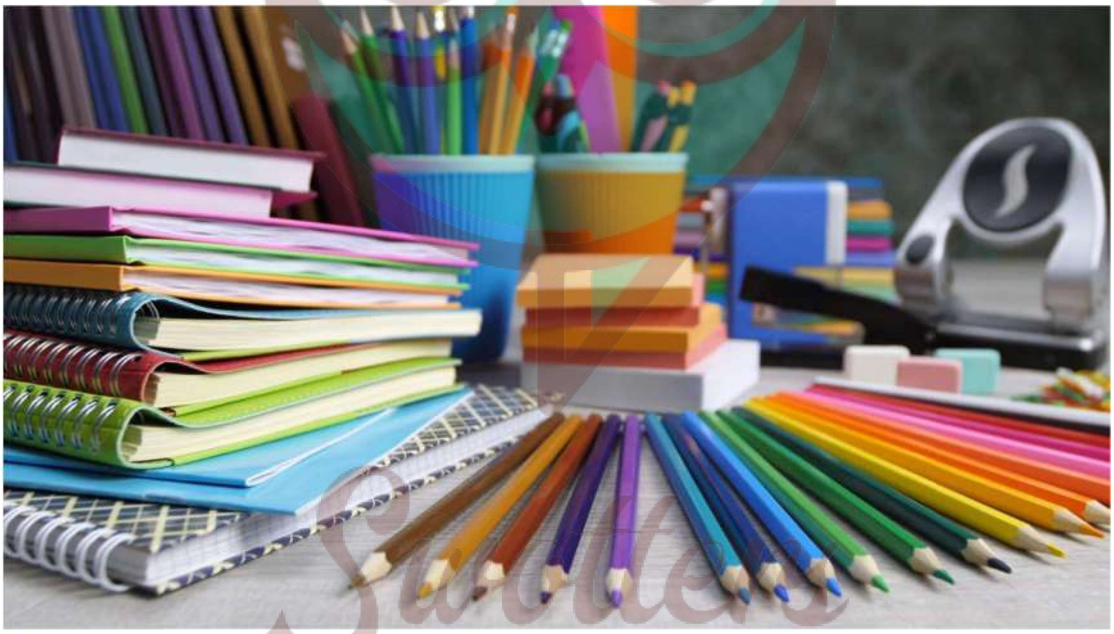
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion (A): Matrix $\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ is a column matrix.

Reason(R): A matrix of order m × 1 is called a column matrix.

Case Study Questions:

1. Three shopkeepers A, B and C go to a store to buy stationary. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 12 and a pencil costs ₹ 3.



Based on the above information, answer the following questions.

(i) The number of items purchased by shopkeepers A, B and C represented in matrix form as:

a. $\begin{matrix} \text{Notebooks} & \text{Pens} & \text{Pencils} \\ \end{matrix}$

$$\begin{bmatrix} 144 & 60 & 72 \\ 120 & 720 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

b. $\begin{matrix} \text{Notebooks} & \text{Pens} & \text{Pencils} \\ \end{matrix}$

$$\begin{bmatrix} 144 & 72 & 60 \\ 120 & 84 & 72 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

c. $\begin{matrix} \text{Notebooks} & \text{Pens} & \text{Pencils} \\ \end{matrix}$

$$\begin{bmatrix} 144 & 72 & 72 \\ 120 & 156 & 84 \\ 132 & 84 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

d. $\begin{matrix} \text{Notebooks} & \text{Pens} & \text{Pencils} \\ \end{matrix}$

$$\begin{bmatrix} 144 & 60 & 60 \\ 120 & 84 & 72 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

(ii) If Y represents the matrix formed by the cost of each item, then XY equals.

a. $\begin{bmatrix} 5741 \\ 6780 \\ 8040 \end{bmatrix}$

b. $\begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

c. $\begin{bmatrix} 5916 \\ 6696 \\ 7440 \end{bmatrix}$

d. $\begin{bmatrix} 6740 \\ 5740 \\ 8140 \end{bmatrix}$

(iii) Bill of A is equal to:

a. ₹ 6740

b. ₹ 8140

c. ₹ 5740

d. ₹ 6696

(iv) If $A^2 = A$, then $(A + 1)^3 - 7A =$

a. A

b. A - I

c. I

d. A + I

(v) If A and B are 3 × 3 matrices such that $A^2 - B^2 = (A - B)(A + B)$, then

a. Either A or B is zero matrix.

b. Either A or B is unit matrix.

c. A = B

d. AB = BA

2. Consider 2 families A and B. Suppose there are 4 men,4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

Based on the above information, answer the following questions.

(i) The requirement of calories and proteins for each person in matrix form can be represented as:

a.

	Calorise	Proteins
Man	2400	45
Woman	1900	55
Children	1800	33

b.

	Calorise	Proteins
Man	1900	55
Woman	2400	45
Children	1800	33

c.

	Calorise	Proteins
Man	1800	33
Woman	1900	55
Children	2400	45

d.

	Calorise	Proteins
Man	2400	33
Woman	1900	55
Children	1800	45

(ii) Requirement of calories of family A is:

a. 24000
b. 24400
c. 15000
d. 15800

(iii) Requirement of proteins for family B is:

a. 560 grams
b. 332 grams
c. 266 grams
d. 300 grams

(iv) If A and Bare two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals.

a. 2AB
b. 2BA
c. A + B
d. AB

(v) If $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{n\times p}$ and $C = (c_{ij})_{p\times q}$ then the product (BC) A is possible only when.

a. m = q
b. n = q
c. p = q
d. m = p

Answer Key-

Multiple Choice questions-

1. Answer: (c) m = n
2. Answer: (b) Not possible to find
3. Answer: (d) 512.
4. Answer: (a) k = 3, p = n
5. Answer: (b) 2 × n
6. Answer: (a) Skew-symmetric matrix
7. Answer: (a) $\frac{\pi}{6}$
8. Answer: (d) AB = BA = I.
9. Answer: (c) $1 - \alpha^2 - \beta\gamma = 0$
10. Answer: (b) A is a zero matrix

Very Short Answer:

1. Solution:

   $1\times8,\ 8\times1,\ 4\times2,\ 2\times4,$

2. Solution: I_n
3. Solution: A matrix in which the no. of rows are equal to no. of columns i.e. m = n
4. Solution: $512=2^9$
5. Solution:

(i) $a_{33} = 9$, $a_{12} = 4$

(ii) 3×3

6. Solution: They are of the same order.

7. Solution: A square matrix in which every non – diagonal element is zero is called diagonal matrix.

8. Solution: Zero.

9. Solution:

$$A+A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix}$$

$$A+A' = I(Given)$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{2}$$

$$\cos\alpha = \cos\frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

10. Solution:

$$A+A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

Short Answer:

1. Solution:

We have $[a_{ij}] = \frac{|i-j|}{2}$

$\therefore a_{23} = \frac{|2-3|}{2} = \frac{|-1|}{2} = \frac{1}{2}$

2. Solution:

We have

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$[1 + 4 + 1 \; 2 + 0 + 0 \; 0 + 2 + 2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$[6 \; 2 \; 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$\Rightarrow [0 + 4 + 4x] = 0$

$\Rightarrow [4 + 4x] = [0]$

$\Rightarrow 4 + 4x = 0.$

Hence, x = -1.

3. Solution:

Here, 2A - 3B + 5C = 0

⇒ 2A = 3B - 5C

$$\Rightarrow 2A = 3\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix} - 5\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$$

$$= \begin{bmatrix}-6 & 6 & 0\\ 9 & 3 & 12\end{bmatrix} + \begin{bmatrix}-10 & 0 & 10\\ -35 & -5 & -30\end{bmatrix}$$

$$= \begin{bmatrix}-6-10 & 6+0 & 0+10\\ 9-35 & 3-5 & 12-30\end{bmatrix}$$

$$= \begin{bmatrix}-16 & 6 & 10\\ -26 & -2 & -18\end{bmatrix}.$$

Hence, $A = \begin{bmatrix}-8 & 3 & 5\\ -13 & -1 & -9\end{bmatrix}$.

4. Solution:

Here $A = \begin{pmatrix}\cos\alpha - \sin\alpha\\ \sin\alpha - \cos\alpha\end{pmatrix}$

Now $A = I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ when

$\cos \alpha = 1$ and $\sin \alpha = 0$.

Hence, $\alpha = 0$.

5. Solution:

We have:

$$2\begin{bmatrix}x & z\\ y & t\end{bmatrix} + 3\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix} = 3\begin{bmatrix}3 & 5\\ 4 & 6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x & 2z\\ 2y & 2t\end{bmatrix} + \begin{bmatrix}3 & -3\\ 0 & 6\end{bmatrix} = \begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2x+3 & 2z-3\\ 2y & 2t+6\end{bmatrix} = \begin{bmatrix}9 & 15\\ 12 & 18\end{bmatrix}$$

$\Rightarrow 2x + 3 = 9$ …………. (1)

$2z - 3 = 15$ …………. (2)

$2y = 12$ …………. (3)

2t + 6 = 18 …………. (4)

From (1), ⇒ 2x = 9 – 3

⇒ 2x = 6

⇒ x = 3.

From (3) 2y = 12

⇒ y = 6.

From (2), ⇒ 2z – 3 = 15

⇒ 2z = 18

⇒ z = 9.

From (4), 2t + 6 = 18

⇒ 2t = 12

⇒ t = 6.

Hence, x = 3, y = 6, z = 9 and t = 6.

6. Solution:

We have $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Then $A^2 = AA$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1-0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}.$$

$$\therefore A^2 - 5A = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}.$$

7. Solution:

We have : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

$$\therefore A^2 = AA = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \quad \ldots (1)$$

Also, $5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ ...(2)

and $kI = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ ...(3)

$\therefore A^2 = 5A + kI$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

[Using (1), (2) & (3)]

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+k & 5 \\ -5 & 10+k \end{bmatrix}$$

⇒ 8 = 15 + k and 3 = 10 + k

⇒ k = -1 and k = -7.

Hence, k – (-7).

8. Solution:

Since A and B are symmetric matrices,

∴ A’ = A and B’ = B …(1)

Now, (AB – BA)’= (AB)’ – (BA)’

= B’A’ – A’B’

= BA – AB [Using (1)]

= – (AB – BA).

Hence, AB – BA is a skew-symmetric matrix.

Long Answer:

1. Solution:

We have

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Comparing the corresponding elements of two given matrices, we get:

2a + b = 4 …(1)

a-2b = – 3 …(2)

5c-d = 11 …(3)

$4c + 3d = 24$ …(4)

Solving (1) and (2):

From (1),

$b = 4 – 2a$ …(5)

Putting in (2), $a – 2 (4 – 2a) = – 3$

$\Rightarrow a – 8 + 4a = -3$

$\Rightarrow 5a = 5$

$\Rightarrow a = 1.$

Putting in (5),

$b = 4 – 2(1) = 4 – 2 = 2.$

Solving (3) and (4):

From (3),

$d = 5c- 11$ …(6)

Putting in (4),

$4c+ 3 (5c- 11) = 24$

$\Rightarrow 4c + 15c – 33 = 24$

$\Rightarrow 19c = 57$

$\Rightarrow c = 3.$

Putting in (6),

$d = 5 (3) – 11 = 15 – 11 = 4.$

Hence, $a = 1$, $b = 2$, $c = 3$ and $d = 4$.

2. Solution:

Let $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}.$$

Then $$\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} a_{11}+1 & a_{12}+2 & a_{13}-1 \\ a_{21}+0 & a_{22}+4 & a_{23}+9 \end{bmatrix}.$$

Comparing:

$9 = a_{11} + 1 - 1 = a_{12} + 2$,

$4 = 1_{13} - 1$, $-2 = a_{21}$

$1 = a_{22} + 4$, and $3 = a_{23} + 9$

$a_{11} = 8$, $a_{12} = -3$,

$a_{13} = 5$, $a_{21} = -2$

$a_{22} = -3$, and $a_{23} = -6$.

Hence, $A = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

3. Solution:

Let $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$.

Then A + B + C = O

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+6 & 2+2 \\ -3+1 & 1+3 \\ 4+0 & 0+4 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+6+c_{11} & 2+2+c_{12} \\ -3+1+c_{21} & 1+3+c_{22} \\ 4+0+c_{31} & 0+4+c_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8+c_{11} & 4+c_{12} \\ -2+c_{21} & 4+c_{22} \\ 4+c_{31} & 4+c_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Comparing:

$8 + c_{11} = 0 \Rightarrow c_{11} = -8$,

$4 + C_{12} = 0 \Rightarrow C_{12} = -4$,

$-2 + C_{21} = 0 \Rightarrow C_{21} = 2$

$4 + C_{22} = 0 \Rightarrow C_{22} = -4$,

$4 + c_{31} = 0 \Rightarrow C_{31} = -4$

and $4 + c_{32} = 0 \Rightarrow C_{32} = -4$.

Hence, $C = \begin{bmatrix} -8 & -4 \\ 2 & -4 \\ -4 & -4 \end{bmatrix}$

4. Solution:

We have: 2A + 3X = 5B

⇒ 2A + 3X-2A = 5B-2A

⇒ 2A-2A + 3X = 5B-2A

⇒ (2A – 2A) + 3X = 5B – 2A

⇒ O + 3X = 5B – 2A

[∵ – 2A is the inverse of2A]

⇒ 3X = 5B – 2A.

[∵ O is the additive identity]

Hence, X = $\frac{1}{3}$(5B – 2A)

$$=\frac{1}{3}\left(5\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}-2\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}\right)$$

$$=\frac{1}{3}\left(\begin{bmatrix}10 & -10\\ 20 & 10\\ -25 & 5\end{bmatrix}+\begin{bmatrix}-16 & 0\\ -8 & 4\\ -6 & -12\end{bmatrix}\right)$$

$$=\frac{1}{3}\begin{bmatrix}10-16 & -10+0\\ 20-8 & 10+4\\ -25-6 & 5-12\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}-6 & -10\\ 12 & 14\\ -31 & -7\end{bmatrix}=\begin{bmatrix}-2 & -10/3\\ 4 & 14/3\\ -31/3 & -7/3\end{bmatrix}$$

Assertion and Reason Answers:

1. (d) A is false and R is true.

Solution:

We know that, $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ is an indentity matrix

∴ Given Assertion [A] is false We know that for identity matrix $a_{ij} = 1$, *ifi = j* and $a_{ij} = 0$, *ifi ≠ j*

∴ Given Reason (R) is true Hence option (d) is the correct answer.

2. a) Both A and R are true and R is the correct explanation of A.

Solution:

We know that order of column matrix is always $m \times 1$

$\therefore \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ is column matrix.

⇒ Assertion (A) is true Also Reason (R) is true and is correct explanation of A. Hence option (a) is the correct answer.

Case Study Answers:

1. Answer :

i. (a)
$$\begin{array}{cccc} \text{Notebooks} & \text{Pens} & \text{Pencils} & \\ \left[\begin{matrix} 144 \\ 120 \\ 132 \end{matrix}\right. & \begin{matrix} 60 \\ 720 \\ 156 \end{matrix} & \left.\begin{matrix} 72 \\ 84 \\ 96 \end{matrix}\right] & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

Solution:

$$X = \begin{array}{cccc} \text{Notebooks} & \text{Pens} & \text{Pencils} & \\ \left[\begin{matrix} 144 \\ 120 \\ 132 \end{matrix}\right. & \begin{matrix} 60 \\ 720 \\ 156 \end{matrix} & \left.\begin{matrix} 72 \\ 84 \\ 96 \end{matrix}\right] & \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$$

ii. (b) $\begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

Solution:

Since, $Y = \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix} \begin{matrix} \text{Notebook} \\ \text{Pens} \\ \text{Pencil} \end{matrix}$

$\therefore \quad XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 5760 + 720 + 216 \\ 4800 + 864 + 252 \\ 5280 + 1872 + 288 \end{bmatrix} = \begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

iii. (d) ₹ 6696

Solution:

Bill of A is ₹ 6696.

iv. (c) I

Solution:

$(A + I)^2 = A^2 + 2A + I = 3A + I$

$\Rightarrow (A + 1)^3 = (3A + I)(A + I)$

$= 3A^2 + 4A + I = 7A + I$

∴∴ $(A + I)^3 - 7A = I$

v. (d) AB = BA

Solution:

$A^2 - B^2 = (A - B)(A + B) = A^2 + AB - BA - B^2$

∴∴ AB = BA

2. Answer :

i. (a)

$$\begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calorise} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

Solution:

Let F be the matrix representing the number of family members and

R be the matrix representing the requirement of calories and proteins

for each person. Then

$$F = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calorise} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

ii. (b) 24400

Solution:

The requirement of calories and proteins for each ofthe two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400 + 1900 + 1800) & 4(45 + 55 + 33) \\ 2(2400 + 1900 + 1800) & 2(45 + 55 + 33) \end{bmatrix}$$

$$FR = \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array}$$

iii. (c) 266 grams
iv. (c) A + B

Solution:

Since, AB = B ... (i)

BA = A ... (ii)

$\therefore A^2 + B^2 = A \times A + B \times B$

= A(BA) + B(AB)

= (AB)A + (BA)B

= BA + AB

= A + B

v. (a) m = q

Solution:

$$A = (a_{ij})_{m\times n},\ \ B = (b_{ij})_{n\times p},\ \ C = (c_{ij})_{p\times q}$$

$$BC = (b_{ij})_{n\times p} \times (c_{ij})_{p\times q} = (d_{ij})_{n\times q}$$

$$(BC)A = (d_{ij})_{n\times q} \times (a_{ij})_{m\times n}$$

Hence, (BC)A is possible only when m = q