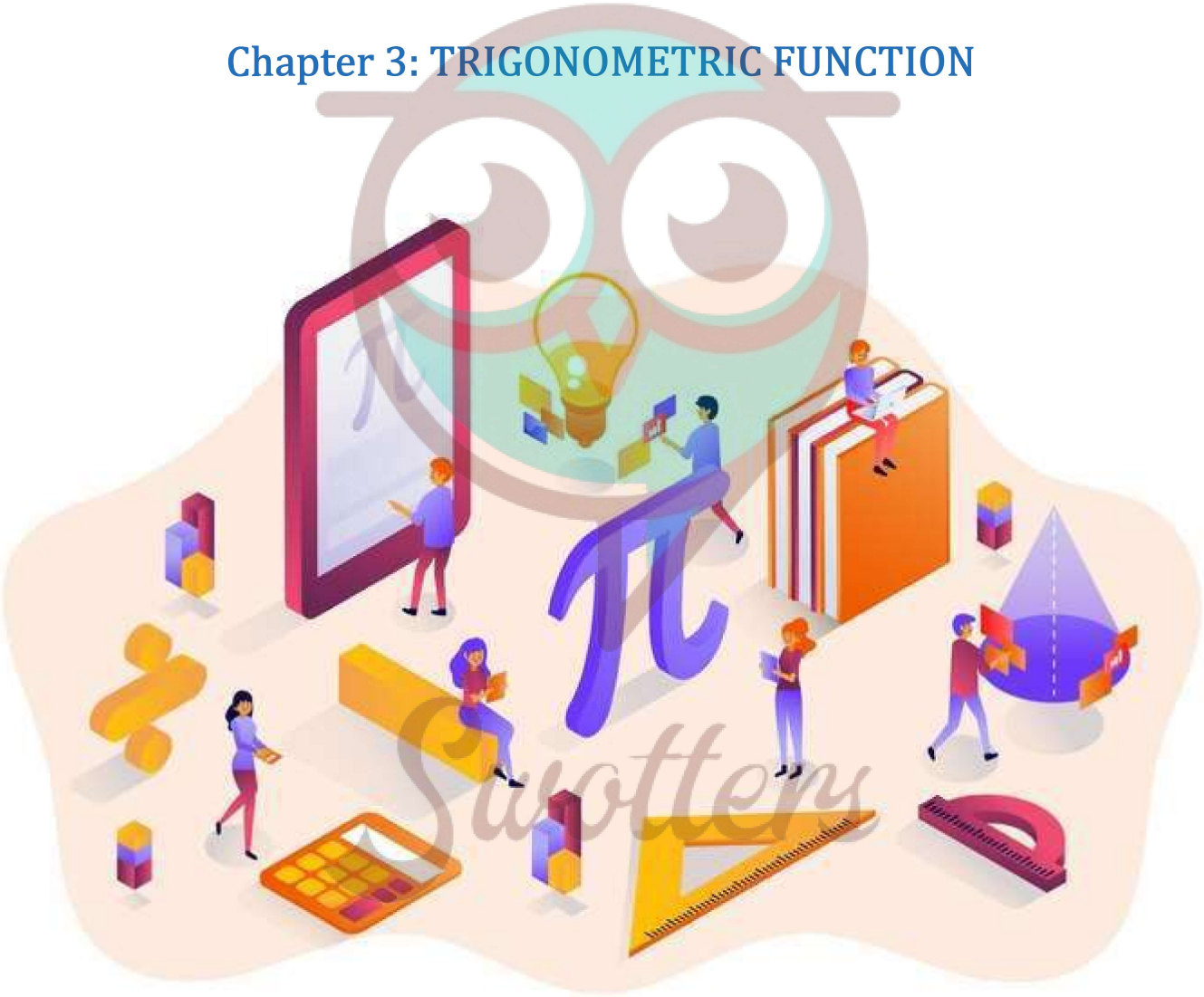
MATHEMATICS

Chapter 3: TRIGONOMETRIC FUNCTION



Important Questions

Multiple Choice questions-

Question 1. The value of sin 15 + cos 15 is

(a) 1

(b) 1/2

(c) √3/2

(d) √3

Question 2. The value of tan A/2 – cot A/2 + 2cot A is

(a) 0

(b) 1

(c) -1

(d) None of these

Question 3. The value of $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3)$ is

(a) sin x

(b) sin 2x

(c) sin 3x

(d) sin 4x

Question 4. If tan x = (cos 9 + sin 9)/(cos 9 – sin 9), then x =

(a) 45

(b) 54

(c) 36

(d) None of these

Question 5. In a triangle ABC, sin A – cos B = cos C, then angle B is

(a) π/2

(b) π/3

(c) π/4

(d) π/6

Question 6. The value of cos 420° is

(a) 0

(b) 1

(c) 1/2

(d) √3/2

Question 7. If in a triangle ABC, tan A + tan B + tan C = 6 then the value of cot A × cot B × cot C is

(a) 1/2

(b) 1/3

(c) 1/4

(d) 1/6

Question 8. If a × cos x + b × cos x = c, then the value of $(a \times \sin x - b \times \cos x)^2$ is

(a) $a^2 + b^2 + c^2$

(b) $a^2 - b^2 - c^2$

(c) $a^2 - b^2 + c^2$

(d) $a^2 + b^2 - c^2$

Question 9. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is

(a) 30°

(b) 60°

(c) 75°

(d) 45°

Question 10. In any triangle ABC, if cos A/a = cos B/b = cos C/c and the side a = 2, then the area of the triangle is

(a) √3

(b) √3/4

(c) √3/2

(d) 1/√3

Very Short:

1. Find the radian measure corresponding to 5° 37" 30""

2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$.

3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

4. Find the value of $\frac{19\pi}{3}$.

5. Find the value of sin(–1125°)
6. Find the value of tan 15°
7. If sin A = $\frac{3}{5}$ and $\frac{\pi}{2}$ < A < π, find cos A
8. If tan A = $\frac{a}{a+1}$ and tan B = $\frac{1}{2a+1}$ then find the value of A + B.
9. Express sin 12θ + sin 4θ as the product of sines and cosines.
10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

Short Questions:

1. The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 minute?
2. Show that tan 3x. tan 2x. tan x = tan 3x – tan 2x – tan x
3. Find the value of tan $\frac{\pi}{8}$.
4. Prove that $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x+\tan y}{\tan x-\tan y}$
5. If in two circles, arcs of the same length subtend angles 600 and 750 at the center find the ratio of their radii.

Long Questions:

1. If Sin α + Sin β = a and Cos α + Cos β = b show that Cos (α + β) = $\frac{b^2-a^2}{b^2+a^2}$
2. Prove that Cos α + Cos β + Cos γ + Cos (α + β + γ)

   $= 4 \cos\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\beta+\gamma}{2}\right).\cos\left(\frac{\gamma+\alpha}{2}\right)$
3. Prove that Sin3x + Sin2x - Sin2x = 4Sinx.cos $\frac{x}{2}$. cos $\frac{3x}{2}$
4. Prove that $2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$
5. Find the value of tan (α + β) Given that.

   $\cot\alpha = \frac{1}{2}, \alpha \in \left(\pi, \frac{3\pi}{2}\right)$ and $\sec\beta = -\frac{5}{3}, \beta \in \left(\frac{\pi}{2}, \pi\right)$

Assertion Reason Questions:

1. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

   Assertion (A) : $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

   Reason (R) : $\sin^{-1}(\sin\theta) = \theta$, if $q \in [(-\pi)/2, \pi/2]$

   (i) Both A and R are true and R is the correct explanation of A

(ii) Both A and R are true but R is NOT the correct explanation of A

(iii) A is true but R is false

(iv) A is false and R is True

2. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

Assertion (A) : Principal value of $\cos^{-1}(1)$ is π

Reason (R) : Value of cos 0° is 1

(i) Both A and R are true and R is the correct explanation of A

(ii) Both A and R are true but R is NOT the correct explanation of A

(iii) A is true but R is false

(iv) A is false and R is True

Answer Key:

MCQ

1. (c) √3/2

2. (a) 0

3. (c) sin 3x

4. (b) 54

5. (a) π/2

6. (c) 1/2

7. (d) 1/6

8. (d) $a^2 + b^2 - c^2$

9. (d) 45°

10. (a) √3

Very Short Answer:

1. $\left(\frac{\pi}{32}\right)^c$

2. 39°22"30""

3. $\frac{5\pi}{12} cm$

4. $\sqrt{3}$

5. $\frac{-1}{\sqrt{2}}$

6. $2-\sqrt{3}$
7. $\frac{-4}{5}$
8. 45°
9. 2 sin8θ cos4θ
10. sin 6x – sin2x

Short Answer:

1. r = 1.5 cm

Angle made in 60 mint = 360˚

Angle made in 1 min = $\frac{360}{60}$ = 60˚

Angle made in 40 mint = 6 × 40

= 240˚

$\Theta = \frac{l}{r}$

$\frac{240}{180} \times \frac{\pi}{3} = \frac{l}{1.5}$

$\frac{4 \times 3.14}{3} = \frac{l}{2}$

$2 \times 3.14 = l$

$6.28 = l$

$l = 6.28$ cm

2.

Let 3x = 2x + x

tan 3x = tan (2x + x)

$\frac{\tan 3x}{1} = \frac{\tan 2x + \tan x}{1 - \tan 2x . \tan x}$

tan 3x (1 - tan 2x. tan x) = tan 2x + tan x

tan 3x - tan 3x. tan 2x. tan x = tan 2x + tan x

tan 3x. tan 2x. tan x = tan 3x - tan 2x - tan x

3.

Let $x = \frac{\pi}{8}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} t$

$\tan\left(2\times\frac{\pi}{8}\right) = \frac{2\tan \pi/8}{1-\tan^2 \pi/8}$

$1 = \frac{2\tan \pi/8}{1-\tan^2 \pi/8}$

put $\tan \pi/8 =$

$\frac{1}{1} = \frac{2t}{1-t^2}$

$2t = 1-t^2$

$t^2 + 2t - 1 = 0$

$t = \frac{-2 \pm 2\sqrt{2}}{2\times 1}$

$= -1 \pm \sqrt{2}$

$= \pm\sqrt{2} - 1$

$= \sqrt{2} - 1$ or $-\sqrt{2} - 1$

$\tan \pi/8 = \sqrt{2} - 1$

4.

$L.H.S = \frac{Sin(x+y)}{Sin(x-y)}$

$= \frac{Sinx.Cosy + Cosx.Siny}{Sinx.Cosy - Cosx.Siny}$

$Dividing N and D by Cosx.Cosy$

$= \frac{\tan x + \tan y}{\tan x - \tan y}$

5.

$$\theta = \frac{l}{r_1}$$

$$60 \times \frac{\pi}{18} = \frac{l}{r_1}$$

$$r_1 = \frac{3l}{\pi} \quad (1)$$

$$\theta = \frac{l}{r_2}$$

$$75 \times \frac{\pi}{18} = \frac{l}{r_2}$$

$$r_2 = \frac{12l}{5\pi} \quad (2)$$

(1) $\div$ (2)

$$\frac{r_1}{r_2} = \frac{\frac{3l}{\pi}}{\frac{12l}{5\pi}}$$

$$= \frac{3l}{\pi} \times \frac{5\pi}{12l}$$

$$= 5 : 4$$

Long Answer:

1.

$$b^2 + a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 + (\text{Sin } \alpha + \text{Sin } \beta)^2$$

$$= \text{Cos}^2 \alpha + \text{Cos}^2 \beta + 2 \text{ Cos } \alpha . \text{ Cos } \beta + \text{Sin}^2 \alpha + \text{Sin}^2 \beta + 2 \text{Sin } \alpha . \text{ Sin } \beta$$

$$= 1 + 1 + 2 (\text{Cos } \alpha . \text{ Cos } \beta + \text{Sin } \alpha . \text{ Sin } \beta)$$

$$= 2 + 2 \text{ Cos } (\alpha - \beta) \quad (1)$$

$$b^2 - a^2 = (\text{Cos } \alpha + \text{Cos } \beta)^2 - (\text{Sin } \alpha + \text{Sin } \beta)^2$$

$$= (\text{Cos}^2 \alpha - \text{Sin}^2 \beta) + (\text{Cos}^2 \beta - \text{Sin}^2 \alpha) + 2 \text{ Cos } (\alpha + \beta)$$

$= \text{Cos}(\alpha+\beta)\ \text{Cos}(\alpha-\beta) + \text{Cos}(\beta+\alpha)\text{Cos}(\alpha-\beta) + 2\,\text{Cos}(\alpha+\beta)$

$= 2\,\text{Cos}(\alpha+\beta).\,\text{Cos}(\alpha-\beta) + 2\,\text{Cos}(\alpha+\beta)$

$= Cos(\alpha+\beta)\left[2\,\text{Cos}(\alpha-\beta)+2\right]$

$= \text{Cos}(\alpha+\beta).(b^2+a^2)$ [from (1)

$$\frac{b^2-a^2}{b^2+a^2} = \text{Cos}(\alpha+\beta)$$

2. L. H. S.

$= \text{Cos}\,\alpha + \text{Cos}\,\beta + \text{Cos}\,\gamma + \text{Cos}(\alpha+\beta+\gamma)$

$= \text{Cos}\,\alpha + \text{Cos}\,\beta) + \left[\text{Cos}\,\gamma + \text{Cos}(\alpha+\beta+\gamma)\right]$

$$= 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right).\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\,\text{Cos}\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right).\text{Cos}\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$= 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right).\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right).\text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[\text{Cos}\left(\frac{\alpha-\beta}{2}\right) + \text{Cos}\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$= 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\,\text{Cos}\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right).\text{Cos}\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right]$$

$$= 2\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right)\left[2\,\text{Cos}\left(\frac{\alpha+\gamma}{2}\right).\text{Cos}\left(\frac{\beta+\gamma}{2}\right)\right]$$

$$= 4\,\text{Cos}\left(\frac{\alpha+\beta}{2}\right).\text{Cos}\left(\frac{\beta+\gamma}{2}\right).\text{Cos}\left(\frac{\gamma+\alpha}{2}\right)$$

3.

$(\sin 3x - \sin x) + \sin 2x$

$$= 2\cos\left(\frac{3x+x}{2}\right).\sin\left(\frac{3x-x}{2}\right) + \sin 2x$$

$= 2\cos 2x.\sin x + \sin 2x$

$= 2\cos 2x.\sin x + 2\sin x\cos x$

$= 2\sin x\left[\cos 2x + \cos x\right]$

$$= 2\sin x\left[2\cos x\frac{3x}{2}.\cos\frac{x}{2}\right]$$

$$= 4\sin x\cos\frac{3x}{2}\cos\frac{x}{2}$$

4. L.H.S.

$$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= \cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right)+\cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= \cos\frac{10\pi}{13}+\cos\frac{8\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= \cos\left(\pi-\frac{3\pi}{13}\right)+\cos\left(\pi-\frac{5\pi}{13}\right)+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= -\cos\frac{3\pi}{13}-\cos\frac{5\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$= 0$$

5.

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta} \quad (1)$$

$$\text{Cot}\ \alpha = \frac{1}{2},$$

$$\Rightarrow \tan\alpha = 2$$

$$1+\tan^2\beta = \text{Sec}^2\beta$$

$$1+\tan^2\beta = \left(\frac{-5}{3}\right)^2 \quad \left[\because \text{Sec}\,\beta = \frac{-5}{3}1\right)$$

$$\tan\beta = \pm\ \frac{4}{3}$$

$$\tan\beta = -\frac{4}{3} \left[\because\ \beta \in \left(\frac{\pi}{2}/x\right)\right.$$

put $\tan\alpha$, and $\tan\beta$ in eq.

$$\tan(\alpha+\beta) = \frac{2-\frac{4}{3}}{1-2\left(-\frac{4}{3}\right)}$$

$$= \frac{2}{11}$$

Assertion Reason Answer:

1. (iv) A is false and R is True

2. (i) Both A and R are true and R is the correct explanation of A