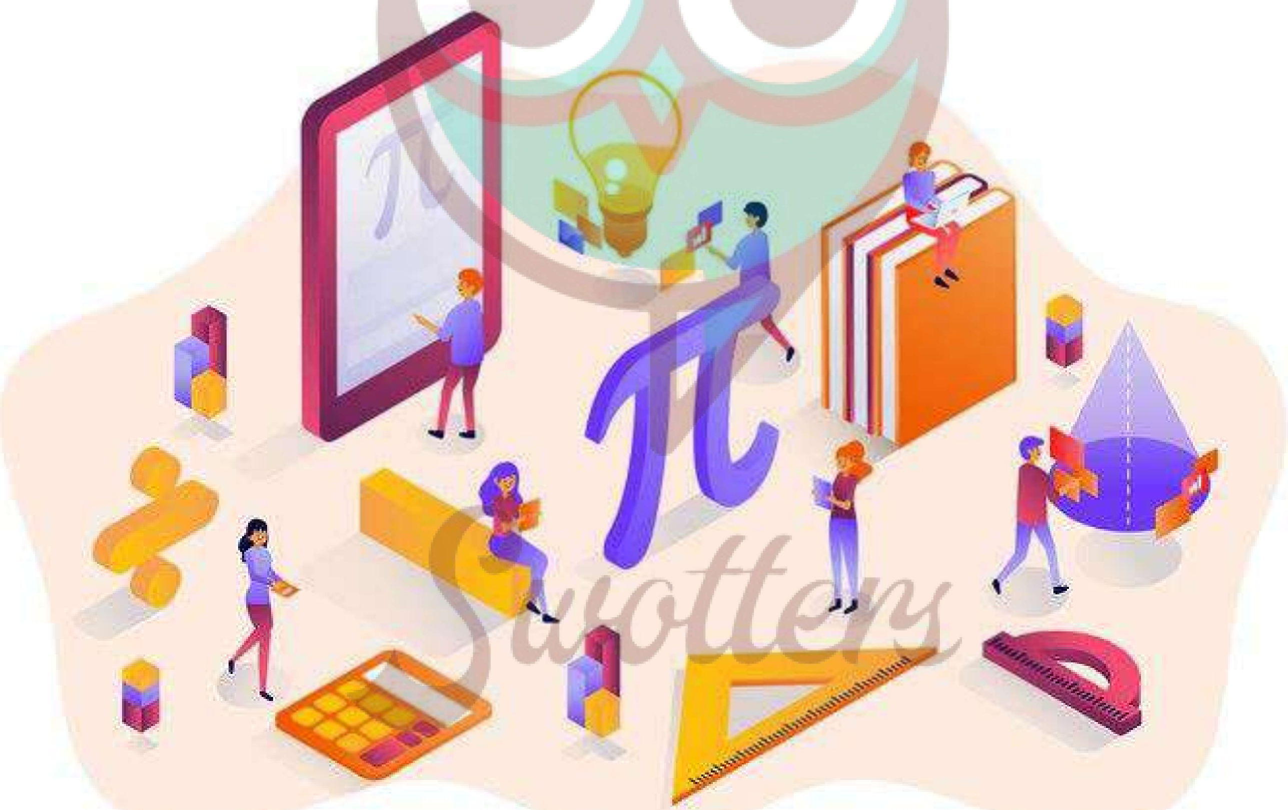
MATHEMATICS



Important Questions

Multiple Choice questions-

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(a) 6

(b) ±6

(c) -6

(d) 6, 6

2. Let A be a square matrix of order 3 × 3. Then |kA| is equal to

(a) k |A|

(b) k^2|A|

(c) k^3|A|

(d) 3k|A|

3. Which of the following is correct?

(a) Determinant is a square matrix

(b) Determinant is a number associated to a matrix

(c) Determinant is a number associated to a square matrix

(d) None of these.

4. If area of triangle is 35 sq. units with vertices (2, -6), (5, 4) and (k, 4). Then k is

(a) 12

(b) -2

(c) -12, -2

(d) 12, -2.

5. If and $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ A_{ij} is co-factors of a_{ij}, then A is given by

(a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(b) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{33}$

(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

6. Let A be a non-singular matrix of order 3 × 3. Then |adj. A| is equal to

(a) $|A|$

(b) $|A|^2$

(c) $|A|^3$

(d) $3|A|$

7. If A is any square matrix of order 3 x 3 such that |a| = 3, then the value of |adj. A| is?

(a) 3

(b) $\frac{1}{3}$

(c) 9

(d) 27

8. If A is an invertible matrix of order 2, then det (A-1) is equal to

(a) det (A)

(b) $\frac{1}{\det(A)}$

(c) 1

(d) 0

9. If a, b, c are in A.P., then determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \text{ is :}$$

(a) 0

(b) 1

(c) x

(d) 2x

10. If x, y, z are non-zero real numbers, then the inverse of matrix A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

(a) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(b) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(c) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(d) $\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Very Short Questions:

1. Find the co-factor of the element a23 of the determinant:

$$\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

(C.B.S.E. 2019 C)

2. If A and B are invertible matrices of order 3, |A| = 2 and $|(AB)^{-1}| = -\frac{1}{6}$ Find |B|. (C.B.S.E. Sample Paper 2018-19)

3. Check whether (l + m + n) is a factor of the determinant $\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ or not. Given reason. (C.B.S.E. Sample Paper 2020)

4. If A is a square matrix of order 3, with |A| = 9, then write the value of |2 . adj. A|. (A.I.C.B.S.E. 2019)

5. If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of IBI. (C.B.S.E. 2019)

6. A is a square matrix with |A| = 4. Then find the value of |A. (adj. A) |. (A.I.C.B.S.E. 2019)

7. If Δ = $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write:

   (i) the minor of the element a23 (C.B.S.E. 2012)

   (ii) the co-factor of the element a32. (C.B.S.E. 2012)

8. Find the adjoint of the matrix A = $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ (A.I.C.B.S.E. 2010)

9. Given A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ compute A^{-1} and show that $2A^{-1} = 9I - A$. (C.B.S.E. 2018)

10. For what value of 'x', the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? (C.B.S.E. 2011)

Long Questions:

1. Using properties of determinants, prove the following:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$

2. If f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$,using properties of determinants, find the value (C.B.S.E. 2015)

3. Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

4. Using properties of determinants, prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

Assertion and Reason Questions-

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion(A): Minor of element 6 in the matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$ is 3.

Reason (R): Minor of an element a_{ij} of a matrix is the determinant obtained by deleting its i^{th}

row.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion (A): For two matrices A and B of order 3, $|A|=3$, $|B|=-4$, then $|2AB|$ is -96.

Reason(R): For a matrix A of order n and a scalar k, $|kA|=k^n|A|$.

Case Study Questions-

1. Raja purchases 3 pens, 2 pencils and 1 mathematics instrument box and pays ₹41 to the shopkeeper. His friends, Daya and Anil purchases 2 pens, 1 pencil, 2 instrument boxes and 2 pens, 2 pencils and 2 mathematical instrument boxes respectively. Daya and Anil pays ₹29 and ₹44 respectively. Based on the above information answer the following:

(i) The cost of one pen is:

a) ₹2

b) ₹5

c) ₹10

d) ₹15

(ii) The cost of one pen and one pencil is:

a) ₹ 5

b) ₹10

c) ₹15

d) ₹17

(iii) The cost of one pen and one mathematical instrument box is:

a) ₹ 7

b) ₹10

c) ₹15

d) ₹18

(iv) The cost of one pencil and one mathematical instrumental box is:

a) ₹ 5

b) ₹10

c) ₹15

d) ₹20

(v) The cost of one pen, one pencil and one mathematical instrumental box is:

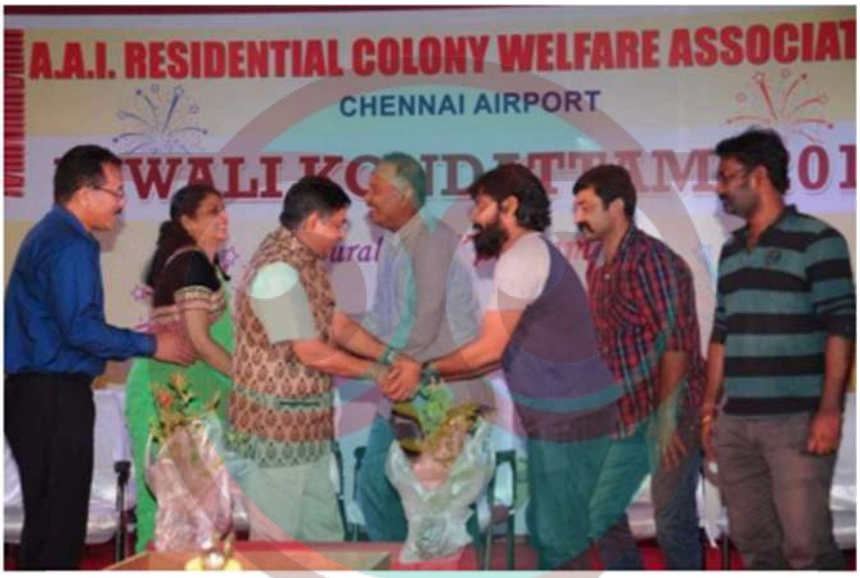
a) ₹ 10

b) ₹15

c) ₹22

d) ₹25

2. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (sayy) for helping others and some others (sayz) for supervising the workers to kept the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. The sum of the number of awardees for honesty and supervision is twice the number of awardees for helping.



(i) Value of x + y + z is

(a) 3

(b) 5

(c) 7

(d) 12

(ii) Value of x − 2 y is

(a) z

(b) -z

(c) 2z

(d) -2z

(iii) The value of z is

(a) 3

(b) 4

(c) 5

(d) 6

(iv) The value of x + 2 y is

(a) 9

(b) 10

(c) 11

(d) 12

(v) The value of 2x + 3y + 5z is

(a) 40

(b) 43

(c) 50

(d) 53

Answer Key-

Multiple Choice questions-

1. Answer: (a) 6
2. Answer: (c) $k^3|A|$
3. Answer: (c) Determinant is a number associated to a square matrix
4. Answer: (d) 12, -2.
5. Answer: (d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$
6. Answer: (b) $|A|^2$
7. Answer: (c) 9
8. Answer: (b) $\frac{1}{\det(A)}$
9. Answer: (a) 0

$$(a)\ \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \end{bmatrix}$$

10.Answer:

Very Short Answer:

1. Solution:

Co-factor of a_{23} = $(-1)^{2+3}\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$

= $(-1)^5$ (5 x 2 – 1 x 3)

= (-1) (10-3)

= (-1) (7) = -7.

2. Solution:

$$|(AB)^{-1}| = -\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{|AB|} = -\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{|A||B|} = -\frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{2|B|} = -\frac{1}{6}$$

Hence |B| = 3

3. Solution:

Given

$$\text{det.} = \begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2$]

$$= (l+m+n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}.$$

Hence, (l + m + n) is a factor of given determinant.

4. Solution:

$| 2 - \text{adj. } A| = 2^3 \, | A |^{3-1}$

$= 8(9)^2$

$= 648.$

5. Solution:

We have: AB = 2I

$\therefore |AB| = |2I|$

$\Rightarrow |A||B| = |2I|$

$\Rightarrow 2|B| = 2(1).$

Hence, $|B| = 1$.

6. Solution:

$|A. (\text{adj. } A) | = |A|^n$

$= 4^n$ or 16 or 64.

7. Solution:

(i) $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$

$= (5)(2) - (1)(3)$

$= 10 - 3 = 7.$

(ii) $a_{32} = (-1)^{3+2} \begin{vmatrix} 5 & 8 \\ 2 & 1 \end{vmatrix}$

$= (-1)^5 [(5)(1) - (2)(8)]$

$= (-1)^5 (5 - 16)$

$= (-1)(-11) = 11.$

8. Solution:

Here $|A| = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

Now A_{11} = Co-factor of 2 = 3,

A_{12} = Co-factor of – 1 = – 4,

A_{21} = Co-factor of 4 = 1

and A_{22} = Co-factor of 3 = 2

∴ Co-factor matrix = $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

Hence, adj. A = $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

9. Solution:

(i) We have: A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

∴ |A| = $\begin{vmatrix} 2 & -3 \\ -4 & 7 \end{vmatrix}$

= (2) (7) – (-4) (-3)

= 14 – 12 = 2 ≠ 0.

∴ A^{-1} exists and

$A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

(ii) RHS = 9I – A

$= 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 4 & -7 \end{bmatrix}$

$= \begin{bmatrix} 9-2 & 0+3 \\ 0+4 & 9-7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= 2 \times \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= 2A^{-1}$ = **LHS.**

10.Solution:

The matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular

$\Rightarrow \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix} = 0$

$\Rightarrow 4(5 - x) - 2(x + 1) = 0$

$\Rightarrow 20 - 4x - 2x - 2 = 0$

$\Rightarrow 18 - 6x = 0$

$\Rightarrow 6x = 18.$

Hence, $x = 3$.

Long Answer:

1. Solution:

$$\text{LHS} = \begin{vmatrix} a+b+c & a+b & a+c \\ -c & a+b & -(a+c) \\ -b & -(a+b) & a+c \end{vmatrix}$$

[Operating $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_1$]

$$= (a+b)(a+c)\begin{vmatrix} a+b+c & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$$

$$= (a+b)(a+c)\begin{vmatrix} a+b+c & 1 & 2 \\ -c & 1 & 0 \\ -b & -1 & 0 \end{vmatrix}$$

[Operating $C_3 \to C_3 + C_2$]

$= (a + b)(a + c)(2)[c + b]$

$= 2(a + b)(b + c)(c + a) = \text{RHS}.$

2. Solution:

We have

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$$= a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$$

[*Taking a common from* C_1]

$$= a\begin{vmatrix} 1 & 0 & 0 \\ x & a+x & -1 \\ x^2 & ax+x^2 & a \end{vmatrix}$$

[Operating $C_2 \to C_2 + C_1$]

$= a[(a + x) a + (ax + x^2)]$

$= a[a^2 + ax + ax + x^2]$

$= a(x^2 + 2 ax + a^2)$

$= a(x + a)^2$.

$f(2x) = a(2x + a)^2$,

$f(2x) - f(x) = a[(2x + a)^2 - (x + a)^2]$

$= a[(4x^2 + 4 ax + a^2) - (x^2 + 2ax + a^2)]$

$= a(3x^2 + 2 ax)$

$= ax(3x + 2a)$.

3. Solution:

$$\textbf{LHS} = \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$$

[*Operating* $C_2 \to C_2 - C_1$ *and* $C_3 \to C_3 - C_1$]

$$= 1.\begin{vmatrix} -3y & -3y \\ 3z & 0 \end{vmatrix} + 3x\begin{vmatrix} 1+3y & -3y \\ 1 & 3z \end{vmatrix}$$

[*Expanding by* R_1]

(0 + 9yz) + 3x(3 z + 9yz + 3y)

= 9(3xyz + xy + yz + zx) = RHS

4. Solution:

$$\textbf{LHS} = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix}$$

$$= \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

[*Operating* $C_1 \to a\,C_1$]

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$$

[*Operating* $C_1 \to C_1 + b\,C_2 + c\,C_3$]

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

[*Taking* $(a^2 + b^2 + c^2)$ *common from* C_1]

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$

[Opertating $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$]

$$= \frac{1}{a}(a^2+b^2+c^2)(1)\begin{vmatrix} c & -a-b \\ a+c & -b \end{vmatrix}$$

$$= \frac{a^2+b^2+c^2}{a}[-bc + a^2 + ac + ba + bc]$$

$$= \frac{(a^2+b^2+c^2)}{a}(a)(a+b+c)$$

$= (a+b+c)(a^2+b^2+c^2) =$ **RHS.**

Case Study Answers-

1.

(i) (a) ₹ 2

(ii) (d) ₹ 17

(iii) (a) ₹ 7

(iv) (d) ₹20

(v) (c) ₹22

2.

(i) (d) 12

(ii) (b)-z

(iii) (c)5

(iv) (c) 11

(v) (b)43

Assertion and Reason Answers-

1. (e) Both A and R are false.

Solution:

Minor of element $6 = M_{13} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 - 4 = -3$

∴ Given Assertion [A] is false Also we know that minor of an element a_{ij} of a matrix is the determinant obtained by deleting its i^{th} row and j^{th} column.

∴ Given Reason (R) is also false

∴ Both Assertion [A] and Reason [R] are false Hence option (e) is the correct Answer.

2. (b) Both A and R are true but R is not the correct explanation of A.

Solution:

Here,

|2AB|=23|AB|=8|A||B|

=8×3×−4=−96

∴ Assertion [A] is true

{∵|kA|=kn|A|and|AB|=|A||B|}

Also we know that |kA|=kn|A|

for matrix A of order n.

∴ Reason (R) is true But |AB|=|A||B|is not mentioned in Reason R.

∴ Both A and R are true but R is not correct explanation of A Hence option (b) is the correct answer.