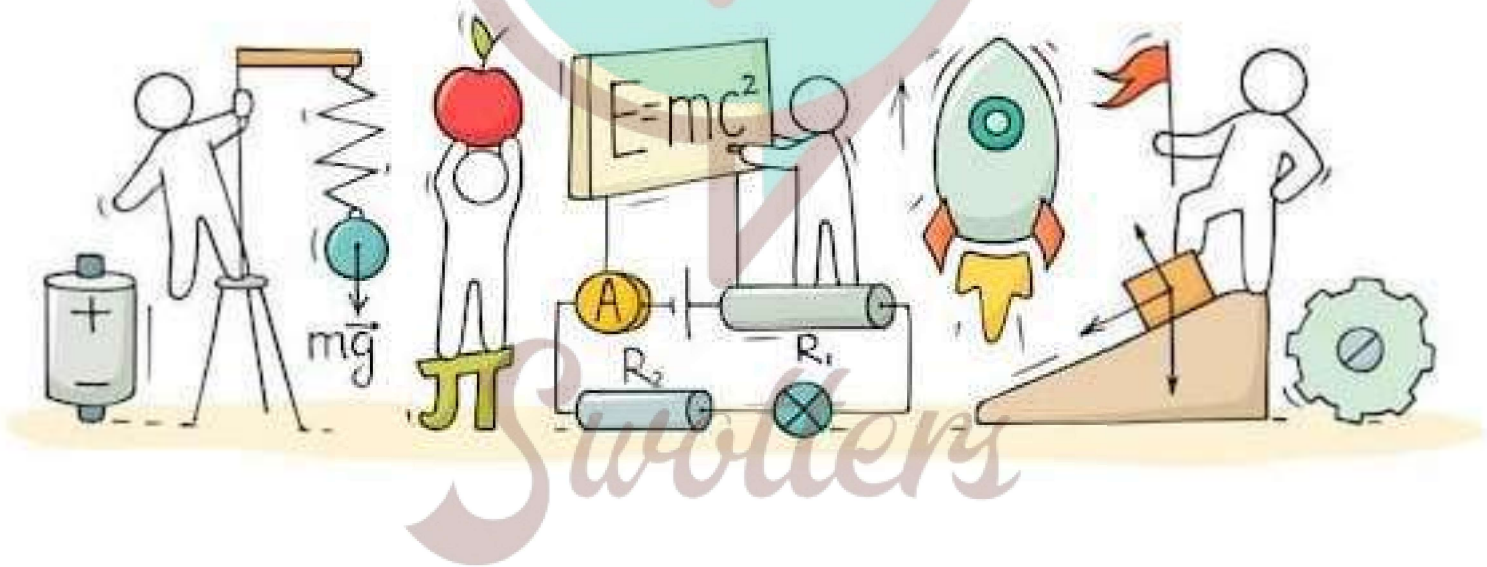


# PHYSICS

## Chapter 4: Motion in a Plane



## Important Questions

### Multiple Choice questions-

1. A body of mass 500 gram is rotating in a vertical circle of radius 1 m. What is the difference in its kinetic energies at the top and the bottom of the circle?
  - (a) 4.9 J
  - (b) 19.8 J
  - (c) 2.8 J
  - (d) 9.8 J
2. A particle has a displacement of 2 units along the x -axis, 1 unit along the y – axis and 2 units along the z – axis. Then the resultant displacement of the particle is
  - (a) 3 units
  - (b) 5 units
  - (c) 4 units
  - (d) 1 units
3. A car is moving on a circular path and takes a turn. If  $R^1$  and  $R^2$  are the reactions on the inner and outer wheels respectively, then
  - (a)  $R^1 = >R^2$
  - (b)  $R^1 = R^2$
  - (c)  $R^1 < R^2$
  - (d)  $R^1 > R^2$
4. The angle between centripetal acceleration and tangential acceleration is?
  - (a)  $180^\circ$
  - (b)  $0^\circ$
  - (c)  $90^\circ$
  - (d)  $45^\circ$
5. Large angle produces?
  - (a) high trajectory
  - (b) curve trajectory
  - (c) flat trajectory
  - (d) straight trajectory
6. He dimensional formula for normal acceleration is

- (a)  $LT^{-1}$
- (b)  $L^2T^2$
- (c)  $L^3T^{-2}$
- (d)  $LT^{-2}$

7. A book is pushed with an initial horizontal velocity of 5.0 meters per second off the top of a desk. What is the initial vertical velocity of the book?

- (a) 10. m/s
- (b) 0 m/s
- (c) 50 m/s
- (d) 2.5 m/s

8. One radian is defined to be the angle subtended where the arc length  $S$  is exactly equal to the?

- (a) radius of the circle.
- (b) diameter of the circle.
- (c) circumference of the circle.
- (d) half of radius of the circle.

9. A body travels along the circumference of a circle of radius 2 m with a linear velocity of 6 m/s. Then its angular velocity is

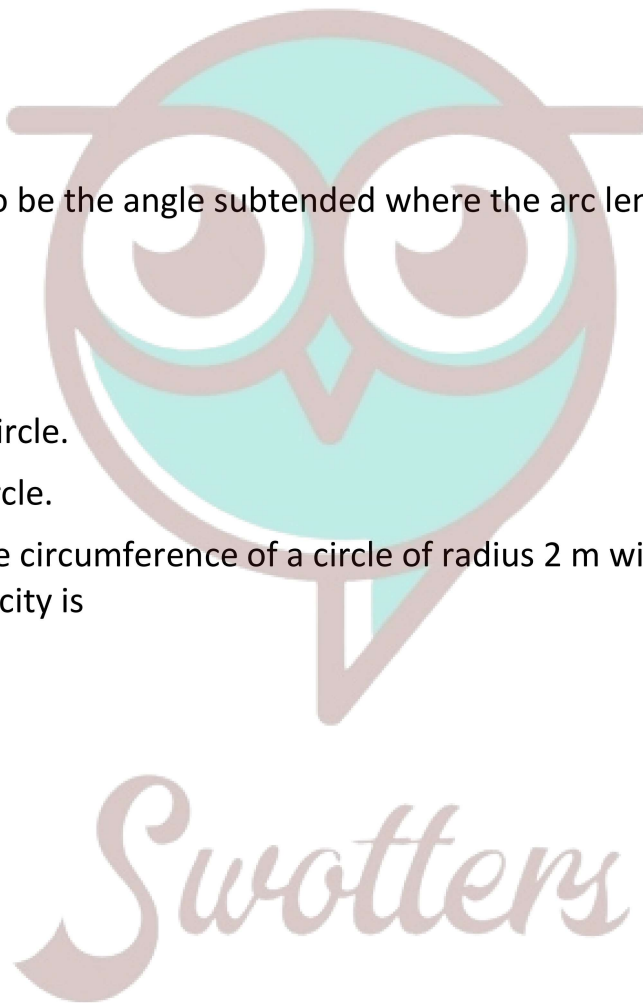
- (a) 6 rad /s
- (b) 3 rad /s
- (c) 2 rad / s
- (d) 4 rad / s

10. One° ( $1^\circ$ ) is equal to?

- (a) 0.1745 rad
- (b) 0.01745 rad
- (c) 0.001745 rad
- (d) 7.1745 rad

### Very Short:

1. Under what condition  $|a + b| = |a| + |b|$  holds good?
2. Under what condition  $|a - b| = |a| - |b|$  holds good?
3. The sum and difference of the two vectors are equal in magnitude



- i. e.  $|a + b| = |a - b|$ . What conclusion do you draw from this?
- What is the angle between  $\vec{A} \times \vec{B}$  and  $\vec{B} \times \vec{A}$ ?
  - What is the minimum number of coplanar vectors of different magnitudes which can give zero resultant?
  - When  $a - b = a + b$  condition holds good then what can you say about  $b$ ?
  - What is the magnitude of the component of the  $9\hat{i} - 9\hat{j} + 19\hat{k}$  vector along the x-axis?
  - Can displacement vector be added to force vector?
  - What is the effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar?
  - (a) What is the angle between  $\hat{i} + \hat{j}$  and  $\hat{i}$  vectors?  
(b) What is the angle between  $\hat{i} - \hat{j}$  and the x-axis?  
(c) What is the angle between  $\hat{i} + \hat{j}$  and  $\hat{i} - \hat{j}$ ?

### Short Questions:

- Name two quantities that are the largest when the maximum height attained by the projectile is largest.
- A stone dropped from the window of a stationary railway carriage takes 2 seconds to reach the ground. At what time the stone will reach the ground when the carriage is moving with
  - the constant velocity of  $80\text{kmh}^{-1}$
  - constant acceleration of  $2\text{ms}^{-2}$ ?
- Can a particle accelerate when its speed is constant? Explain.
- (a) Is circular motion possible at a constant speed or at constant velocity? Explain.  
(b) Define frequency and time period.
- When the component of a vector A along the direction of vector B is zero, what can you conclude about the two vectors?
- Comment on the statement whether it is true or false "Displacement vector is fundamentally a position vector." Why?
- Does the nature of a vector changes when it is multiplied by a scalar?
- Can the walk of a man be an example of the resolution of vectors? Explain.

### Long Questions:

- Discuss the problem of a swimmer who wants to cross the river in the shortest time.
- State and prove parallelogram law of vector addition. Discuss some special cases.



3. Derive the relation between linear velocity and angular velocity. Also, deduce its direction.

### Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
  - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
  - (c) Assertion is correct, reason is incorrect
  - (d) Assertion is incorrect, reason is correct.

**Assertion:** In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is  $180^\circ$ .

**Reason:** At the highest point, velocity of projectile will be in horizontal direction only.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
  - (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
  - (c) Assertion is correct, reason is incorrect
  - (d) Assertion is incorrect, reason is correct.

**Assertion:** Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.

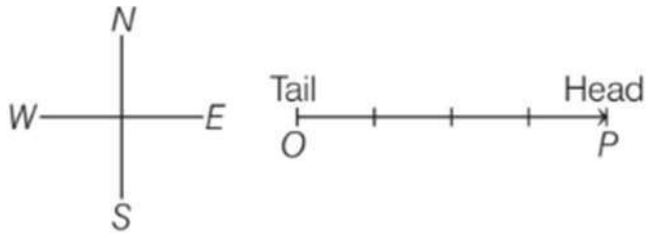
**Reason:** The maximum height of projectile is independent of particle mass.

### Case Study Questions:

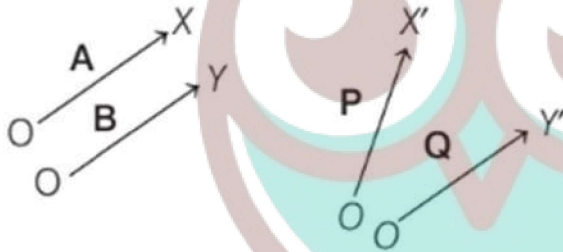
1. Vectors are the physical quantities which have both magnitudes and directions and obey the triangle/parallelogram laws of addition and subtraction. It is specified by giving its magnitude by a number and its direction. e.g., Displacement, acceleration, velocity, momentum, force, etc. A vector is represented by a bold face type and also by an arrow placed over a letter, i.e.

$$\mathbf{F}, \mathbf{a}, \mathbf{b} \text{ or } \vec{F}, \vec{a}, \vec{b}.$$

The length of the line gives the magnitude, and the arrowhead gives the direction. The point P is called head or terminal point and point O is called tail or initial point of the vector **OP**.



- i. Amongst the following quantities, which is not a vector quantity?
- Force
  - Acceleration
  - Temperature
  - Velocity
- ii. Set of vectors A and B, P and Q are as shown below

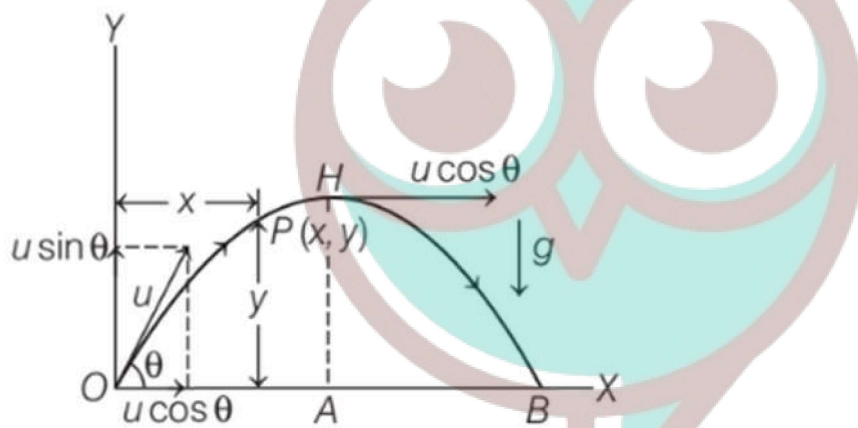


Length of A and B is equal, similarly length of P and Q is equal. Then, the vectors which are equal, are

- A and P
  - P and Q
  - A and B
  - B and Q
- iii.  $|\lambda A| = \lambda |A|$ , if
- $\lambda > 0$
  - $\lambda < 0$
  - $\lambda = 0$
  - $\lambda \neq 0$
- iv. Among the following properties regarding null vector which is incorrect?
- $A + 0 = A$
  - $\lambda 0 = \lambda$
  - $0A = 0$
  - $A - A = 0$

- v. The x and y components of a position vector P have numerical values 5 and 6, respectively. Direction and magnitude of vector P are
- $\tan^{-1}\left(\frac{6}{5}\right)$  and  $\sqrt{61}$
  - $\tan^{-1}\left(\frac{5}{6}\right)$  and  $\sqrt{61}$
  - $60^\circ$  and 8
  - $30^\circ$  and 9

2. Projectile motion is a form of motion in which an object or particle is thrown with some initial velocity near the earth's surface, and it moves along a curved path under the action of gravity alone. The path followed by a projectile is called its trajectory, which is shown below. When a projectile is projected obliquely, then its trajectory is as shown in the figure below



Here velocity  $u$  is resolved into two components, we get (a)  $u \cos \theta$  along  $OX$  and (b)  $u \sin \theta$  along  $OY$

- The example of such type of motion is
  - Motion of car on a banked road
  - Motion of boat in sea
  - A javelin thrown by an athlete
  - Motion of ball thrown vertically upward
- The acceleration of the object in horizontal direction is
  - Constant
  - Decreasing
  - Increasing
  - Zero
- The vertical component of velocity at point H is
  - Maximum

- (b) Zero  
 (c) Double to that at O  
 (d) Equal to horizontal component
- iv. A cricket ball is thrown at a speed of 28 m/s in a direction  $30^\circ$  with the horizontal. The time taken by the ball to return to the same level will be
- (a) 2.0 s  
 (b) 3.0 s  
 (c) 4.0 s  
 (d) 2.9 s
- v. In above case, the distance from the thrower to the point where the ball returns to the same level will be
- (a) 39 m  
 (b) 69 m  
 (c) 68 m  
 (d) 72 m



✓ Answer Key:

### Multiple Choice Answers-

1. Answer: (d) 9.8 J
2. Answer: (a) 3 units
3. Answer: (c)  $R^1 < R^2$
4. Answer: (c)  $90^\circ$
5. Answer: (a) high trajectory
6. Answer: (d)  $LT^{-2}$
7. Answer: (b) 0 m/s
8. Answer: (a) radius of the circle.
9. Answer: (b) 3 rad /s
10. Answer: (b) 0.01745 rad

### Very Short Answers:

1. Answer: When a and b act in the same direction i. e. when  $\theta = 0$  between • them, then  $|a + b| = |a| + |b|$ .
2. Answer: The condition  $|a - b| = |a| - |b|$  holds goods when a and b act in the opposite

direction.

3. Answer: The two vectors are equal in magnitude and are perpendicular to each other.
4. Answer: The given vectors act along two parallel lines in opposite directions i.e. they are anti-parallel, so the angle between them is  $180^\circ$ .
5. Answer: 3, If three vectors can be represented completely by the three sides of a triangle taken in the same order, then their resultant is zero.
6. Answer: For  $a - b = a + b$  condition to hold good,  $b$  must be a null vector.
7. Answer: 9.
8. Answer: No.
9. Answer: There is no effect on the dimensions of a vector if it is multiplied by a non-dimensional scalar.
10. Answer:
  - (a)  $45^\circ$
  - (b)  $45^\circ$
  - (c)  $90^\circ$

### Short Questions Answers:

1. Answer: Time of flight and the vertical component of velocity are the two quantities that are the largest when the maximum height attained by the projectile is the largest.
2. Answer: The time taken by the freely falling stone to reach the ground is given by

$$t = \sqrt{\frac{2h}{g}}$$

In both cases, the stone will fall through the same height as it is falling when the railway carriage is stationary. Hence the stone will reach the ground after 2 seconds.

3. Answer: Yes. A particle can be accelerated if its velocity changes. A particle having uniform circular motion has constant speed but its direction of motion changes continuously. Due to this, there is a change in velocity and hence the particle is moving with variable velocity. Thus, particle is accelerating.
4. Answer:
  - (a) Circular motion is possible at a constant speed because, in a circular motion, the magnitude of the velocity i.e. speed remains constant while the direction of motion changes continuously.
  - (b) Frequency is defined as the number of rotations completed by a body in one second and the time period is defined as the time taken by an object to complete one rotation.
5. Answer:

The two vectors A and B are perpendicular to each other.

Explanation: Let  $\theta$  = angle between the two vectors A and B component of vector A along the direction of B is obtained by resolving A i.e.  $A \cos \theta$ .

Now according to the statement

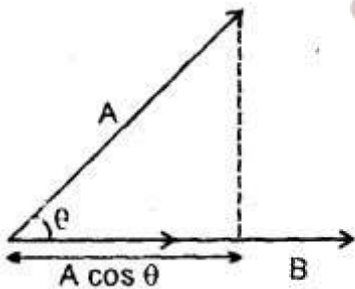
$$A \cos \theta = 0$$

or

$$\cos \theta = 0 = \cos 90^\circ$$

$$\theta = 90^\circ$$

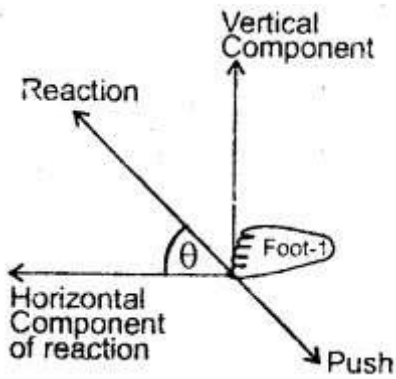
i.e.  $A \perp B$



Hence proved.

6. Answer: The given statement is true. The displacement vector gives the position of a point just like the position vector. The only difference between the displacement and the position vector is that the displacement vector gives the position of a point with reference to a point other than the origin, while the position vector gives the position of a point with reference to the origin. Since the choice of origin is quite arbitrary, so the given statement.
7. Answer: The nature of a vector may or may not be changed when it is multiplied. For example, when a vector is multiplied by a pure number like 1, 2, 3, ... etc., then the nature of the vector does not change. On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes. For example, when acceleration (vector) is multiplied by a mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration.
8. Answer: Yes, when a man walks, he pushes the ground with his foot. In return, an equal and opposite reaction acts on his foot. The reaction is resolved into two components: horizontal and vertical components. The horizontal component of the reaction helps the man to move forward while the vertical component balances the weight of the man.



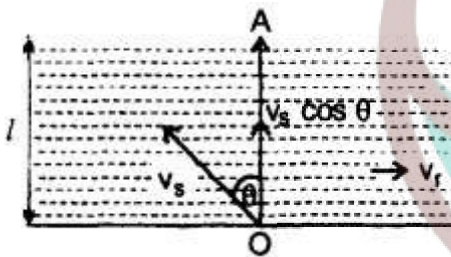


## Long Questions Answers:

1. Answer:

Let  $v_s$  and  $v_r$  be the velocities of swimmer and river respectively.

Let  $v$  = resultant velocity of  $v_s$  and  $v_r$



1. Let the swimmer begins to swim at an angle  $\theta$  with the line OA where OA is  $\perp$  to the flow of the river.

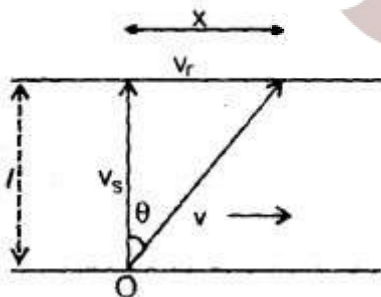
If  $t$  = time taken to cross the river, then

$$t = \frac{l}{v_s \cos \theta} \dots (i)$$

where  $l$  = breadth of the river

For  $t$  to be minimum,  $\cos \theta$  should be maximum.

i.e.,  $\cos \theta = 1$



This is possible if  $\theta = 0$

Thus, we conclude that the swimmer should swim in a direction perpendicular to the direction of the flow of the river.

$$2. v = \sqrt{v_s^2 + v_r^2}$$

where  $v\vec{v}$  is the resultant velocity of  $v_s$  and  $v_r$ .

$$3. \tan \theta = \frac{v_r}{v_s} = \frac{x}{l}$$

or

$$x = l \frac{v_r}{v_s}$$

$$4. t = \frac{l}{v_s}$$

2. Answer:

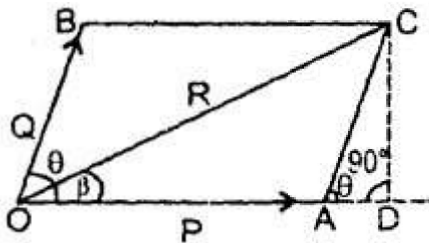
It states that if two vectors can be represented completely (i.e. both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point.

Proof: Let P and Q be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB s.t.

$$\vec{OA} = P, \vec{OB} = Q$$

or

$$|\vec{OA}| = |P|, |\vec{OB}| = |Q|$$



$\theta$  = angle between them =  $\angle AOB$

If R be their resultant, then it will be represented completely by the diagonal OC through point O s.t.  $OC = R$

The magnitude of R: Draw  $CD \perp$  to OA produced,

$$\therefore \angle DAC = \angle AOB = \theta$$

Now in right angled triangle ODC,

$$\begin{aligned} OC^2 &= OD^2 + DC^2 \\ &= (OA + AD)^2 + DC^2 \\ &= OA^2 + AD^2 + 2.OA.AD + DC^2 \\ &= OA^2 + (AD^2 + DC^2) + 2OA.AD \quad \dots (i) \end{aligned}$$

Also in r.t.  $\Delta ADC$ ,

$$AC^2 = AD^2 + DC^2 \quad \dots (ii)$$

Also  $\frac{AD}{AC} = \cos \theta$

or  $AD = AC \cos \theta \quad \dots (iii)$

and  $\frac{DC}{AC} = \sin \theta$

or  $DC = AC \sin \theta \quad \dots (iv)$

$\therefore$  from equation (i), (ii), (iii), we get

$$OC^2 = OA^2 + AC^2 + 2.OA.AC \cos \theta$$

or  $OC = \sqrt{OA^2 + AC^2 + 2.OA.AC \cos \theta} \quad \dots (v)$

As  $OC = R$ ,  $OA = P$ ,  $AC = OB = Q \quad \dots (vi)$

$\therefore$  from (v) and (vi), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad \dots (vii)$$

eqn. (vii) gives the magnitude of R.

The direction of R: Let  $\beta$  be the angle made by R with P

$\therefore$  in r.t.  $\Delta ODC$ ,

$$\tan \beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC \sin \theta}{OA + AC \cos \theta} \quad [\text{by using (iii) and (iv)}]$$

$$\tan \beta = \frac{Q \sin \theta}{P + Q \cos \theta} \quad \dots (viii)$$

Special cases: (a) When two vectors are acting in the same direction:

Then  $\theta = 0^\circ$

$$\therefore R = \sqrt{(P+Q)^2} = P+Q$$

$$\text{and } \tan \beta = \frac{Q \cdot 0}{P+Q} = 0 \text{ or } \beta = 0^\circ$$

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction, and their resultant acts in the direction of P and Q.

(b) When two vectors act in the opposite directions:

Then  $\theta = 180^\circ$

$$\therefore \cos \theta = -1 \text{ and } \sin \theta = 0$$

$$\begin{aligned} \therefore R &= \sqrt{P^2 + Q^2 + 2PQ(-1)} \\ &= \sqrt{P^2 + Q^2 - 2PQ} \\ &= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2} \\ &= (P-Q) \text{ or } (Q-P) \end{aligned}$$

$$\text{and } \tan \beta = \frac{Q \times 0}{P+Q(-1)} = 0 = \tan 0^\circ \text{ or } \tan 180^\circ$$

$$\therefore \beta = 0^\circ \text{ or } 180^\circ$$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of the bigger vector.

(c) If  $\theta = 90^\circ$  i.e. if  $P \perp Q$ ,

then  $\cos 90^\circ = 0$

and

$\sin 90^\circ = 1$

$$R = \sqrt{P^2 + Q^2}$$

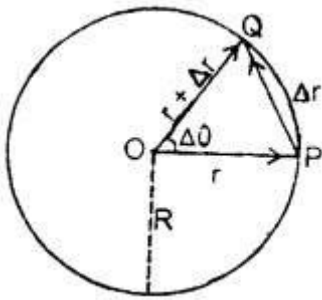
and

$$\tan \beta = \frac{Q}{P}$$

3. Answer:

Let R be the radius of the circular path of centre O on which an object is moving with uniform angular velocity  $\omega$ . Let v = its linear velocity. Let the object move from point P at

time  $t$  to point  $Q$  at time  $t + \Delta t$ . If  $\mathbf{r}$  and  $\mathbf{r} + \Delta \mathbf{r}$  be its position vectors at point  $P$  and  $Q$  respectively, then



$$\vec{OP} = \mathbf{r}$$

and

$$\vec{OQ} = \mathbf{r} + \Delta \mathbf{r}$$

Also

$$|\mathbf{r}| = |\mathbf{r} + \Delta \mathbf{r}| = R \\ = \text{radius of circle.}$$

$\therefore$  Linear displacement of the particle from  $P$  to  $Q$  in small time interval  $\Delta t = \Delta r$ .

Let  $\Delta \theta =$  its angular displacement

$$\therefore \omega = \frac{\Delta \theta}{\Delta t}$$

or

$$\Delta \theta = \omega \Delta t \dots (1)$$

$$\text{Also we know that } \Delta \theta = \frac{\widehat{PQ}}{R} \dots (2)$$

$\therefore$  from (1) and (2), we get

$$\omega \Delta t = \frac{\widehat{PQ}}{R} \text{ or } \frac{\widehat{PQ}}{\Delta t} = \omega R \dots (3)$$

Now when  $\Delta t \rightarrow 0$ , then from eqn. (1)  $\Delta \theta \rightarrow 0$

so arc  $PQ = \widehat{PQ} =$  chord  $PQ$

Thus eqn. (3) reduces to

$$\frac{PQ}{\Delta t} = \omega R$$

or

$$\mathbf{v} = R\omega$$

where  $\mathbf{v} = \frac{PQ}{\Delta t}$  is the linear velocity of the object.

Direction of velocity vector: In isosceles  $\Delta OPQ$ ,



$$\angle P Q O + \angle O P Q + \angle Q O P = 180^\circ$$

As

$$\angle Q = \angle P$$

 $\therefore$ 

$$2\angle P + \Delta\theta = \pi$$

or

$$\angle Q P O = \frac{\pi}{2} - \frac{\Delta\theta}{2}$$

$$= \frac{\pi}{2} - \frac{\omega\Delta t}{2} \quad \dots (4)$$

when  $\Delta t \rightarrow 0$ ,  $\angle Q P O \rightarrow \frac{\pi}{2}$

i.e.  $\vec{OP}$  tends to become  $\perp$  to  $\vec{OP}$

or

$\vec{OP}$  tends to lie along the tangent at P. Hence velocity vector at P is directed along the tangent to the circle in the direction of motion

### Assertion Reason Answer:

- (d) Assertion is incorrect, reason is correct.
- (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

### Case Study Answer:

- i (c) Temperature

#### Explanation:

Temperature is not a vector quantity because it has magnitude only. However, force, acceleration and velocity have both a magnitude and a direction. So, these are vectors in nature.

- ii (c) A and B

#### Explanation:

Two vectors are said to be equal, if and only if they have the same magnitude and direction. Among the given vectors A and B are equal vectors as they have same magnitude (length) and direction. However, P and Q are not equal even though they are of same magnitude because their directions are different.

- iii (a)  $\lambda > 0$

#### Explanation:

$|\lambda A| = \lambda |A|$ , if  $\lambda > 0$  as multiplication of vector A with a positive number  $\lambda$  gives a vector whose magnitude is changed by the factor  $\lambda$  but the direction is same as that of A.

- iv (b)  $\lambda 0 = 0$



**Explanation:**

Null vector  $0$  is a vector, whose magnitude is zero and its direction cannot be specified. So, it means,  $|0| = 0$ . Thus,  $\lambda 0 = 0$ . Hence, property given in option (b) is incorrect.

$$v \text{ (a) } \tan^{-1}\left(\frac{6}{5}\right) \text{ and } \sqrt{61}$$

**Explanation:**

Let  $\mathbf{P}$  be as shown in the figure, then according to the given information

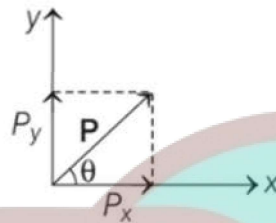
$$P_x = 5, P_y = 6$$

$$\therefore |\mathbf{P}| = \sqrt{P_x^2 + P_y^2}$$

$$= \sqrt{25 + 36}$$

$$\Rightarrow |\mathbf{P}| = \sqrt{61}$$

$$\text{and } \tan \theta = \frac{P_y}{P_x} = \frac{6}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{6}{5}\right)$$



2. i (c) a javelin thrown by an athlete

**Explanation:**

A javelin thrown by an athlete is an example of projectile motion.

ii (d) zero

**Explanation:**

The horizontal component of velocity ( $u \cos \theta$ ) is constant throughout the motion, so there will be no acceleration in horizontal direction.

iii (b) zero

**Explanation:**

As the vertical components of velocity ( $u \sin \theta$ ) decreases continuously with height, from  $O$  to  $H$ , due to downward force of gravity and becomes zero at  $H$

iv (d) 2.9 s

**Explanation:**

The time taken by the ball to return to the same level,

$$T = \frac{2v_0 \sin \theta}{g} = \frac{2 \times 28 \times \sin 30^\circ}{9.8} \approx 2.9 \text{ s}$$

v (b) 69 m

**Explanation:**

The distance from the thrower to the point where the ball returns to the same level is

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{28 \times 28 \times \sin 60^\circ}{9.8} \approx 69 \text{ m}$$



*Swotters*