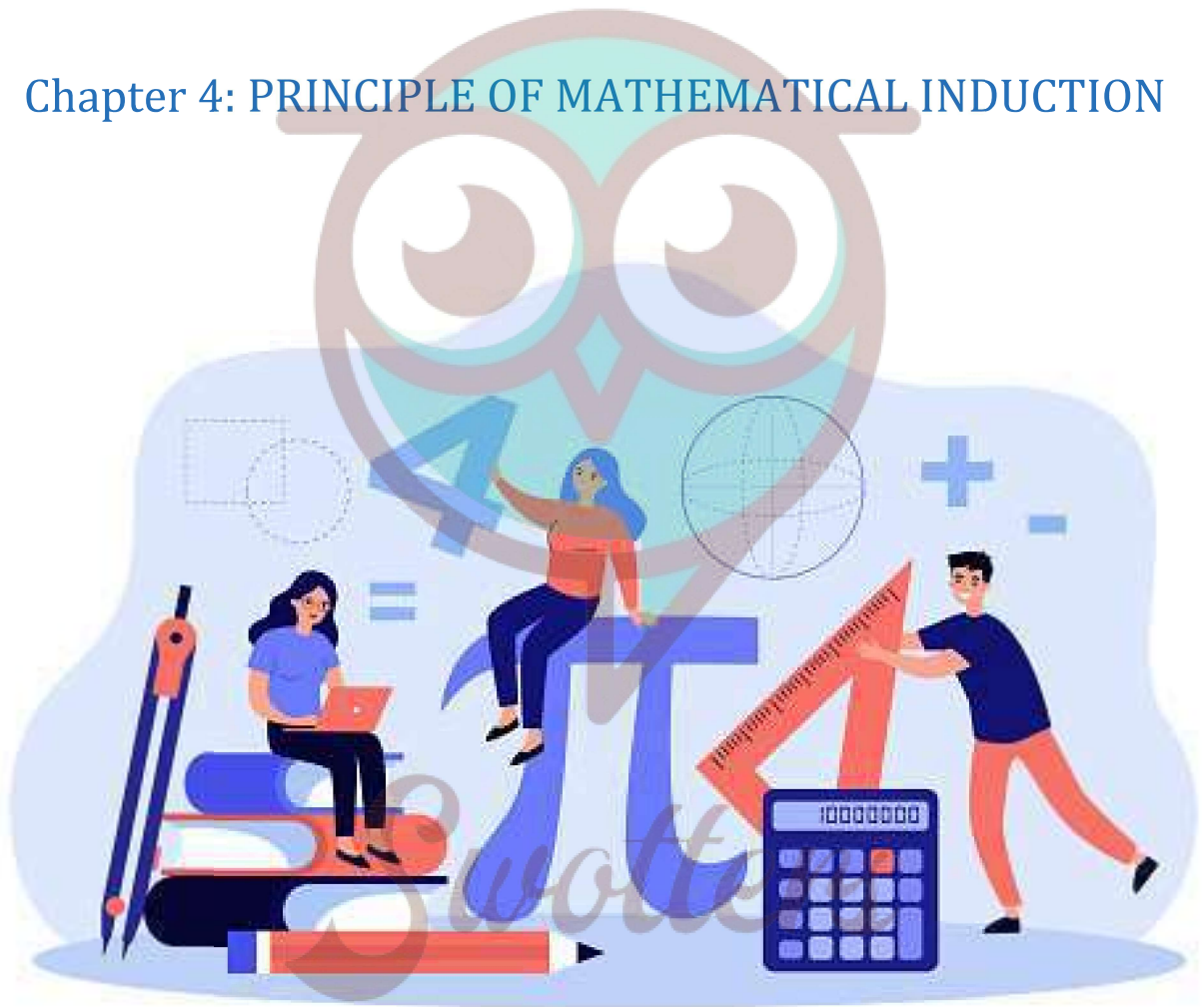


MATHEMATICS

Chapter 4: PRINCIPLE OF MATHEMATICAL INDUCTION



Important Questions

Multiple Choice questions-

Question 1. For all $n \in \mathbb{N}$, $3n^5 + 5n^3 + 7n$ is divisible by

- (a) 5
- (b) 15
- (c) 10
- (d) 3

Question 2. $\{1 - (1/2)\}\{1 - (1/3)\}\{1 - (1/4)\} \dots \dots \{1 - 1/(n + 1)\} =$

- (a) $1/(n + 1)$ for all $n \in \mathbb{N}$.
- (b) $1/(n + 1)$ for all $n \in \mathbb{R}$
- (c) $n/(n + 1)$ for all $n \in \mathbb{N}$.
- (d) $n/(n + 1)$ for all $n \in \mathbb{R}$

Question 3. For all $n \in \mathbb{N}$, $3^{2n} + 7$ is divisible by

- (a) non of these
- (b) 3
- (c) 11
- (d) 8

Question 4. The sum of the series $1 + 2 + 3 + 4 + 5 + \dots \dots n$ is

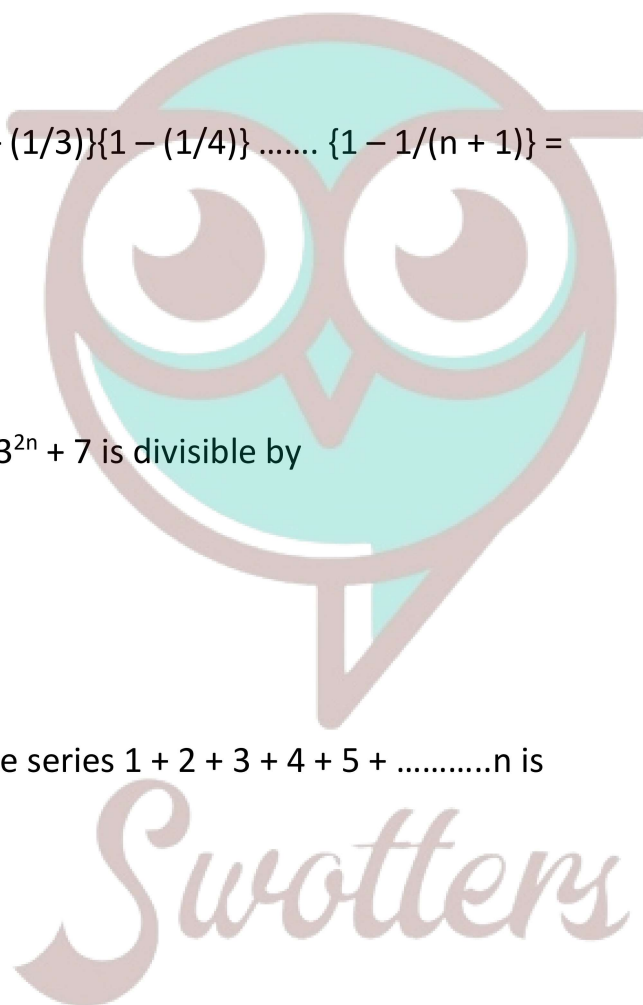
- (a) $n(n + 1)$
- (b) $(n + 1)/2$
- (c) $n/2$
- (d) $n(n + 1)/2$

Question 5. The sum of the series $1^2 + 2^2 + 3^2 + \dots \dots n^2$ is

- (a) $n(n + 1) (2n + 1)$
- (b) $n(n + 1) (2n + 1)/2$
- (c) $n(n + 1) (2n + 1)/3$
- (d) $n(n + 1) (2n + 1)/6$

Question 6. For all positive integers n , the number $n(n^2 - 1)$ is divisible by:

- (a) 36



- (b) 24
- (c) 6
- (d) 16

Question 7. If n is an odd positive integer, then $a^n + b^n$ is divisible by :

- (a) $a^2 + b^2$
- (b) $a + b$
- (c) $a - b$
- (d) none of these

Question 8. $n(n + 1)(n + 5)$ is a multiple of ____ for all $n \in \mathbb{N}$

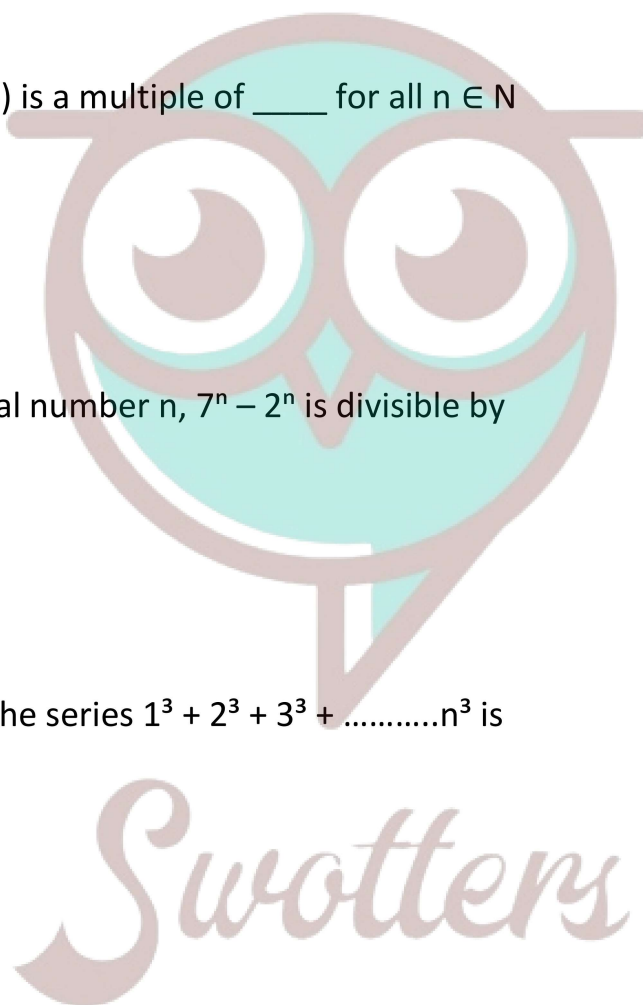
- (a) 2
- (b) 3
- (c) 5
- (d) 7

Question 9. For any natural number n , $7^n - 2^n$ is divisible by

- (a) 3
- (b) 4
- (c) 5
- (d) 7

Question 10. The sum of the series $1^3 + 2^3 + 3^3 + \dots + n^3$ is

- (a) $\{(n + 1)/2\}^2$
- (b) $\{n/2\}^2$
- (c) $n(n + 1)/2$
- (d) $\{n(n + 1)/2\}^2$



Very Short:

- 1.

Short Questions:

1. For every integer n , prove that $7n - 3n$ divisible by 4.
2. Prove that $n(n + 1)(n + 5)$ is multiple of 3.
3. Prove that $10^{2n-1} + 1$ is divisible by 11.
4. Prove that $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \dots \left(1 + \frac{1}{n}\right) = (n + 1)$

5. Prove $1.2 + 2.3 + 3.4 + \dots + n(n + 1) = \frac{n(n+1)(n+2)}{3}$

Long Questions:

1. Prove $(2n+7) < (n + 3)^2$

2. Prove that:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

3. Prove $1.2 + 2.22 + 3.23 + \dots + n.2^n = (n - 1)^{2n+1} + 2$

4. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 $\forall n \in N$.

5. Prove that $41^n - 14^n$ is a multiple of 27.

Answer Key:

MCQ:

1. (b) 15
2. (a) $1/(n + 1)$ for all $n \in N$.
3. (d) 8
4. (d) $n(n + 1)/2$
5. (d) $n(n + 1) (2n + 1)/6$
6. (c) 6
7. (b) $a + b$
8. (b) 3
9. (c) 5
10. (d) $\{n(n + 1)/2\}^2$

Very Short Answer:

1. $\left(\frac{\pi}{32}\right)^c$
2. $39^\circ 22' 30''$
3. $\frac{5\pi}{12} cm$
4. $\sqrt{3}$
5. $\frac{-1}{\sqrt{2}}$
6. $2 - \sqrt{3}$

7. $\frac{-4}{5}$

8. 45°

9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x - \sin 2x$

Short Answer:

1. $P(n) : 7^n - 3^n$ is divisible by 4

For $n = 1$

$P(1) : 7^1 - 3^1 = 4$ which is divisible by Thus, $P(1)$ is true

Let $P(k)$ be true

$7^k - 3^k$ is divisible by 4

$7^k - 3^k = 4\lambda$, where $\lambda \in \mathbb{N}$ (i)

we want to prove that $P(k+1)$ is true whenever $P(k)$ is true

$7^{k+1} - 3^{k+1} = 7^k \cdot 7 - 3^k \cdot 3$

$= (4\lambda + 3^k) \cdot 7 - 3^k \cdot 3$ (from i)

$= 28\lambda + 7 \cdot 3^k - 3^k \cdot 3$

$= 28\lambda + 3^k(7 - 3)$

$= 4(7\lambda + 3^k)$

Hence

$7^{k+1} - 3^{k+1}$ is divisible by 4

thus $P(k+1)$ is true when $P(k)$ is true.

Therefore by P.M.I. the statement is true for every positive integer n .

2.

$P(n) : n(n+1)(n+5)$ is multiple of 3

for $n=1$

$P(1) : 1(1+1)(1+5) = 12$ is multiple of 3

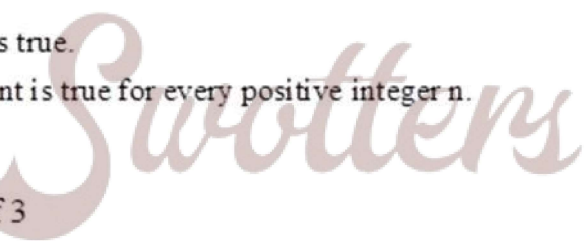
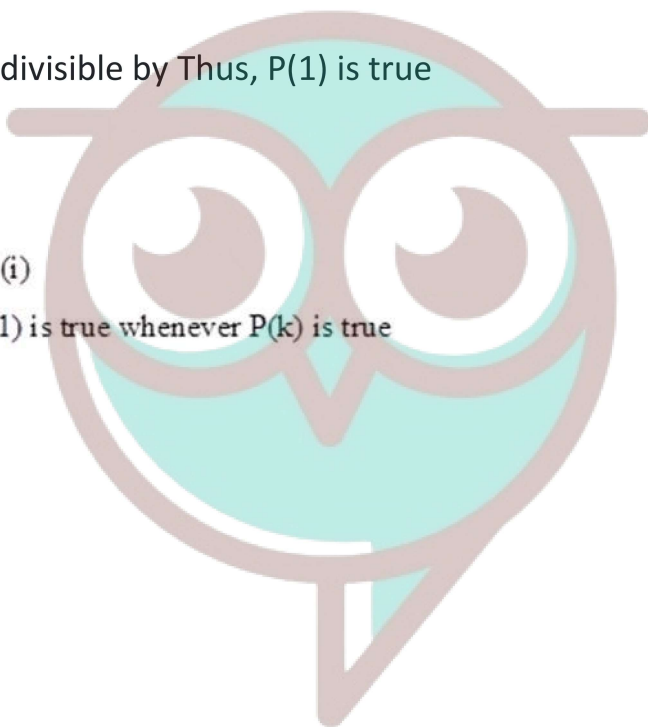
let $P(k)$ be true

$P(k) : k(k+1)(k+5)$ is multiple of 3

$\Rightarrow k(k+1)(k+5) = 3\lambda$ where $\lambda \in \mathbb{N}$ (i)

we want to prove that result is true for $n=k+1$

$P(k+1) : (k+1)(k+2)(k+6)$



$$\begin{aligned} \Rightarrow (K+1)(k+2)(k+6) &= [(k+1)(k+2)](k+6) \\ &= k(k+1)(k+2)+6(k+1)(k+2) \\ &= k(k+1)(k+5-3)+6(k+1)(k+2) \\ &= k(k+1)(k+5)-3k(k+1)+6(k+1)(K+2) \\ &= k(k+1)(k+5)+(k+1)[6(k+2)-3k] \\ &= k(k+1)(k+5)+(k+1)(3k+12) \\ &= k(k+1)(k+5)+3(k+1)(k+4) \\ &= 3\lambda + 3(k+1)(k+4) \text{ (from i)} \\ &= 3[\lambda + (K+1)(K+4)] \text{ which is multiple of three} \end{aligned}$$

Hence $P(k+1)$ is multiple of 3 .

3.

$P(n): 10^{2n-1} + 1$ is divisible by 11

for $n=1$

$P(1) = 10^{2 \cdot 1 - 1} + 1 = 11$ is divisible by 11 Hence result is true for $n=1$

let $P(k)$ be true

$P(k): 10^{2k-1} + 1$ is divisible by 11

$\Rightarrow 10^{2k-1} + 1 = 11\lambda$ where $\lambda \in \mathbb{N}(i)$

we want to prove that result is true for $n= k+$

$= 10^{2(k+1)-1} + 1 = 10^{2k+2-1} + 1$

$= 10^{2k+1} + 1$

$= 10^{2k} \cdot 10^1 + 1$

$= (110\lambda - 10) \cdot 10 + 1$ (from i)

$= 1100\lambda - 100 + 1$

$= 1100\lambda - 99$

$= 11(100\lambda - 9)$ is divisible by 11

Hence by P.M.I. $P(k+1)$ is true whenever $P(k)$ is true.

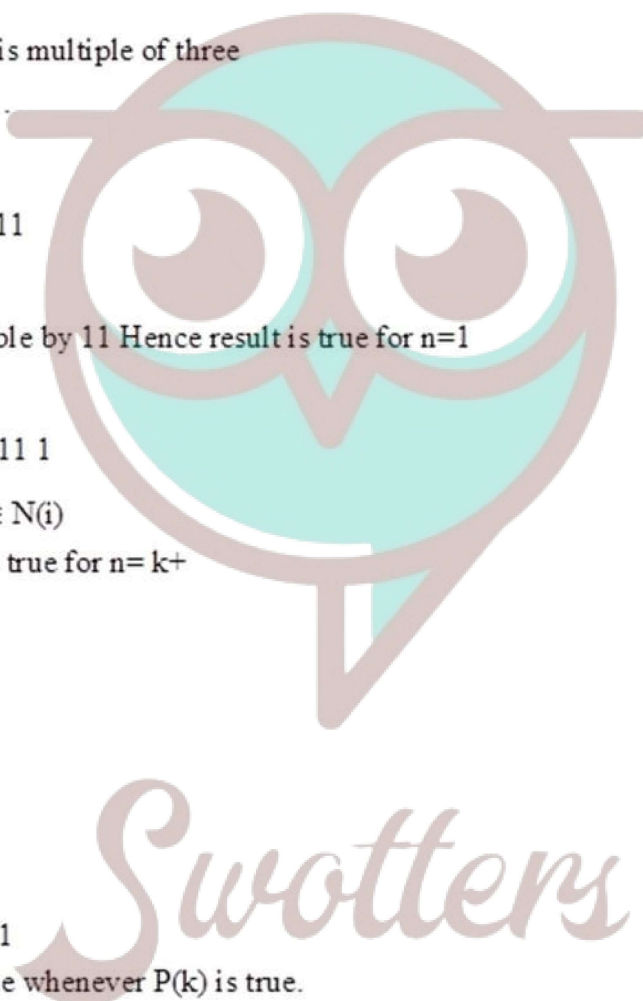
4.

let $P(n): \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \dots \left(1 + \frac{1}{n}\right) = (n+1)$

for $n=1$

$P(1): \left(1 + \frac{1}{1}\right) = (1+1) = 2$

which is true



let $P(k)$ be true

$$P(k) : \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\dots\left(1 + \frac{1}{k}\right) = (k+1)$$

we want to prove that $P(k+1)$ is true

$$P(k+1) : \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\dots\left(1 + \frac{1}{k+1}\right) = (k+2)$$

$$L.H.S. = \left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\dots\left(1 + \frac{1}{k}\right)\left(1 + \frac{1}{k+1}\right)$$

$$= (k+1)\left(1 + \frac{1}{k+1}\right) \quad [from(1)]$$

$$= (k+1)\left(\frac{k+1+1}{k+1}\right)$$

$$= (k+2)$$

thus $P(k+1)$ is true whenever

$P(k)$ is true.

5.

$$p(n) : 1.2 + 2.3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

for $n=1$

$$p(1) : 1(1+1) = \frac{1(1+1)(1+2)}{3}$$

$$p(1) = 2 = 2$$

hence $p(1)$ be true

$$p(k) : 1.2 + 2.3 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3} \dots\dots\dots (i)$$

we want to prove that

$p(k+1)$:

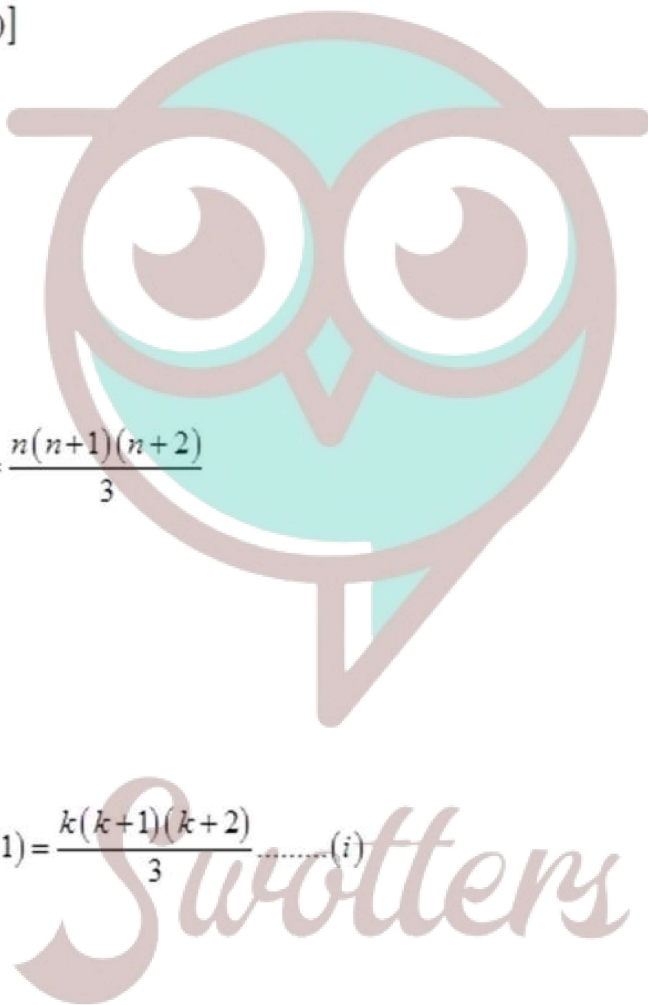
$$1.2 + 2.3 + \dots + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

L.H.S.

$$= 1.2 + 2.3 + \dots + k(k+1) + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + \frac{(k+1)(k+2)}{1} \quad [from(i)]$$

$$\frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$



$$\frac{(k+1)(k+2)[k+3]}{3}$$

hence $p(k+1)$ is true whenever $p(k)$ is true

Long Answer:

1.

$$p(n) : (2n+7) < (n+3)^2$$

for $n=1$

$$9 < (4)^2$$

$$9 < 16$$

which is true

let $p(k)$ be true

$$(2k+7) < (k+3)^2$$

now

$$2(k+1)+7 = (2k+7)+2$$

$$< (k+3)^2 + 2 = k^2 + 6k + 11$$

$$< k^2 + 8k + 16 = (k+4)^2$$

$$= (k+3+1)^2$$

$$\therefore p(k+1) : 2(k+1)+7 < (k+1+3)^2$$

$\Rightarrow p(k+1)$ is true, whenever $p(k)$ is true

hence by PMI $p(k)$ is true for all $n \in \mathbb{N}$



Swotters

2.

$$p(n) : \frac{1}{1.4} + \frac{1}{4.7} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}$$

for $n=1$

$$p(1) : \frac{1}{(3-2)(3+1)} = \frac{1}{(3+1)} = \frac{1}{4}$$

which is true

let $p(k)$ be true

$$p(k) : \frac{1}{1.4} + \frac{1}{4.7} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{(3k+1)} \dots (i)$$

we want to prove that $p(k+1)$ is true

$$p(k+1): \frac{1}{1.4} + \frac{1}{4.7} + \dots + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{(3k+4)}$$

L.H.S.

$$= \frac{1}{1.4} + \frac{1}{4.7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} \quad [\text{from.....(i)}]$$

$$= \frac{k(3k+4)+1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2+4k+1}{(3k+1)(3k+4)} = \frac{\cancel{(3k+1)}(k+1)}{\cancel{(3k+1)}(3k+4)}$$

$p(k+1)$ is true whenever $p(k)$ is true.

3.

$$p(n): 1.2 + 2.2^2 + 3.2^3 + \dots + n.2^n = (n-1)2^{n+1} + 2$$

$$p(n): 1.2 + 2.2^2 + 3.2^3 + \dots + n.2^n = (n-1)2^{n+1} + 2$$

for $n=1$

$$p(1): 1.2^1 = (1-1)2^2 + 2$$

$$2 = 2 \text{ which is true}$$

let $p(k)$ be true

$$p(k): 1.2 + 2.2^2 + \dots + k.2^k = (k-1)2^{k+1} + 2 \dots \dots \dots (i)$$

we want to prove that $p(k+1)$ is true

$$p(k+1): 1.2 + 2.2^2 + \dots + (k+1)2^{k+1} = k.2^{k+2} + 2$$

L.H.S.

$$1.2 + 2.2^2 + \dots + k.2^k + (k+1)2^{k+1} \quad [\text{from.....(i)}]$$

$$= (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$$

$$= 2^{k+1}(k-1 + k+1) + 2$$

$$= 2^{k+2}k + 2$$

This $p(k+1)$ is true whenever $p(k)$ is true

4. $P(n) : 2.7^n + 3.5^n - 5$ is divisible by 24

for $n = 1$

$$P(1) : 2.7^1 + 3.5^1 - 5 = 24 \text{ is divisible by 24}$$

Hence result is true for $n = 1$

Let P (K) be true

$$P (K) : 2 \cdot 7^K + 3 \cdot 5^K - 5$$

$$\Rightarrow 2 \cdot 7^K + 3 \cdot 5^K - 5 = 24\lambda \text{ when } \lambda \in \mathbb{N}$$

we want to prove that P (K+!) is True whenever P (K) is true

$$2 \cdot 7^{K+1} + 3 \cdot 5^{K+1} - 5 = 2 \cdot 7^K \cdot 7 + 3 \cdot 5^K \cdot 5 - 5$$

$$= 7[2 \cdot 7^K + 3 \cdot 5^K - 5 - 3 \cdot 5^K + 5] + 3 \cdot 5^K \cdot 5 - 5$$

$$= 7[24\lambda - 3 \cdot 5^K + 5] + 15 \cdot 5^K - 5 \text{ (from i)}$$

$$= 7 \times 24\lambda - 21 \cdot 5^K + 35 + 15 \cdot 5^K - 5$$

$$= 7 \times 24\lambda - 6 \cdot 5^K + 30$$

$$= 7 \times 24\lambda - 6(5^K - 5)$$

$$= 7 \times 24\lambda - 6 \cdot 4p \text{ [} \because 5^K - 5 \text{ is multiple of 4]}$$

$$= 24(7\lambda - p), \quad 24 \text{ is divisible by 24}$$

Hence by P M I p (n) is true for all $n \in \mathbb{N}$.

5. P (n) : $41^n - 14^n$ is a multiple of 27

for $n = 1$

$$P (1) : 41^1 - 14 = 27, \text{ which is a multiple of 27}$$

Let P (K) be True

$$P (K) : 41^K - 14^K$$

$$\Rightarrow 41^K - 14^K = 27\lambda, \text{ where } \lambda \in \mathbb{N}$$

we want to prove that result is true for $n = K + 1$

$$41^{K+1} - 14^{K+1} = 41^K \cdot 41 - 14^K \cdot 14$$

$$= (27\lambda + 14^K) \cdot 41 - 14^K \cdot 14 \text{ (from i)}$$

$$= 27\lambda \cdot 41 + 14^K \cdot 41 - 14^K \cdot 14$$

$$= 27\lambda \cdot 41 + 14^K(41 - 14)$$

$$= 27\lambda \cdot 41 + 14^K(27)$$

$$= 27(41\lambda + 14^K) \text{ is a multiple of 27}$$

Hence by PMI p (n) is true for all $n \in \mathbb{N}$.

