MATHEMATICS

Chapter 4: PRINCIPLE OF MATHEMATICAL INDUCTION

Important Questions

Multiple Choice questions-

Question 1. For all n ∈ N, $3n^5 + 5n^3 + 7n$ is divisible by

(a) 5

(b) 15

(c) 10

(d) 3

Question 2. {1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. {1 – 1/(n + 1)} =

(a) 1/(n + 1) for all n ∈ N.

(b) 1/(n + 1) for all n ∈ R

(c) n/(n + 1) for all n ∈ N.

(d) n/(n + 1) for all n ∈ R

Question 3. For all n ∈ N, $3^{2n} + 7$ is divisible by

(a) non of these

(b) 3

(c) 11

(d) 8

Question 4. The sum of the series 1 + 2 + 3 + 4 + 5 + ……….n is

(a) n(n + 1)

(b) (n + 1)/2

(c) n/2

(d) n(n + 1)/2

Question 5. The sum of the series $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots n^2$ is

(a) n(n + 1) (2n + 1)

(b) n(n + 1) (2n + 1)/2

(c) n(n + 1) (2n + 1)/3

(d) n(n + 1) (2n + 1)/6

Question 6. For all positive integers n, the number $n(n^2 - 1)$ is divisible by:

(a) 36

(b) 24

(c) 6

(d) 16

Question 7. If n is an odd positive integer, then $a^n + b^n$ is divisible by :

(a) $a^2 + b^2$

(b) a + b

(c) a – b

(d) none of these

Question 8. n(n + 1) (n + 5) is a multiple of ____ for all n ∈ N

(a) 2

(b) 3

(c) 5

(d) 7

Question 9. For any natural number n, $7^n – 2^n$ is divisible by

(a) 3

(b) 4

(c) 5

(d) 7

Question 10. The sum of the series $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots n^3$ is

(a) $\{(n + 1)/2\}^2$

(b) $\{n/2\}^2$

(c) n(n + 1)/2

(d) $\{n(n + 1)/2\}^2$

Very Short:

1.

Short Questions:

1. For every integer n, prove that 7n – 3n divisible by 4.

2. Prove that n (n + 1) (n + 5) is multiple of 3.

3. Prove that $10^{2n-1} + 1$ is divisible by 11.

4. Prove that $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots.\left(1+\frac{1}{n}\right)=(n+1)$

5. Prove 1.2 + 2.3 + 3.4 + _ _ _ + n $(n+1) = \frac{n(n+1)(n+2)}{3}$

Long Questions:

1. Prove $(2n+7) < (n+3)^2$
2. Prove that:

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$$

3. Prove $1.2 + 2.22 + 3.23 +...+ n.2^n = (n-1)^{2n+1} + 2$
4. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 $\forall\, n \epsilon N$.
5. Prove that $41^n - 14^n$ is a multiple of 27.

Answer Key:

MCQ:

1. (b) 15
2. (a) 1/(n + 1) for all n ∈ N.
3. (d) 8
4. (d) n(n + 1)/2
5. (d) n(n + 1) (2n + 1)/6
6. (c) 6
7. (b) a + b
8. (b) 3
9. (c) 5
10. (d) $\{n(n+1)/2\}^2$

Very Short Answer:

1. $\left(\frac{\pi}{32}\right)^c$
2. 39°22"30""
3. $\frac{5\pi}{12} cm$
4. $\sqrt{3}$
5. $\frac{-1}{\sqrt{2}}$
6. $2-\sqrt{3}$

7. $\frac{-4}{5}$

8. $45°$

9. 2 sin8θ cos4θ

10. sin 6x – sin2x

Short Answer:

1. P(n) : 7^n - 3^n is divisible by 4

For n = 1

P(1) : 7^1 - 3^1 =4 which is divisible by Thus, P(1) is true

Let P(k) be true

$7^k - 3^k$ *is* divisible by 4

$7^k - 3^k = 4\lambda$, where $\lambda \in N$ (i)

we want to prove that P (k+1) is true whenever P(k) is true

$7^{k+1} - 3^{k+1} = 7^k.7 - 3^k.3$

$= (4\lambda + 3^k).7 - 3^k.3$(from i)

$= 28\lambda + 7.3^k - 3^k.3$

$= 28\lambda + 3^k(7-3)$

$= 4(7\lambda + 3^k)$

Hence

$7^{k+1} - 3^{k+1}$ is divisible by 4

thus P (k+1) is true when P(k) is true.

Therefore by P.M.I. the statement is true for every positive integer n.

2.

$P(n) : n(n+1)(n+5)$ is multiple of 3

for n=1

P(1) : 1(1+1) (1+5) = 12 is multiple of 3

let P(k) be true

P(k) : K (k+1) (k+5) is muetiple of 3

$\Rightarrow$ k(k+1)(k+5)=3λ where $\lambda \in N$ (i)

we want to prove that result is true for n=k+1

P(k+1) : (k+1)(k+2)(k+6)

$\Rightarrow$ (K+1)(k+2)(k+6)= $\left[(k+1)(k+2)\right](k+6)$

= k(k+1)(k+2)+6(k+1)(k+2)

=k(k+1)(k+5-3)+6(k+1)(k+2)

=k(k+1)(k+5)-3k(k+1)+6(k+1)(K+2)

=k(k+1)(k+5)+(k+1)$\left[6(k+2)-3k\right]$

=k(k+1)(k+5)+(k+1)(3k+12)

=k(k+1)(k+5)+3(k+1)(k+4)

=3λ+3 (k+1)(k+4) (from i)

=$3\left[\lambda+(K+1)(K+4)\right]$ which is multiple of three

Hence P(k+1) is multiple of 3 .

3.

$P(n):10^{2n-1}+1$ is divisible by 11

for n=1

P(1) $=10^{2\times1-1}+1=11$ is divisible by 11 Hence result is true for n=1

let P(k) be true

P(k) : $10^{2k-1}+1$ is divisible by 11 1

$\Rightarrow$ $10^{2k-1}+1=11\lambda$ where $\lambda \in$ N(i)

we want to prove that result is true for n= k+

$=10^{2(k+1)-1}+1=10^{2k+2-1}+1$

$=10^{2k+1}+1$

$=10^{2k}.10^{1}+1$

$=(110\lambda-10).10+1($*from* i)

$=1100\lambda-100+1$

$=1100\lambda-99$

$=11(100\lambda-9)$*is* divisible by 11

Hence by P.M.I. P (k+1) is true whenever P(k) is true.

4.

$letP(n):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{n}\right)=(n+1)$

for n=1

P(1): $\left(1+\frac{1}{1}\right)=(1+1)=2$

which is true

let P(k) be true

$$P(k):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{k}\right)=(k+1)$$

we want to prove that P(k+1) is true

$$P(k+1):\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)..\left(1+\frac{1}{k+1}\right)=(k+2)$$

$$L.H.S.=\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)....\left(1+\frac{1}{k}\right)\left(1+\frac{1}{k+1}\right)$$

$$=(k+1)\left(1+\frac{1}{k+1}\right) \quad [from(1)]$$

$$=(k+1)\left(\frac{k+1+1}{K+1}\right)$$

$$=(K+2)$$

thus P(k+1) is true whenever

P (K) is true.

5.

$$p(n):1.2+2.3+--n(n+1)=\frac{n(n+1)(n+2)}{3}$$

for $n=1$

$$p(1):1(1+1)=\frac{1(1+1)(1+2)}{3}$$

$$p(1)=2=2$$

hence $p(1)$ be true

$$p(k):1.2+2.3+---+k(k+1)=\frac{k(k+1)(k+2)}{3}.........(i)$$

we want to prove that

$p(k+1)$:

$$1.2+2.3+---+(k+1)(k+2)=\frac{(k+1)(k+2)(k+3)}{3}$$

$L.H.S.$

$$=1.2+2.3+---+k(k+1)+(k+1)(k+2)$$

$$=\frac{k(k+1)(k+2)}{3}+\frac{(k+1)(k+2)}{1} \quad [from(i)]$$

$$\frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$$

$\frac{(k+1)(k+2)[k+3]}{3}$

hence $p(k+1)$ is true whenenes $p(k)$ is true

Long Answer:

1.

$p(n):(2n+7)<(n+3)^2$

for $n=1$

$9<(4)^2$

$9<16$

which is true

let $p(k)$ be true

$(2k+7)<(k+3)^2$

now

$2(k+1)+7=(2k+7)+2$

$<(k+3)^2+2=k^2+6k+11$

$<k^2+8k+16=(k+4)^2$

$=(k+3+1)^2$

$\therefore p(k+1):2(k+1)+7<(k+1+3)^2$

$\Rightarrow p(k+1)$ is true, when ever $p(k)$ is true

hence by PMI $p(k)$ is true for all $n\in N$

2.

$p(n):\frac{1}{1.4}+\frac{1}{4.7}+---+\frac{1}{(3n-2)(3n+1)}=\frac{n}{(3n+1)}$

for $n=1$

$p(1):\frac{1}{(3-2)(3+1)}=\frac{1}{(3+1)}=\frac{1}{4}$

which is true

let $p(k)$ be true

$p(k):\frac{1}{1.4}+\frac{1}{4.7}+---+\frac{1}{(3k-2)(3k+1)}=\frac{k}{(3k+1)}$ ....(i)

we want to prove that $p(k+1)$ is true

$p(k+1): \frac{1}{1.4}+\frac{1}{4.7}+---+\frac{1}{(3k+1)(3k+4)}=\frac{k+1}{(3k+4)}$

$L.H.S.$

$=\frac{1}{1.4}+\frac{1}{4.7}+---+\frac{1}{(3k-2)(3k+1)}+\frac{1}{(3k+1)(3k+4)}$

$=\frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)}$ [from.......(i)]

$=\frac{k(3k+4)+1}{(3k+1)(3k+4)}$

$=\frac{3k^2+4k+1}{(3k+1)(3k+4)}=\frac{(3k+1)(k+1)}{(3k+1)(3k+4)}$

$p(k+1)$ is true whenever $p(k)$ is true.

3.

$p(n): 1.2+2.2^2+3.2^3+---+n.2^n=(n-1)2^{n+1}+2$

$p(n): 1.2+2.2^2+3.2^3+---+n.2^n=(n-1)2^{n+1}+2$

for $n=1$

$p(1): 1.2^1=(1-1)2^2+2$

$2=2$ which is true

let $p(k)$ be true

$p(k): 1.2+2.2^2+---+k.2^k=(k-1)2.^{k+1}+2$..........(i)

we want to prove that $p(k+1)$ is true

$p(k+1): 1.2+2.2^2+---+(k+1)2^{k+1}=k.2^{k+2}+2$

$L.H.S.$

$1.2+2.2^2+---+k.2^k+(k+1)2^{k+1}$ [from...........(i)]

$=(k-1)2^{k+1}+2+(k+1)2^{k+1}$ c

$=2^{k+1}(k-1+k+1)+2$

$=2^{k+2}k+2$

This $p(k+1)$ is true whenever $p(k)$ is true

4. P(n) : $2.7^n + 3.5^n - 5$ is divisible by 24

for n = 1

P (1) : $2.7^1 + 3.5^1 - 5 = 24$ is divisible by 24

Hence result is true for n = 1

Let P (K) be true

P (K) : $2.7^K + 3.5^K - 5$

$\Rightarrow 2.7^K + 3.5^K - 5 = 24\lambda$ when $\lambda \in N$

we want to prove that P (K+!) is True whenever P (K) is true

$2.7^{K+1} + 3.5^{K+1} - 5 = 2.7^K.7^1 + 3.5^K.5^1 - 5$

$= 7\left[2.7^K + 3.5^K - 5 - 3.5^K + 5\right] + 3.5^K.5^1 - 5$

$= 7\left[24\lambda - 3.5^K + 5\right] + 15.5^K - 5$ (from i)

$= 7 \times 24\lambda - 21.5^K + 35 + 15.5^K - 5$

$= 7 \times 24\lambda - 6.5^K + 30$

$= 7 \times 24\lambda - 6(5^K - 5)$

$= 7 \times 24\lambda - 6.4p\ \left[\because 5^K - 5 \text{ is multiple of } 4\right]$

$= 24(7\lambda - p)$, 24 is divisible by 24

Hence by P M I p (n) is true for all n $\in$ N.

5. P (n) : $41^n - 14^n$ is a multiple of 27

for n = 1

P (1) : $41^1 - 14 = 27$, which is a multiple of 27

Let P (K) be True

P (K) : $41^K - 14^K$

$\Rightarrow 41^K - 14^K = 27\lambda$, where $\lambda \in N$

we want to prove that result is true for n = K + 1

$41^{K+1} - 14^{k+1} = 41^K.41 - 14^K.14$

$= (27\lambda + 14^K).41 - 14^K.14$ (*from* i)

$= 27\lambda.41 + 14^K.41 - 14^K.14$

$= 27\lambda.41 + 14^K(41 - 14)$

$= 27\lambda.41 + 14^K(27)$

$= 27(41\lambda + 14^K)$ is a multiple of 27

Hence by PMI p (n) is true for ace n $\in$ N.