Swotters Academy

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Test / Exam Name: Ch4 - Principle Of Mathematical Induction | **Standard: 11th Science** | **Subject: Mathematics**

Student Name: .... **Section:** .... **Roll No.:** ....

Questions: 26	Time: 01:00 hh:mm	Marks: 50

Instructions

1. Rough work at the last page should be in proper manner too
2. New section on new page
3. Honesty is the best policy.

SECTION-A

Q1. If p(n): 2n < (1 × 2 × 3 × ... × n). Then the smallest positive integer for which p(n) is true is: **1 Mark**

A 1 **B** 2 **C** 3 **D** 4

Q2. If $10^n + 3 \times 4^{n+2} + \lambda$ is divisible by 9 for all $n \in N$, then the least positive integer value of λ is **1 Mark**

A 5 **B** 3 **C** 7 **D** 1

Q3. If p(n): $49^n + 16^n\lambda$ is divisible by 64 for $n \in N$ is true, then the least negative integral value of λ is: **1 Mark**

A -3 **B** -2 **C** -1 **D** -4

Q4. For all $n \in N$, $3 \times 5^{2n+1} + 2^{3n+1}$ is divisible by: **1 Mark**

A 19 **B** 17 **C** 23 **D** 25

Q5. Choose the correct answer. **1 Mark**
For all n ∈ N, $3 \times 5^{2n+1} + 2^{3n+1}$ is divisible by:

A 19 **B** 17 **C** 23 **D** 25

Q6. Choose the correct answer. **1 Mark**
If x^n - 1 is divisible by x - k, then the least positive integral value of k is:

A 1 **B** 2 **C** 3 **D** 4

Q7. If $i^2 = -1$, then the sum $i + i^2 + i^3 + ...$ upto 1000 terms is equal to: **1 Mark**

A 1 **B** -1 **C** i **D** 0

Q8. Fill in the blanks: **1 Mark**
If P(n)P: 2n < n!, n ∈ N, then P(n) is true for all $n \geq$ ________.

Q9. State whether the following statement is true or false. Justify: **1 Mark**
Let P(n) be a statement and let P(k) ⇒ P(k + 1), for some natural number k, then P(n) is true for all n ∈ N.

Q10. Given an example of a statement P(n) such that it is true for all $n \in N$. **1 Mark**

Q11. If P(n) is the statement "n(n + 1) is even", then what is P(3)? **1 Mark**

Q12. State the first principle of mathematical induction. **1 Mark**

Q13. If p(n): $2 \times 4^{2n-1} + 3^{3n+1}$ is divisible by λ for all $n \in N$ is true, then find the value of λ **1 Mark**

Q14. Write the set of value if n for which the statement p(n): 2n < n! is true. **1 Mark**

Q15. State the second principle of mathematical induction. **1 Mark**

Q16. If P(n) is the statement "n^3 + n is divisible by 3", prove that P(3) is true but P(4) is not true. **2 Marks**

Q17. Prove the following statement by principle of mathematical induction: **2 Marks**
For any natural number n, $7^n - 2^n$ is divisible by 5.

Q18. If P(n) is the statement "$2^n \geq 3n$" and if P(r) is true, prove that P(r + 1) is true. **2 Marks**

Q19. If P(n) is the statement "n^2 + n is even", and if P(r) is true, then P(r + 1) is true. **2 Marks**

SECTION-B

Q20. If P(n) is the statement "n^2 - n + 41 is prime", prove that P(1), P(2) and P(3) are true. Prove also that P(41) is not true. **3 Marks**

Q21. Give an example of a statement P(n) which is true for all $n \geq 4$ but P(1), P(2) and P(3) are not true. Justify your answer. **3 Marks**

SECTION-C

Q22. Prove the following by the principle of mathematical induction: **4 Marks**
$1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$

Q23. Prove the following by the principle of mathematical induction: **4 Marks**
$1.3 + 2.4 + 3.5 + ... + n(n+2)$
$= \frac{1}{6}n(n+1)(2n+7)$

Q24. A sequence a_1, a_2, a_3, ... is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$ for all natural numbers $k \geq 2$. Show that $a_n = 3.7^{n-1}$ for all $n \in N$. **4 Marks**

Q25. Prove the following by the principle of mathematical induction: **4 Marks**
$2 + 5 + 8 + 11 + ... + (3n-1) = \frac{1}{2}n(3n+1)$

Q26. Prove that $\underbrace{7 + 77 + 777 + ... + 777\ldots7}_{\text{n-digits}} = \frac{7}{81}(10^{n+1} - 9n - 10)$ for all $n \in N$. **5 Marks**