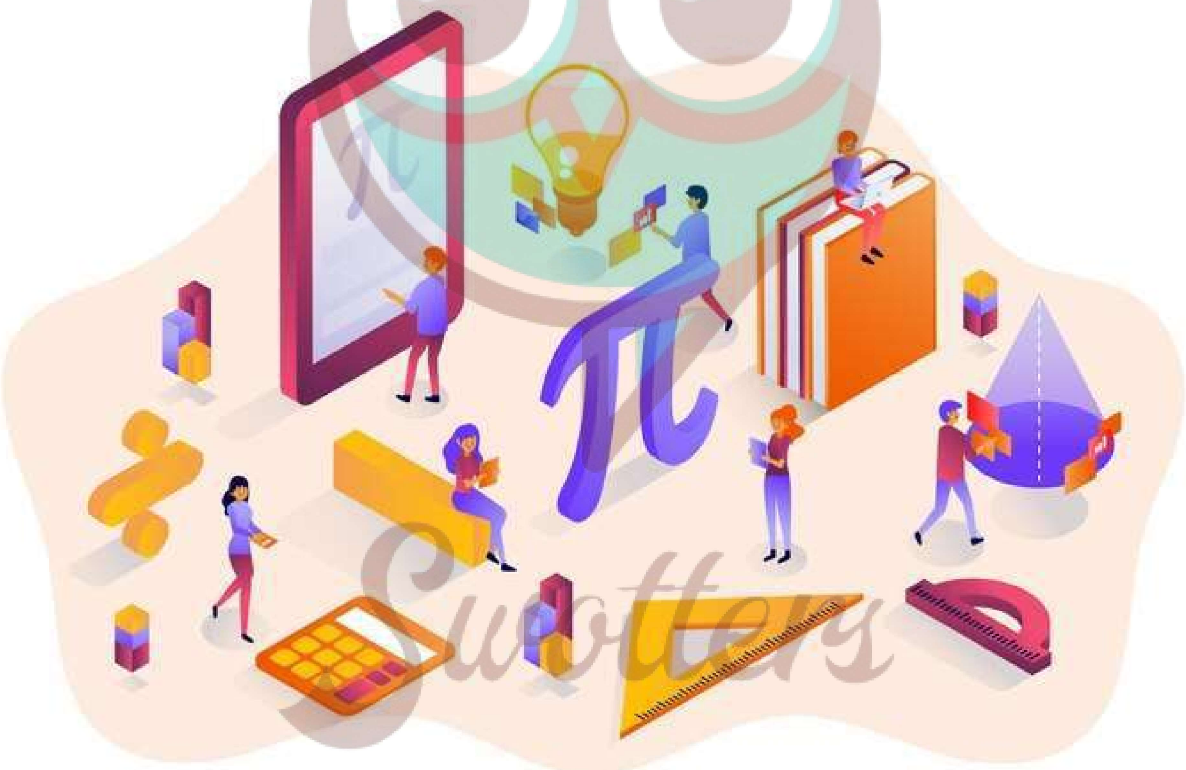
MATHEMATICS



Important Questions

Multiple Choice questions-

1. Which of the following is not a quadratic equation

(a) $x^2 + 3x – 5 = 0$

(b) $x^2 + x3 + 2 = 0$

(c) $3 + x + x^2 = 0$

(d) $x^2 – 9 = 0$

2. The quadratic equation has degree

(a) 0

(b) 1

(c) 2

(d) 3

3. The cubic equation has degree

(a) 1

(b) 2

(c) 3

(d) 4

4. A bi-quadratic equation has degree

(a) 1

(b) 2

(c) 3

(d) 4

5. The polynomial equation $x (x + 1) + 8 = (x + 2) \{x – 2)$ is

(a) linear equation

(b) quadratic equation

(c) cubic equation

(d) bi-quadratic equation

6. The equation $(x - 2)^2 + 1 = 2x - 3$ is a

(a) linear equation

(b) quadratic equation

(c) cubic equation

(d) bi-quadratic equation

7. The quadratic equation whose roots are 1 and

(a) $2x^2 + x - 1 = 0$

(b) $2x^2 - x - 1 = 0$

(c) $2x^2 + x + 1 = 0$

(d) $2x^2 - x + 1 = 0$

8. The quadratic equation whose one rational root is $3 + \sqrt{2}$ is

(a) $x^2 - 7x + 5 = 0$

(b) $x^2 + 7x + 6 = 0$

(c) $x^2 - 7x + 6 = 0$

(d) $x^2 - 6x + 7 = 0$

9. The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is

(a) $\pm\sqrt{6}$

(b) ± 4

(c) $\pm 3\sqrt{2}$

(d) $\pm 2\sqrt{6}$

10. The sum of the roots of the quadratic equation $3×2 - 9x + 5 = 0$ is

(a) 3

(b) 6

(c) -3

(d) 2

Very Short Questions:

1. What will be the nature of roots of quadratic equation $2x^2 + 4x – n = 0$?

2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx – 54 = 0$, then find the value of k.

3. If $ax^2 + bx + c = 0$ has equal roots, find the value of c.

4. If a and b are the roots of the equation $x^2 + ax – b = 0$, then find a and b.

5. Show that $x = – 2$ is a solution of $3x^2 + 13x + 14 = 0$.

6. Find the discriminant of the quadratic equation $4\sqrt{2}x^2 + 8x + 2\sqrt{2} = 0$).

7. State whether the equation $(x + 1)(x – 2) + x = 0$ has two distinct real roots or not. Justify your answer.

8. Is 0.3 a root of the equation $x^2 – 0.9 = 0$? Justify.

9. For what value of k, is 3 a root of the equation $2x^2 + x + k = 0$?

10. Find the values of k for which the quadratic equation $9x^2 – 3kx + k = 0$ has equal roots.

Short Questions :

1. Find the roots of the following quadratic equations by factorisation:

(i) $\sqrt{2}x2 + 7x + 5\sqrt{2} = 0$ (ii) $2x2 – x + \frac{1}{8} = 0$

2. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 + x – 4 = 0$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

3. Find the roots of the following quadratic equations by applying the quadratic formula.

(i) $2x^2 - 7x + 3 = 0$

(ii) $4x^2 + 4\sqrt{3}x + 3 = 0$

4. Using quadratic formula solve the following quadratic equation:

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

5. Find the roots of the following equation:

$$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20};\quad x \neq -3, 6$$

6. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $3x^2 - 4\sqrt{3}x + 4 = 0$) (ii) $2x^2 - 6x + 3 = 0$

7. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

8. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

9. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$.

10. If $\sin\theta$ and $\cos\theta$ are roots of the equation $ax^2 + bx + c = 0$, prove that $a^2 - b^2 + 2ac = 0$.

Long Questions :

1. Using quadratic formula, solve the following equation for x:

$abx^2 + (b^2 - ac)x - bc = 0$

2. Find the value of p for which the quadratic equation

$(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

3. Solve for

$$x: \frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3}; x \neq 5, 7$$

4. The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is Find his present age.

5. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

6. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

7. The sum of two numbers is 15 and the sum of their reciprocals is 3. Find the numbers.

8. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.

9. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

10. The sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Case Study Question:

1. If p(x) is a quadratic polynomial i.e., p(x) = $ax^2 + bx + c$, $a\neq0a\neq0$ then p(x) = 0 is called a quadratic equation. Now, answer the following questions.

   i. Which of the following is correct about the quadratic equation $ax^2 + bx + c = 0$?

      a. a, b and c are real numbers $c \neq 0$
      b. a, b and c are rational numbers, $a \neq 0$
      c. a, b and c are integers, a, band $c \neq 0$
      d. a, b and c are real numbers $a \neq 0$

   ii. The degree of a quadratic equation is:

      a. 1
      b. 2
      c. 3
      d. Other than 1

   iii. Which of the following is a quadratic equation?

      a. $x(x + 3) + 7 = 5x - 11$
      b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$
      c. $x^2(2x + 1) - 4 = 5x^2 - 10$
      d. $x(x - 1)(x + 7) = x(6x - 9)$

iv. Which of the following is incorrect about the quadratic equation $ax^2 + bx + c = 0$?

a. If $a\alpha^2 + b\alpha + c = 0$ then $x = -\alpha$ is the solution of the given quadratic equation.
b. The additive inverse of zeroes of the polynomial $ax^2 + bx + c$ is the roots of the given equati
c. If α is a root of the given quadratic equation, then its other root is $-\alpha$
d. All of these.

v. Which of the following is not a method of finding solutions of the given quadratic equation:

a. Factorisation method
b. Completing the square method
c. Formula method
d. None of these

2. Quadratic equations started around 3000 B.C. with the Babylonians. They were one of the world's first civilisation, and came up with some great ideas like agriculture, irrigation and writing. There were many reasons why Babylonians needed to solve quadratic equations. for example to know what amount of crop you can grow on the square field. Based on the above information, represent the following questions in th form of quadratic equation.

i. The sum of squares of two consecutive integers is 650.

a. $x^2 + 2x - 650 = 0$
b. $2x^2 + 2x - 649 = 0$
c. $x^2 - 2x - 650 = 0$
d. $2x^2 + 6x - 550 = 0$

ii. The sum of two numbers is 15 and the sum of their reciprocals is 310310.

a. $x^2 + 10x - 150 = 0$
b. $15x^2 - x + 150 = 0$
c. $x^2 - 15x + 50 = 0$
d. $3x^2 - 10x + 15 = 0$

iii. Two numbers differ by 3 and their product is 504.

a. $3x^2 - 504 = 0$
b. $x^2 - 504x + 3 = 0$
c. $504x^2 + 3 = x$
d. $x^2 + 3x - 504 = 0$

iv. A natural number whose square diminished by 84 is thrice of 8 more of given number.

a. $x^2 + 8x - 84 = 0$
b. $3x^2 - 84x + 3 = 0$
c. $x^2 - 3x - 108 = 0$
d. $x^2 - 11x + 60 = 0$

v. A natural number when increased by 12, equals 160 times its reciprocal.

a. $x^2 - 12x + 160 = 0$
b. $x^2 - 160x + 12 = 0$
c. $12x^2 - x - 160 = 0$
d. $x^2 + 12x - 160 = 0$

Assertion Reason Questions-

1. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: The product of two successive positive integral multiples of 5 is 300, then the two numbers are 15 and 20.

Reason: The product of two consecutive integrals is a multiple of 2.

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both A and R are true and R is the correct explanation for A.

b. Both A and R are true and R is the correct explanation for A.

c. A is true but R is false.

d. A is false but R is true.

Assertion: The roots of the quadratic equation x2 + 2x + 2 = 0 are imaginary.

Reason: If discriminant D = b2 – 4ac < 0 then the roots of the quadratic equation ax2 + bx + c = 0 are imaginary.

Answer Key-

Multiple Choice questions-

1. (b) -10
2. (b) $x^2 + x3 + 2 = 0$
3. (c) 2
4. (c) 3
5. (d) 4
6. (a) linear equation
7. (b) quadratic equation
8. (b) $2x^2 – x – 1 = 0$
9. (d) $x^2 – 6x + 7 = 0$
10. (d) $\pm 2\sqrt{6}$
11. (c) -3

Very Short Answer :

1. $D = b^2 – 4ac$

$\Rightarrow 42 – 4 \times 2 (-7)$

$\Rightarrow 16 + 56 = 72 > 0$

Hence, roots of quadratic equation are real and unequal.

2. $\therefore \frac{1}{2}$ is a root of quadratic equation.

$\therefore$ It must satisfy the quadratic equation.

$$x^2 + kx - \frac{5}{4} = 0$$

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \quad \Rightarrow \quad \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0 \quad \Rightarrow \quad 2k - 4 = 0$$

$$\Rightarrow \quad k = 2$$

3. For equal roots D = 0

i.e., b2 – 4ac = 0

⇒ b2 = 4 ac

⇒ $c = \frac{b^2}{4a}$

4. Sum of the roots = a + b = $-\frac{B}{A}$ = – a

Product of the roots = ab = $\frac{B}{A}$ = – b

= a + b = – a and ab = -b

⇒ 2a = -b and a = -1

⇒ b = 2 and a = -1

5. Put the value of x in the quadratic equation,

⇒ LHS = $3x^2 + 13x + 14$

⇒ $3(-2)^2 + 13(-2) + 14$

⇒ 12 – 26 + 14 = 0

⇒ RHS Hence, x = -2 is a solution.

6. D = 62 – 4ac = $(8)^2 - 4(4\sqrt{2})(2\sqrt{2})$

⇒ 64 – 64 = 0

7. (x + 1)(x – 2) + x = 0

⇒ $x^2 - x - 2 + x = 0$

⇒ $x^2 - 2 = 0$

D = $b^2 - 4ac$

⇒ (-4(1)(-2) = 8 > 0

∴ Given equation has two distinct real roots.

8. ∵ 0.3 is a root of the equation $x^2 - 0.9 = 0$

∴ $x^2 - 0.9 = (0.3)^2 - 0.9 = 0.09 - 0.9 \neq 0$

Hence, 0.3 is not a root of given equation.

9. 3 is a root of $2x^2 + x + k = 0$, when

$\Rightarrow 2(3)^2 + 3 + k = 0$

$\Rightarrow 18 + 3 + k = 0$

$\Rightarrow k = -21$

10. For equal roots:

$D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-3k)^2 - 4 \times 9 \times k = 0$

$\Rightarrow 9k^2 = 36k$

$\Rightarrow k = 4$

Short Answer :

1. (i) We have, $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$= \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$= (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\therefore$ Either $\sqrt{2}x + 5 = 0$ or $x + \sqrt{2} = 0$

$\therefore x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Hence, the roots are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(ii) We have, $2x^2 - x + 18 = 0$

$\Rightarrow \quad \frac{16x^2 - 8x + 1}{8} = 0 \quad \Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0 \quad \Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow \quad (4x - 1)(4x - 1) = 0$

So, either $4x - 1 = 0$ or $4x - 1 = 0$

$$x = \frac{1}{4} \quad \text{or} \quad x = \frac{1}{4}$$

Hence, the roots of the given equation are $\frac{1}{4}$ and $\frac{1}{4}$.

2. (i) We have, $2x^2 + x - 4 = 0$

On dividing both sides by 2, we have

$$x^2 + \frac{x}{2} - 2 = 0$$

$$\Rightarrow \quad x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0 \qquad \left[b = \frac{1}{2}(\text{coefficient of } x) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\right]$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0 \quad \Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2 = \frac{1 + 32}{16} = \frac{33}{16} > 0$$

$\Rightarrow$ Roots exist.

$$\therefore \quad x + \frac{1}{4} = \pm\sqrt{\frac{33}{16}} \quad \Rightarrow \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow \quad x + \frac{1}{4} = \frac{\sqrt{33}}{4} \quad \text{or} \quad x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$$

$$\therefore \quad x = -\frac{1}{4} + \frac{\sqrt{33}}{4} \quad \text{or} \quad x = -\frac{1}{4} - \frac{\sqrt{33}}{4}$$

$$\Rightarrow \quad x = \frac{\sqrt{33} - 1}{4} \quad \text{or} \quad x = \frac{-(\sqrt{33} + 1)}{4}$$

Hence, roots of given equation are $\frac{\sqrt{33} - 1}{4}$ and $\frac{-(\sqrt{33} + 1)}{4}$.

(ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

On dividing both sides by 4, we have

$$x^2 + \sqrt{3}x + \frac{3}{4} = 0 \quad \Rightarrow \quad x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0 \quad \Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \qquad \ldots(i)$$

$$\Rightarrow \quad \text{Roots exist. } \therefore (i) \quad \Rightarrow \quad x = -\frac{\sqrt{3}}{2}, \; -\frac{\sqrt{3}}{2}$$

Hence, roots of given equation are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

3. (i) We have, $2x^2 - 7x + 3 = 0$

Here, a = 2, b = -7 and c = 3

Therefore, D = b+ – 4ac

$\Rightarrow D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$

$\because D > 0, \therefore$ roots exist.

Thus, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-(-7) \pm \sqrt{25}}{2 \times 2} = \dfrac{7 \pm 5}{4}$

$x = \dfrac{7+5}{4}$ or $\dfrac{7-5}{4}$

$= 3$ or $\dfrac{1}{2}$

So, the roots of given equation are 3 and $\frac{1}{2}$.

(ii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

Here, a = 4, b = $4\sqrt{3}$ and c = 3

Therefore, $D = b^2 - 4ac = (4\sqrt{3})2 - 4 \times 4 \times 3 = 48 - 48 = 0$

$\therefore D = 0$, roots exist and are equal.

Thus, $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-4\sqrt{3} \pm 0}{2 \times 4} = \dfrac{-\sqrt{3}}{2}$

Hence, the roots of given equation are $\dfrac{-\sqrt{3}}{2}$ and $\dfrac{-\sqrt{3}}{2}$.

4. We have, $p^2x^2 + (p^2 - q^2)\,x - q^2 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = p^2$, $b = p^2 - q^2$ and $c = -q^2$

$\therefore D = b^2 - 4ac$

$\Rightarrow (p^2 - q)^2 - 4 \times p^2 \times (-q^2)$

$\Rightarrow (p^2 - q^2)^2 + 4p^2q^2$

$\Rightarrow (p^2 + q^3)2 > 0$

So, the given equation has real roots given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(p^2 - q^2) + (p^2 + q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(p^2 - q^2) - (p^2 + q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

5.

Given, $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}$; $x \neq -3, 6$

$\Rightarrow \quad \frac{(x-6)-(x+3)}{(x+3)(x-6)} = \frac{9}{20} \quad \Rightarrow \quad \frac{-9}{(x+3)(x-6)} = \frac{9}{20}$

$\Rightarrow (x + 3)(x – 6)$

$\Rightarrow -20$ or $x^2 – 3x + 2 = 0$

$\Rightarrow x^2 – 2x -x + 2 = 0$

$\Rightarrow x(x – 2) -1(x – 2) = 0)$

$\Rightarrow (x – 1)(x – 2) = 0$

$\Rightarrow x = 1$ or $x = 2$

Both x = 1 and x = 2 are satisfying the given equation. Hence, x = 1, 2 are the solutions of the equation.

6. (i) We have, $3x^2 – 4\sqrt{3}x + 4 = 1$

Here, a = 3, b = $– 4\sqrt{3}$ and c = 4

Therefore,

$D = b^2 – 4ac$

$\Rightarrow (- 4\sqrt{3})^2 – 4 \times 3 \times 4$

$\Rightarrow 48 – 48 = 0$

Hence, the given quadratic equation has real and equal roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\sqrt{3}) \pm \sqrt{0}}{2 \times 3} = \frac{2\sqrt{3}}{3}$

Hence, equal roots of given equation are $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$.

(ii) We have, $2x^2 - 6x + 3 = 0$

Here, a = 2, b = -6, c = 3

Therefore, $D = b^2 - 4ac$

= (-6)2 4 × 2 × 3 = 36 – 24 = 12 > 0

Hence, given quadratic equation has real and distinct roots.

Thus, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence, roots of given equation are $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$

7. (i) We have, $2x^2 + kx + 3 = 0$

Here, a = 2, b = k, c = 3

$D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$ For equal roots

D = 0

i.e., $k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm \sqrt{24}$

$\Rightarrow k = + 2\sqrt{6}$

(ii) We have, kx(x – 2) + 6 = 0

⇒ kx2 – 2kx + 6 = 0

Here, a = k, b = – 2k, c = 6

For equal roots, we have

D = 0

i.e., b2 – 4ac = 0

⇒ (-2k)2 – 4 × k × 6 = 0

⇒ 4k2 – 24k = 0

⇒ 4k (k – 6) = 0

Either 4k = 0 or k – 6 = 0

⇒ k = 0 or k = 6

But k = 0 6) because if k = 0 then given equation will not be a quadratic equation).

So, k = 6.

8. Since the equation $(a – b)x^2 + (b – c) x + (c – a) = 0$ has equal roots, therefore discriminant

$D = (b – c)^2 – 4(a – b) (c – a) = 0$

$\Rightarrow b^2 + c^2 – 2bc – 4(ac – a^2 – bc + ab)$

$\Rightarrow b^2 + c^2 – 2bc – 4ac + 4a^2 + 4bc – 4ab = 0$

$\Rightarrow 4a^2 + b^2 + c^2 – 4ab + 2bc – 4ac = 0$

$\Rightarrow (2a)^2 + (- b)^2 + (-c)^2 + 2(2a) (-b) + 2(-b) (-c) + 2(-c) 2a = 0$

$\Rightarrow (2a – b – c)^2 = 0$

$\Rightarrow 2a – b – c = 0$

$\Rightarrow 2a = b + c$. Hence Proved.

9. The given equation is $(1 + m^2) x^2 + (2mc) x + (c^2 – a^2) = 0$

Here, $A = 1 + m^2$, $B = 2mc$ and $C = c^2 – a^2$

Since the given equation has equal roots, therefore $D = 0 = B^2 – 4AC = 0$.

$\Rightarrow (2mc)^2 – 4(1 + m^2) (c^2 – a^2) = 0$

$\Rightarrow 4m^2c^2 – 4(c^2 – a^2 + m^2c^2 – m^2a^2) = 0$

$\Rightarrow m^2c^2 – c^2 + a^2 – m^2c^2 + m^2a^2 = 0$. [Dividing throughout by 4]

$\Rightarrow - c^2 + a^2 (1 + m^2) = 0$

$\Rightarrow c^2 = a(1 + m^2)$ Hence Proved

10.

Sum of the roots $= \frac{-B}{A} \quad \Rightarrow \quad \sin\theta + \cos\theta = \frac{-b}{a}$ ...(i)

Product of the roots $= \frac{C}{A} \quad \Rightarrow \quad \sin\theta \cdot \cos\theta = \frac{c}{a}$ ...(ii)

Now, we have, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1 \quad \Rightarrow \quad \left(\frac{-b}{a}\right)^2 - 2.\frac{c}{a} = 1$

$\Rightarrow \quad \frac{b^2}{a^2} - \frac{2c}{a} = 1 \quad$ or $\quad b^2 - 2ac = a^2$

$\Rightarrow \quad a^2 - b^2 + 2ac = 0$

Long Answer :

1. We have, $abx^2 + (b^2 - ac)x - bc = 0$

Here, $A = ab$, $B = b^2 - ac$, $C = - bc$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 - 4(ab)(-bc)}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 - ac)^2 + 4ab^2c}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^4 - 2ab^2c + a^2c^2 + 4ab^2c}}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2ab} \quad \Rightarrow \quad x = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$\Rightarrow \quad x = \frac{-(b^2 - ac) + (b^2 + ac)}{2ab} \quad \text{or} \quad x = \frac{-(b^2 - ac) - (b^2 + ac)}{2ab}$$

$$x = \frac{2ac}{2ab} \quad \text{or} \quad x = \frac{-2b^2}{2ab} \quad \Rightarrow \quad x = \frac{c}{b} \quad \text{or} \quad x = \frac{-b}{a}$$

2. Since the quadratic equation has equal roots, D = 0 i.e., $b^2 - 4ac = 0$

In $(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$

Here, $a = (2p + 1)$, $b = -(7p + 2)$, $c = (7p - 3)$

For $p = -\frac{4}{7}$

$$\left(2 \times \left(\frac{-4}{7}\right) + 1\right)x^2 - \left(7 \times \left(\frac{-4}{7}\right) + 2\right)x + \left(7 \times \left(\frac{-4}{7}\right) - 3\right) = 0$$

$\Rightarrow \frac{-1}{7}x^2 + 2x - 7 = 0 \quad \Rightarrow x^2 - 14x + 49 = 0$

$\Rightarrow x^2 - 7x - 7x + 49 = 0 \quad \Rightarrow x(x - 7) - 7(x - 7) = 0$

$\Rightarrow (x - 7)^2 = 0 \quad \Rightarrow x = 7, 7$

For $p = 4$,

$(2 \times 4 + 1)x^2 - (7 \times 4 + 2)x + (7 \times 4 - 3) = 0$

$\Rightarrow 9x^2 - 30x + 25 = 0 \quad \Rightarrow 9x^2 - 15x - 15x + 25 = 0$

$\Rightarrow 3x(3x - 5) - 5(3x - 5) = 0 \quad \Rightarrow (3x - 5)(3x - 5) = 0$

$\Rightarrow x = \frac{5}{3}, \frac{5}{3}$

3.

$\frac{x-4}{x-5} + \frac{x-6}{x-7} = \frac{10}{3} \quad \Rightarrow \frac{(x-4)(x-7) + (x-6)(x-5)}{(x-5)(x-7)} = \frac{10}{3}$

$\Rightarrow \frac{x^2 - 7x - 4x + 28 + x^2 - 5x - 6x + 30}{x^2 - 7x - 5x + 35} = \frac{10}{3}$

$\Rightarrow \frac{2x^2 - 22x + 58}{x^2 - 12x + 35} = \frac{10}{3} \quad \Rightarrow \frac{x^2 - 11x + 29}{x^2 - 12x + 35} = \frac{5}{3}$

$\Rightarrow 3x^2 - 33x + 87 = 5x^2 - 60x + 175 \quad \Rightarrow 2x^2 - 27x + 88 = 0$

$\Rightarrow 2x^2 - 16x - 11x + 88 = 0 \quad \Rightarrow 2x(x - 8) - 11(x - 8) = 0$

$\Rightarrow (2x - 11)(x - 8) = 0 \quad \Rightarrow 2x - 11 = 0$ or $x - 8 = 0$

$\Rightarrow x = \frac{11}{2}$ or $x = 8$

4. Let the present age of Rehman be x years.

So, 3 years ago, Rehman's age = $(x - 3)$ years

And 5 years from now, Rehman's age = $(x + 5)$ years

Now, according to question, we have

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$ $\quad\Rightarrow \frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$

$\Rightarrow 6x+6=(x-3)(x+5)$ $\quad\Rightarrow 6x+6=x^2+5x-3x-15$

$\Rightarrow x^2+2x-15-6x-6=0$ $\quad\Rightarrow x^2-4x-21=0$

$\Rightarrow x^2-7x+3x-21=0$ $\quad\Rightarrow x(x-7)+3(x-7)=0$

$\Rightarrow (x-7)(x+3)=0$ $\quad\Rightarrow x=7$ or $x=-3$

But x ≠ -3 (age cannot be negative)

Therefore, present age of Rehman = 7 years.

5. Let the two natural numbers be x and y such that x > y.

According to the question,

Difference of numbers, x – y = 5 ⇒ x = 5 + y …..(i)

Difference of the reciprocals,

$$\frac{1}{y}-\frac{1}{x}=\frac{1}{10} \quad \ldots(ii)$$

Putting the value of (*i*) in (*ii*)

$\frac{1}{y}-\frac{1}{5+y}=\frac{1}{10}$ $\quad\Rightarrow \frac{5+y-y}{y(5+y)}=\frac{1}{10}$

$\Rightarrow 50=5y+y^2$ $\quad\Rightarrow y^2+5y-50=0$

$\Rightarrow y^2+10y-5y-50=0$ $\quad\Rightarrow y(y+10)-5(y+10)=0$

$\Rightarrow (y-5)(y+10)=0$

∴ y is a natural number.

∵ y = 5

Putting the value of y in (i), we have

⇒ x = 5 + 5

⇒ x = 10

The required numbers are 10 and 5.

6. Let the two consecutive odd numbers be x and x + 2.

$\Rightarrow \quad x^2 + (x+2)^2 = 394 \qquad \Rightarrow \quad x^2 + x^2 + 4 + 4x = 394$

$\Rightarrow \quad 2x^2 + 4x + 4 = 394 \qquad \Rightarrow \quad 2x^2 + 4x - 390 = 0$

$\Rightarrow \quad x^2 + 2x - 195 = 0 \qquad \Rightarrow \quad x^2 + 15x - 13x - 195 = 0$

$\Rightarrow \quad x(x+15) - 13(x+15) = 0 \qquad \Rightarrow \quad (x-13)(x+15) = 0$

$\Rightarrow \quad x - 13 = 0 \text{ or } x + 15 = 0 \qquad \Rightarrow \quad x = 13 \text{ or } x = -15$

Hence, the numbers are 13 and 15 or -15 and -13.

7. Let the numbers be x and 15 – x.

According to given condition,

$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10} \qquad \Rightarrow \quad \frac{15-x+x}{x(15-x)} = \frac{3}{10}$

$\Rightarrow 150 = 3x(15 - x)$

$\Rightarrow 50 = 15x - x^2$

$\Rightarrow x^2 - 15x + 50 = 0$

$\Rightarrow x^2 - 5x - 10x + 50 = 0$

$\Rightarrow x(x - 5) - 10(x - 5) = 0$

$\Rightarrow (x - 5)(x - 10) = 0$

$\Rightarrow x = 5 \text{ or } 10.$

When x = 5, then 15 – x = 15 – 5 = 10

When x = 10, then 15 – x = 15 – 10 = 5

Hence, the two numbers are 5 and 10.

8. Let Shefali's marks in Mathematics be x.

Therefore, Shefali's marks in English is (30 – x).

Now, according to question,

$\Rightarrow (x + 2)(30 - x - 3) = 210$

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow 25x - x^2 + 54 - 210 = 0$

$\Rightarrow 25x - x^2 - 156 = 0$

$\Rightarrow -(x^2 - 25x + 156) = 0$

$\Rightarrow x^2 - 25x + 156 = 0$

$= x^2 - 13x - 12x + 156 = 0$

$\Rightarrow x(x - 13) - 12(x - 13) = 0$

$\Rightarrow (x - 13)(x - 12) = 0$

Either $x - 13$ or $x - 12 = 0$

$\therefore x = 13$ or $x = 12$

Therefore, Shefali's marks in Mathematics = 13

Marks in English = 30 – 13 = 17

or Shefali's marks in Mathematics = 12

marks in English = 30 – 12 = 18.

9. Let the uniform speed of the train be x km/h.

Then, time taken to cover 360 km $= \frac{360}{x}$ h

Now, new increased speed = $(x + 5)$ km/h

So, time taken to cover 360 km $= \frac{360}{x+5}$ h

According to question, $\frac{360}{x} - \frac{360}{x+5} = 1$

$\Rightarrow \quad 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1 \qquad \Rightarrow \quad \frac{360(x+5-x)}{x(x+5)} = 1$

$\Rightarrow \quad \frac{360 \times 5}{x(x+5)} = 1 \qquad \Rightarrow \quad 1800 = x^2 + 5x$

$\therefore \quad x^2 + 5x - 1800 = 0 \qquad \Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0 \qquad \Rightarrow \quad (x + 45)(x - 40) = 0$

Either $x + 45 = 0$ or $x - 40 = 0$

$\therefore \quad x = -45$ or $x = 40$

But x cannot be negative, so $x \neq -45$

therefore, $x = 40$

Hence, the uniform speed of train is 40 km/h

10. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 468$ ...(i)

Let x be the length of the side of the bigger square.

$4x - 4y = 24$

$\Rightarrow x - y = 6$ or $x = y + 6$ ...(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow y^2 + 12y + 36 + y^2 = 468$ or $232 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

Either $y + 18 = 0$ or $y - 12 = 0$

$\Rightarrow y = -18$ or $y = 12$

But, sides cannot be negative, so $y = 12$

Therefore, $x = 12 + 6 = 18$

Hence, sides of two squares are 18 m and 12 m.

Case Study Answers:

1. Answer :

i. (d) a, b and c are real numbers $a \neq 0$
ii. (b) 2
iii. (a) $x(x + 3) + 7 = 5x - 11$

Solution:

a. $x(x + 3) + 7 = 5x - 11$

$\Rightarrow x^2 + 3x + 7 = 5x - 11$

$\Rightarrow x^2 - 2x + 18 = 0$ is a quadratic equation.

b. $(x - 1)^2 - 9 = (x - 4)(x + 3)$

$\Rightarrow x^2 - 2x - 8 = x^2 - x - 12$

$\Rightarrow x - 4 = 0$ is not a quadratic equation.

c. $x^2(2x + 1) - 4 = 5x^2 - 10$

$\Rightarrow 2x^3 + x^2 - 4 = 5x^2 - 10$

$\Rightarrow 2x^3 - 4x^2 + 6 = 0$ is not a quadratic equation.

d. $x(x - 1)(x + 7) = x(6x - 9)$

$x^3 + 6x^2 - 7x = 6x^2 - 9x$

$x^3 + 2x = 0$ is not a quadratic equation.

iv. (d) All of these.

v. (d) None of these

2. Answer :

i. (b) $2x^2 + 2x - 649 = 0$

Solution:

Let two consecutive integers be x, x + 1. Given, $x^2 + (x + 1)^2 = 650$.

$\Rightarrow 2x^2 + 2x + 1 - 650 = 0$

$\Rightarrow 2x^2 + 2x - 649 = 0$

ii. (c) $x^2 - 15x + 50 = 0$

Solution:

Let the two numbers be x and 15 - x. Given 1x+115−x=310

$\Rightarrow$ 10(15 - x + x) = 3x(15 - x)

$\Rightarrow$ 50 = 15x - x^2

$\Rightarrow$ x^2 -15x + 50 = 0

iii. (d) x^2 + 3x - 504 = 0

Solution:

Let the numbers be x and x + 3. Given, x(x + 3) = 504

$\Rightarrow$ x^2 + 3x - 504 = 0

iv. (c) x^2 - 3x - 108 = 0

Solution:

Let the number be x. According to question, x^2 - 84 = 3(x + 8)

$\Rightarrow$ x^2 - 84 = 3x + 24

$\Rightarrow$ x^2 - 3x - 108 = 0

v. (d) x^2 + 12x - 160 = 0

Solution:

Let the number be x .Accoding to question x+12=160xx+12=160x

$\Rightarrow$ x^2 + 12x - 160 = 0

Assertion Reason Answer-

1. (b) Both A and R are true and R is the correct explanation for A.
2. (c) A is true but R is false.