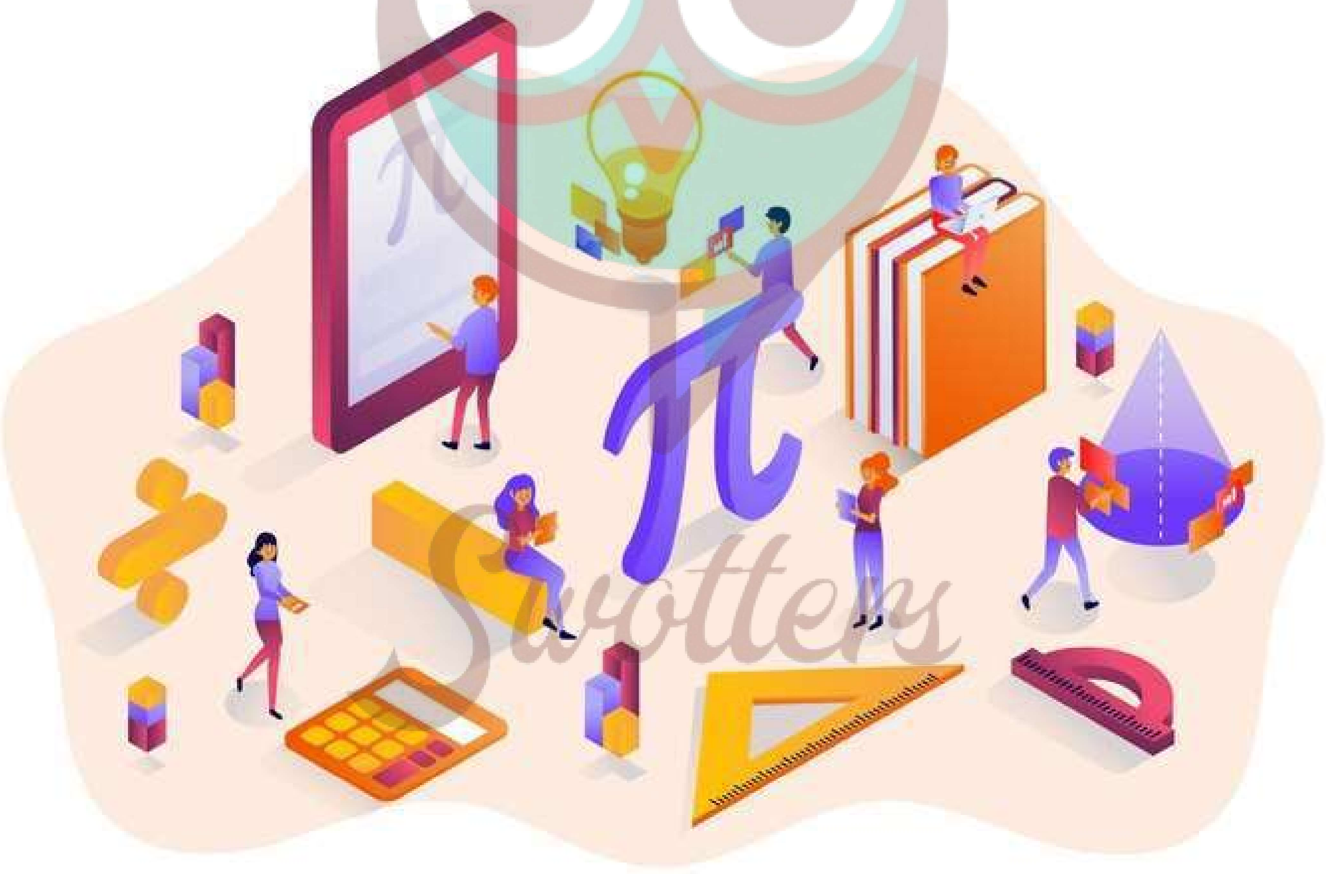
MATHEMATICS

Chapter 5: COMPLEX NUMBERS & QUADRATIC EQUATIONS



Important Questions

Multiple Choice questions-

Question 1. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further assume that the origin, z_1 and z_2 form an equilateral triangle. Then

(a) $a^2 = b$

(b) $a^2 = 2b$

(c) $a^2 = 3b$

(d) $a^2 = 4b$

Question 2. The value of i^i is

(a) 0

(b) $e^{-\pi}$

(c) $2e^{-\pi/2}$

(d) $e^{-\pi/2}$

Question 3. The value of $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)}$ is

(a) 13 i

(b) -13 i

(c) 17 i

(d) -17 i

So, $\sqrt{(-25)} + 3\sqrt{(-4)} + 2\sqrt{(-9)} = 17i$

Question 4. If the cube roots of unity are 1, ω and ω², then the value of $(1 + \omega / \omega^2)^3$ is

(a) 1

(b) -1

(c) ω

(d) ω^2

Question 5. If $\{(1 + i)/(1 - i)\}^n = 1$ then the least value of n is

(a) 1

(b) 2

(c) 3

(d) 4

Question 6. The value of $[i^{19} + (1/i)^{25}]^2$ is

(a) -1

(b) -2

(c) -3

(d) -4

Question 7. If z and w be two complex numbers such that |z| ≤ 1, |w| ≤ 1 and |z + iw| = |z – iw| = 2, then z equals {w is congugate of w}

(a) 1 or i

(b) i or – i

(c) 1 or – 1

(d) i or – 1

Question 8. The value of $\{-\sqrt{(-1)}\}^{4n+3}$, n ∈ N is

(a) i

(b) -i

(c) 1

(d) -1

Question 9. Find real θ such that (3 + 2i × sin θ)/(1 – 2i × sin θ) is real

(a) π

(b) nπ

(c) nπ/2

(d) 2nπ

Question 10. If i = √(-1) then $4 + 5(-1/2 + i\sqrt{3}/2)^{334} + 3(-1/2 + i\sqrt{3}/2)^{365}$ is equals to

(a) 1 – i√3

(b) -1 + i√3

(c) i√3

(d) -i√3

Very Short Questions:

Evaluate i^{-39}

1. Solved the quadratic equation $x^2 + x\frac{1}{\sqrt{2}} = 0$

2. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

3. Evaluate $(1+ i)^4$
4. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+1}$
5. Express in the form of a + ib. $(1+3i)^{-1}$
6. Explain the fallacy in -1 = i. i. = $\sqrt{-1}.\sqrt{-1} = \sqrt{-1(-1)} = \sqrt{1} = 1.$
7. Find the conjugate of $\frac{1}{2-3i}$
8. Find the conjugate of – 3i – 5.
9. Let $z_1 = 2 - i$, $z_2 = -2 + i$ Find Re $\left(\frac{z_1 z_2}{z_1}\right)$

Short Questions:

1. If $x + iy = \frac{a+ib}{a-ib}$ Prove that $x^2 + y^2 = 1$
2. Find real θ such that $\frac{3+2i \sin \theta}{1-2i \sin \theta}$ is purely real.
3. Find the modulus of $\frac{(1+i)(2+i)}{3+i}$
4. If $|a + ib| = 1$ then Show that $\frac{1+b+a}{1+b-ai} = b + ai$
5. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ Prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Long Questions:

1. If z = x + i y and w = $\frac{1-i^2}{Z-i}$ Show that $|w| = 1 \Rightarrow$ z is purely real.
2. Convert into polar form $\frac{-16}{1+i\sqrt{3}}$
3. Find two numbers such that their sum is 6 and the product is 14.
4. Convert into polar form $z = \frac{i-1}{\cos\frac{\pi}{3} + i\,s\ \frac{\pi}{3}}$
5. If α and β are different complex number with $|\beta| = 1$ Then find $\left|\frac{\beta-\alpha}{1-\alpha\beta}\right|$

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A): If $i = \sqrt{-1}$, then $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$ and $i^{4k+3} = -i$.

   Reason (R) : $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3} = 1$.

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A): Simplest form of i^{-35} is $-i$.

Reason (R) : Additive inverse of $(1 - i)$ is equal to $-1 + i$.

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (c) $a^2 = 3b$

2. (d) $e^{-\pi/2}$

3. (c) 17 i

4. (b) -1

5. (d) 4

6. (d) -4

7. (c) 1 or – 1

8. (a) i

9. (b) nπ

10. (c) i√3

Very Short Answer:

1.

$$i^{-39} = \frac{1}{i^{39}} = \frac{1}{(i^4)^9\, i^3}$$

$$= \frac{1}{1\times(-i)} \qquad \left[\because i^4 = 1 \atop i^3 = -i\right.$$

$$= \frac{1}{-i}\times\frac{i}{i}$$

$$= \frac{i}{-i^2} = \frac{i}{-(-1)} = i \qquad [\because i^2 = -1$$

2.

$$\frac{x^2}{1}+\frac{x}{1}+\frac{1}{\sqrt{2}} = 0$$

$$\frac{\sqrt{2}x^2+\sqrt{2}x+1}{\sqrt{2}} = \frac{0}{1}$$

$$\sqrt{2}x^2+\sqrt{2}x+1 = 0$$

$$x = \frac{-b\pm\sqrt{D}}{2a}$$

$$= \frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}$$

$$= \frac{-\sqrt{2}\pm\sqrt{2}\sqrt{1-2\sqrt{2}}}{2\sqrt{2}}$$

$$= \frac{-1\pm\sqrt{2\sqrt{2}-1}\, i}{2}$$

3.

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{1+i^2+2i}{1-i^2}\right)^m = 1$$

$$\left(\frac{\cancel{1}-\cancel{1}+2i}{2}\right)^m = 1 \qquad [\because i^2 = -1$$

$$i^m = 1$$

m=4

4.

$(1+i)^4=\left[(1+i)^2\right]^2$

$=\left(1+i^2+2i\right)^2$

$=(1-1+2i)^2$

$=(2i)^2=4i^2$

$=4(-1)=-4$

5.

Let z = $\frac{1+i}{1-i}-\frac{1-i}{1+i}$

$=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$

$=\frac{4i}{2}$

$=2i$

$z=0+2i$

$|z|=\sqrt{(0)^2+(2)^2}$

$=2$

6.

$(1+3i)^{-1}=\frac{1}{1+3i}\times\frac{1-3i}{1-3i}$

$=\frac{1-3i}{(1)^2-(3i)^2}$

$=\frac{1-3i}{1-9i^2}$

$=\frac{1-3i}{1+9}$ $\quad [i^2=-1$

$=\frac{1-3i}{10}$

$=\frac{1}{10}-\frac{3i}{10}$

7.

$1=\sqrt{1}=\sqrt{(-1)(-1)}$ is okay but

$\sqrt{(-1)(-1)}=\sqrt{-1}\sqrt{-1}$ is wrong.

8.

Let z = $\frac{1}{2-3i}$

$z = \frac{1}{2-3i} \times \frac{2+3i}{2+3i} i$

$= \frac{2+3i}{(2)^2-(3i)^2}$

$= \frac{2+3i}{4+9}$

$= \frac{2+3i}{13}$

$z = \frac{2}{13} + \frac{3}{13} i$

$\bar{z} = \frac{2}{13} - \frac{3}{13}$

9. Let z = 3i – 5

$\bar{z} = 3i - 5$

10. $z_1 z_2$ = (2 – i)(-2 + i)

$= -4 + 2i + 2i - i^2$

$= -4 + 4i + 1$

$= 4i - 3$

$\overline{z_1} = 2 + i$

$\frac{z_1 z_2}{\overline{z_1}} = \frac{4i-3}{2+i} \times \frac{2-i}{2-i}$

$= \frac{8i - 6 - 4i^2 + 3i}{4 - i^2}$

$= \frac{11i - 2}{5}$

$\frac{z_1 z_2}{z_1} = \frac{11}{5} i - \frac{2}{5}$

$\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = -\frac{2}{5}$

Short Answer:

1.

$x + iy = \frac{a+ib}{a-ib}$ (i) (Given) taking conjugate both side

$$x - iy = \frac{a-ib}{a+ib} \quad \text{(ii)}$$

(i) × (ii)

$$(x+iy)(x-iy) = \left(\frac{a+ib}{a-ib}\right) \times \left(\frac{a-ib}{a+ib}\right)$$

$$(x)^2 - (iy)^2 = 1$$

$x^2 + y^2 = 1$

$[i^2 = -1]$

2.

$$\frac{3+2i\ Sin\theta}{1-2i\ Sin\theta} = \frac{3+2i\ Sin\theta}{1-2i\ Sin\theta} \times \frac{1+2i\ Sin\theta}{1+2i\ Sin\theta}$$

$$= \frac{3+6i\ Sin\theta + 2i\ Sin\theta - 4Sin^2\theta}{1+4Sin^2\theta}$$

$$= \frac{3-4\ Sin^2\theta}{1+4\ Sin^2\theta} + \frac{8i\ Sin\theta}{1+4\ Sin^2\theta}$$

For purely real

Im (z) = 0

$$\frac{8Sin\theta}{1+4Sin^2\theta} = 0$$

$$Sin\theta = 0$$

$$\theta = n\pi$$

3.

$$\left|\frac{(1+i)(2+i)}{3+i}\right| = \frac{|(1+i)||2+i|}{|3+i|}$$

$$= \frac{\left(\sqrt{1^2+1^2}\right)\left(\sqrt{4+1}\right)}{\sqrt{(3)^2+(1)^2}}$$

$$= \frac{\left(\sqrt{2}\right)\left(\sqrt{5}\right)}{\sqrt{10}}$$

$$= \frac{\sqrt{2}\times\sqrt{5}}{\sqrt{2}\times\sqrt{5}}$$

$$= 1$$

4.

$|a+ib|=1$

$\sqrt{a^2+b^2}=1$

$a^2+b^2=1$

$\frac{1+b+ai}{1+b-ai}=\frac{(1+b)+ai}{(1+b)-ai}\times\frac{(1+b)+ai}{(1+b)+ai}$

$=\frac{(1+b)^2+(ai)^2+2(1+b)(ai)}{(1+b)^2-(ai)^2}$

$=\frac{1+b^2+2b-a^2+2ai+2abc}{1+b^2+2a-a^2}$

$=\frac{(a^2+b^2)+b^2+2b-a^2+2ai+2abi}{(a^2+b^2)+b^2+2b-a^2}$

$=\frac{2b^2+2b+2ai+2abi}{2b^2+2b}$

$=\frac{b^2+b+ai+abi}{b^2+b}$

$=\frac{b(b+1)+ai(b+1)}{b(b+1)}$

$=b+ai$

5.

$x-iy=\sqrt{\frac{a-ib}{c-id}}$ (1) (Given)

Taking conjugate both side

$x+iy=\sqrt{\frac{a+ib}{c+id}}$ (ii)

(i) × (ii)

$(x-iy)\times(x+iy)=\sqrt{\frac{a-ib}{c-id}}\times\sqrt{\frac{a+ib}{c+id}}$

$(x)^2-(iy)^2=\sqrt{\frac{(a)^2-(ib)^2}{(c)^2-(id)^2}}$

$x^2+y^2=\sqrt{\frac{a^2+b^2}{c^2+d^2}}$

squaring both side

$(x^2+y^2)^2=\frac{a^2+b^2}{c^2+d^2}$

Long Answer:

1.

w = $\frac{1-iz}{z-i}$

$=\frac{1-i(x+iy)}{x+iy-i}$

$=\frac{1-ix-i^2y}{x+i(y-1)}$

$=\frac{(1+y)-ix}{x+i(y-1)}$

$\therefore\ |w|=1$

$\Rightarrow\left|\frac{(1+y)-ix}{x+i(y-1)}\right|=1$

$\frac{|(1+y)-ix|}{|x+i(y-1)|}=1$

$\frac{\sqrt{(1+y)^2+(-x)^2}}{\sqrt{x^2+(y-1)^2}}=1$

$1+y^2+2y+x^2=x^2+y^2+1-2y$

$4y=0$

$y=0$

$\therefore$ z = x + i

is purely real

2.

$\frac{-16}{1+i\sqrt{3}}=\frac{-16}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$=\frac{-16\left(1-i\sqrt{3}\right)}{(1)^2-\left(i\sqrt{3}\right)^2}$

$=\frac{-16\left(1-i\sqrt{3}\right)}{1+3}$

$=-4\left(1-i\sqrt{3}\right)$

$z=-4+i4\sqrt{3}$

$r=\ |z|=\sqrt{(-4)^2+\left(4\sqrt{3}\right)^2}$

$= \sqrt{16+48}$

$= \sqrt{64}$

$= 8$

Let α be the acute $\angle S$

$\tan\alpha = \left|\frac{\cancel{4}\sqrt{3}}{-\cancel{4}}\right|$

$\tan\alpha = \tan\frac{\pi}{3}$

$\alpha = \pi/3$

Since Re (z) < o, and Im (z) > o

$\theta = \pi - \alpha$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$z = 8\left(Cos\ \frac{2\pi}{3} + i\ \text{Sin}\ \frac{2\pi}{3}\right)$

3.

Let x and y be the no.

$x + y = 6$

$xy = 14$

$x^2 - 6x + 14 = 0$

$D = -20$

$x = \frac{-(-6) \pm \sqrt{-20}}{2\times1}$

$= \frac{6 \pm 2\sqrt{5}\ i}{2}$

$= 3 \pm \sqrt{5}\ i$

$x = 3+\sqrt{5}\ i$

$y = 6 - \left(3 + \sqrt{5}\ i\right)$

$= 3 - \sqrt{5}\ i$

when $x = 3-\sqrt{5}\ i$

$y = 6 - \left(3 - \sqrt{5}\ i\right)$

$= 3 + \sqrt{5}\ i$

4.

$$z=\frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}$$

$$=\frac{2(i-1)}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z=\frac{\sqrt{3}-1}{2}+\frac{\sqrt{3}+1}{2}i$$

$$r=|z|=\left(\frac{\sqrt{3}-1}{2}\right)^2+\left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$r=2$$

Let α be the acule $\angle$s

$$\tan\ \alpha=\left|\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right|$$

$$=\left|\frac{\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\left(1-\frac{1}{\sqrt{3}}\right)}\right|$$

$$=\left|\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right|$$

$$\tan\alpha=\left|\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right|$$

$$\alpha=\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{12}$$

$$z=2\left(Cos\frac{5\pi}{12}+i\ \text{Sin}\ \frac{5\pi}{12}\right)$$

5.

$$\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|^2=\left(\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\overline{\alpha}\beta}}\right) \quad \left[\because |z|^2=z\overline{z}\right.$$

.

$$=\left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\alpha\bar{\beta}}\right)$$

$$=\left(\frac{\beta\bar{\beta}-\beta\bar{\alpha}-\alpha\bar{\beta}+\alpha\bar{\alpha}}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+\alpha\bar{\alpha}\beta\bar{\beta}}\right)$$

$$=\left(\frac{|\beta|^2-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2|\beta|^2}\right)$$

$$=\left(\frac{1-\beta\bar{\alpha}-\alpha\bar{\beta}+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2}\right) \quad [\because |\beta|=1$$

$$=1$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\sqrt{1}$$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=1$$

Assertion Reason Answer:

1. (iii) Assertion is true but reason is false.
2. (iv) Assertion is false but reason is true.