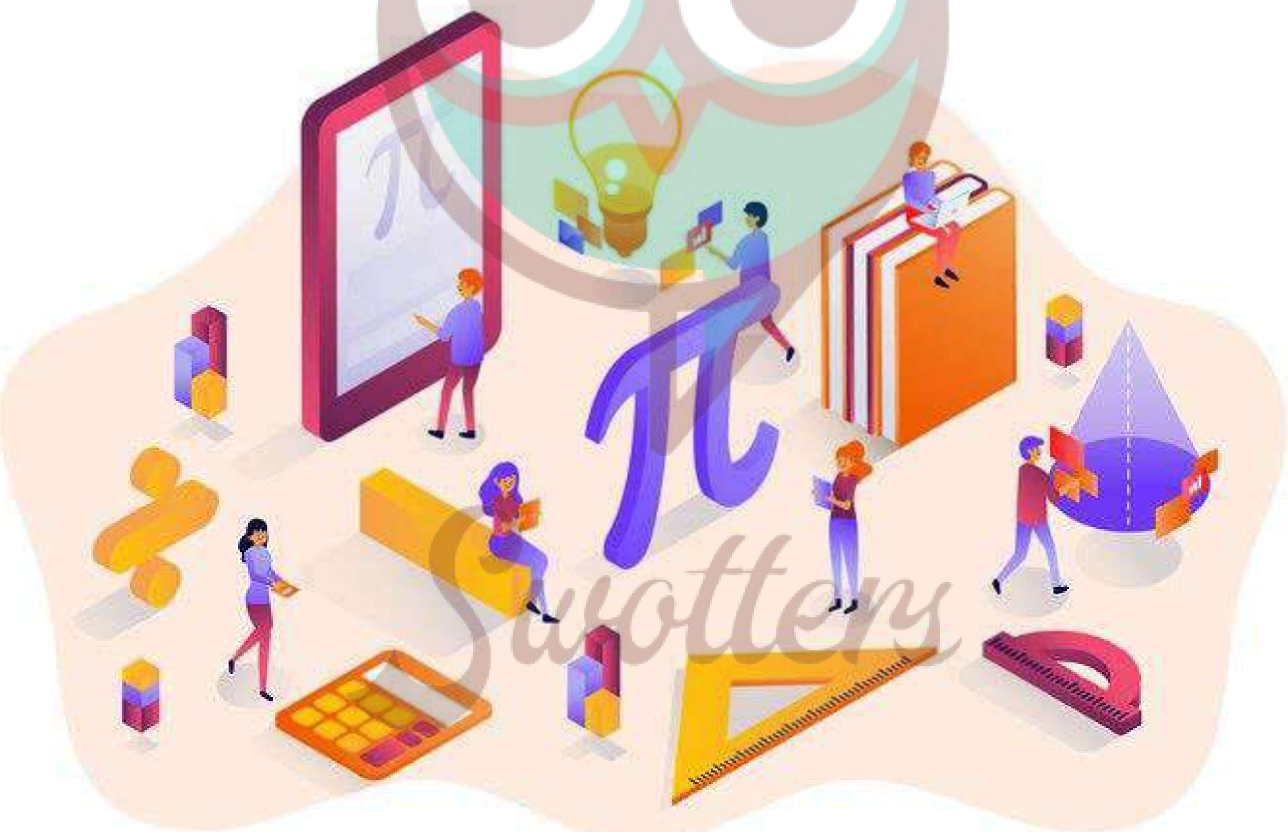
MATHEMATICS



Important Questions

Multiple Choice questions-

1. The function

$$F(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

is continuous at x = 0, then the value of 'k' is:

(a) 3

(b) 2

(c) 1

(d) 1.5.

2. The function f(x) = [x], where [x] denotes the greatest integer function, is continuous at:

(a) 4

(b)-2

(c) 1

(d) 1.5.

3. The value of 'k' which makes the function defined by

$$f(x) = \begin{cases} \sin\dfrac{1}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0, \end{cases}$$

continuous at x = 0 is

(a) -8

(b) 1

(c) -1

(d) None of these.

4. Differential coefficient of sec $(\tan^{-1} x)$ w.r.t. x is

(a) $\frac{x}{\sqrt{1+x^2}}$

(b) $\frac{x}{1+x^2}$

(c) $x\sqrt{1+x^2}$

(d) $\frac{1}{\sqrt{1+x^2}}$

5. If y = log ($\frac{1-x_2}{1+x_2}$) then $\frac{dy}{dx}$ is equal to:

(a) $\frac{4x^3}{1-x^4}$

(b) $\frac{-4x}{1-x^4}$

(c) $\frac{1}{4-x^4}$

(d) $\frac{-4x^3}{1-x^4}$

6.

If y = $\sqrt{sinx + y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{cosx}{2y-1}$

(b) $\frac{cosx}{1-2y}$

(c) $\frac{sinx}{1-2y}$

(d) $\frac{sinx}{2y-1}$

7. If u = sin-1 ($\frac{2x}{1+x_2}$) and u = tan-1 ($\frac{2x}{1-x_2}$) then $\frac{dy}{dx}$ is

(a) 12

(b) x

(c) $\frac{1-x^2}{1+x^2}$

(d) 1

8. If x = t², y = t³, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{3}{2}$

(b) $\frac{3}{4t}$

(c) $\frac{3}{2t}$

(d) $\frac{3t}{2}$

9. The value of ‘c’ in Rolle’s Theorem for the function $f(x) = x^3 – 3x$ in the interval $[0, \sqrt{3}]$ is

(a) 1

(b) -1

(c) $\frac{3}{2}$

(d) $\frac{1}{3}$

10. The value of ‘c’ in Mean Value Theorem for the function $f(x) = x (x – 2)$, $x \in [1, 2]$ is

(a) $\frac{3}{2}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{3}{4}$

Very Short Questions:

1. If y = log (cos ex), then find $\frac{dy}{dx}$ (Delhi 2019)
2. Differentiate $\cos \{\sin (x)_2\}$ w.r.t. x. (Outside Delhi 2019)
3. Differentiate $\sin^2(x^2)$ w.r.t. x^2. (C.B.S.E. Sample Paper 2018-19)
4. Find $\frac{dy}{dx}$, if y + siny = cos or.
5. If $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$ then find $\frac{dy}{dx}$.
6. Is it true that $x = e^{\log x}$ for all real x? (N.C.E.R.T.)
7. Differentiate the following w.r.t. x : 3^{x+2}. (N.C.E.R.T.)

8. Differentiate log $(1 + \theta)$ w.r.t. $\sin^{-1}\theta$.

9. If $y = x^x$, find $\frac{dy}{dx}$.

10.

If $y = \sqrt{2^x + \sqrt{2^x + \sqrt{2^x + \ldots\ldots + 0\infty}}}$ then prove that: $(2y - 1)\frac{dy}{dx} = 2^x \log 2$.

Short Questions:

1. Discuss the continuity of the function: $f(x) = |x|$ at $x = 0$. (N.C.E.R.T.)

2. If $f(x) = x + 1$, find $\frac{d}{dx}$ (fof)(x). (C.B.S.E. 2019)

3. Differentiate $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x. (C.B.S.E. 2018 C)

4. Differentiate: $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x. (C.B.S.E. 2018)

5. Write the integrating factor of the differential equation:

   $(\tan^{-1} y - x)\,dy = (1 + y^2)\,dx$. (C.B.S.E. 2019 (Outside Delhi))

6. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1 - x^2}}{13}\right]$ (A.I.C.B.S.E. 2016)

7. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1 - 4x^2}}{5}\right]$ (A.I.C.B.S.E. 2016)

8. If $y = \{x + \sqrt{x^2 + a^2}\}^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

Long Questions:

1. Find the value of 'a' for which the function 'f' defined as:

$$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x + 1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

   s continuous at x = 0 (CBSE 2011)

2. Find the values of 'p' and 'q' for which:

$$\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ p, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \dfrac{\pi}{2}. \end{cases}$$

is continuous at x = 2 (CBSE 2016)

3. Find the value of ‘k’ for which

$$f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \dfrac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$

is continuous at x = 0 (A.I.C.B.S.E. 2013)

4. For what values of ‘a’ and ‘b\ the function ‘f’ defined as:

$$f(x) = \begin{cases} 3ax+b & \text{if } x < 1 \\ 11 & \text{if } x = 1 \\ 5ax-2b & \text{if } x > 1 \end{cases}$$

is continuous at x = 1. (CBSE 2011)

Assertion and Reason Questions-

1. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion(A): $f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right), & x \ne 0 \\ 0 & x = 0 \end{cases}$ is continuous at x = 0.

Reason (R): Both $h(x) = x^2$, $g(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$ are continuous at x = 0.

2. Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes(a), (b), (c) and (d) as given below.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false and R is true.
e) Both A and R are false.

Assertion (A): The function $f(x) = \begin{cases} |x| + \sqrt{x - |x|}, & x \geq 0 \\ \sin x & x < 0 \end{cases}$ is continuous everywhere.

Reason (R): f(x) is periodic function.

Case Study Questions-

1. If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x. When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$, then we differentiate every term of the given relation w.r.t. x, remembering that a tenn in y is first differentiated w.r.t. y and then multiplied by $\frac{dy}{dx}$.

Based on the ab:ve information, find the value of $\frac{dy}{dx}$ in each of the following questions.

i. $x^3 + x^2y + xy^2 + y^3 = 81$

a. $\frac{(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$

b. $\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$

c. $\frac{(3x^2+2xy-y^2)}{x^2-2xy+3y^2}$

d. $\frac{3x^2+xy+y^2}{x^2+xy+3y^2}$

ii. $x^y = e^{x-y}$

a. $\frac{x-y}{(1+\log x)}$

b. $\frac{x+y}{(1+\log x)}$

c. $\frac{x-y}{x(1+\log x)}$

d. $\frac{x+y}{x(1+\log x)}$

iii. $e^{\sin y} = xy$

a. $\frac{-y}{x(y\cos y-1)}$

b. $\frac{y}{y\cos y-1}$

c. $\frac{y}{y\cos y+1}$

d. $\frac{y}{x(y\cos y-1)}$

iv. $\sin^2 x + \cos^2 y = 1$

a. $\frac{\sin 2y}{\sin 2x}$

b. $-\frac{\sin 2x}{\sin 2y}$

c. $-\frac{\sin 2y}{\sin 2x}$

d. $\frac{\sin 2x}{\sin 2y}$

v. $y = (\sqrt{x})^{\sqrt{x}^{\sqrt{x}^{\dots\infty}}}$

a. $\frac{-y^2}{x(2-y\log x)}$

b. $\frac{y^2}{2+y\log x}$

c. $\frac{y^2}{x(2+y\log x)}$

d. $\frac{y^2}{x(2-y\log x)}$

1. If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x, then y = f(g(x)] is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. This rule is also known as CHAIN RULE.

Based on the above information, find the derivative of functions w.r.t. x in the following questions.

i. $\cos\sqrt{x}$

a. $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

b. $\frac{\sin\sqrt{x}}{2\sqrt{x}}$

c. $\sin\sqrt{x}$

d. $-\sin\sqrt{x}$

ii. $7^{x+\frac{1}{x}}$

a. $\left(\frac{x^2-1}{x^2}\right)\cdot 7^{x+\frac{1}{x}}\cdot \log 7$

b. $\left(\frac{x^2+1}{x^2}\right)\cdot 7^{x+\frac{1}{x}}\cdot \log 7$

c. $\left(\frac{x^2-1}{x^2}\right)\cdot 7^{x-\frac{1}{x}}\cdot \log 7$

d. $\left(\frac{x^2+1}{x^2}\right)\cdot 7^{x-\frac{1}{x}}\cdot \log 7$

iii. $\sqrt{\frac{1-\cos x}{1+\cos x}}$

a. $\frac{1}{2}\sec^2\frac{x}{2}$

b. $-\frac{1}{2}\sec^2\frac{x}{2}$

c. $\sec^2\frac{x}{2}$

d. $-\sec^2\frac{x}{2}$

iv. $\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right)+\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

a. $\frac{-1}{x^2+b^2}+\frac{1}{x^2+a^2}$

b. $\frac{1}{x^2+b^2}+\frac{1}{x^2+a^2}$

c. $\frac{1}{x^2+b^2}-\frac{1}{x^2+a^2}$

d. None of these.

v. $\sec^{-1} x + \operatorname{cosec}^{-1} \frac{x}{\sqrt{x^2-1}}$

a. $\frac{2}{\sqrt{x^2-1}}$

b. $\frac{-2}{\sqrt{x^2-1}}$

c. $\frac{1}{|x|\sqrt{x^2-1}}$

d. $\frac{2}{|x|\sqrt{x^2-1}}$

Answer Key-

Multiple Choice questions-

1. Answer: (b) 2
2. Answer: (d) 1.5.
3. Answer: (d) None of these.
4. Answer:

   (a) $\frac{x}{\sqrt{1+x^2}}$

5. Answer:

   (b) $\frac{-4x}{1-x^4}$

6. Answer:

   (a) $\frac{\cos x}{2y-1}$

7. Answer: (d) 1
8. Answer: (b) $\frac{3}{4t}$
9. Answer: (a) 1
10. Answer: (a) $\frac{3}{2}$

Very Short Answer:

1. Solution:

We have: $y = \log(\cos e^x)$

$\therefore \frac{dy}{dx} = \frac{1}{\cos e^x}(-\sin e^x) \cdot e^x$

$= -e^x \tan e^x$

2. Solution:

Let $y = \cos\{\sin(x)^2\}$.

$\therefore \frac{dy}{dx} = -\sin\{\sin(x)^2\}. \frac{dy}{dx}\{\sin(x)^2\}$

$= -\sin\{\sin(x)^2\}. \cos(x)^2 \frac{dy}{dx}(x^2)$

$= -\sin\{\sin(x)^2\}. \cos(x)^2 2x$

$= -2x \cos(x)^2 \sin\{\sin(x)^2\}$.

3. Solution:

Let $y = \sin^2(x^2)$.

$\therefore \frac{dy}{dx} = 2\sin(x^2)\cos(x^2) = \sin(2x^2)$.

4. Solution:

We have: $y + \sin y = \cos x$.

Differentiating w.r,t. x, we get:

$\frac{dy}{dx} + \cos y. \frac{dy}{dx} = -\sin x$

$(1 + \cos y)\frac{dy}{dx} = -\sin x$

Hence, $\frac{dy}{dx} = -\frac{\sin x}{1+\cos y}$

where $y \neq (2n + 1)\pi, n \in Z$.

5. Solution:

Here $y = \sin^{-1}\left(6x\sqrt{1-9x^2}\right)$

Put 3x = sin θ.

$y = \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta) = 2\theta$

$= 2\sin^{-1} 3x$

$\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$

6. Solution:

The given equation is $x = e^{\log x}$

This is not true for non-positive real numbers.

[∵ Domain of log function is R+]

Now, let $y = e^{\log x}$

If y > 0, taking logs.,

$\log y = \log(e^{\log x}) = \log x.\log e$

$= \log x . 1 = \log x$

$\Rightarrow y = x.$

Hence, $x = e^{\log x}$ is true only for positive values of x.

7. Solution:

Let $y = 3^{x+2}$.

$\frac{dy}{dx} = {}^{3x+2}.\log 3. \frac{d}{dx}(x + 2)$

$= 3^{x+2} .\log 3.(1 + 0)$

$= 3^{x+2}. \log 3 = \log 3\,(3^{x+2})$.

8. Solution:

Let $y = \log(1 + \theta)$ and $u = \sin^{-1}\theta$.

$\therefore \frac{dy}{d\theta} = \frac{1}{1+\theta}$ and $\frac{du}{d\theta} = \frac{1}{\sqrt{1-\theta^2}}$.

$\therefore \qquad \frac{dy}{du} = \frac{dy/d\theta}{du/d\theta}$

$$= \frac{\frac{1}{1+\theta}}{\frac{1}{\sqrt{1-\theta^2}}} = \sqrt{\frac{1-\theta}{1+\theta}}$$

9. Solution:

Here $y = x^x$ ...(1)

Taking logs., $\log y = \log x^x$

$\Rightarrow \log y = x \log x.$

Differentiating w.r.t. x, we get:

$\frac{1}{y} \cdot \frac{dy}{dx}$ = x 1x + logx. (1)

= 1 + log x.

Hence, $\frac{dy}{dx}$ = y (1 + log x) dx

$= x^x (1 + \log x)$. [Using (1)]

10. Solution:

The given series can be written as:

$y = \sqrt{2^x + y}$

Squaring, $y^2 = 2^x + y$

$\Rightarrow y^2 - y = 2^x$.

Diff. w.r.t. x, $(2y - 1)\frac{dy}{dx} = 2^x \log 2$.

Short Answer:

1. Solution:

By definition, $f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0. \end{cases}$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x)$$

$$= \lim_{h \to 0} (-(0-h))$$

$$= \lim_{h \to 0} (h) = 0.$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x)$$

$$= \lim_{h \to 0} (0+h)$$

$$= \lim_{h \to 0} (h) = 0.$$

Also $f(0) = 0$.

Thus $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$.

[$\because$ *Each = 0*]

Hence 'f' is continuous at x = 0.

2. Solution:

We have : f(x) = x + 1 ...(1)

$\therefore$ fof(x) = f (f(x)) =f(x)+ 1

= (x + 1) + 1 = x + 2.

$\therefore \frac{d}{dx}$ (fof)(x).) = $\frac{d}{dx}$ (x + 2) = 1 + 0 = 1.

3. Solution:

Let $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

$$= \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

[*Dividing num. & denom. by cos x*]

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right) = \frac{\pi}{4} - x$$

Differentiating (1) w.r.t. x,

$$\Rightarrow \quad \frac{dy}{dx} = -1$$

4. Solution:

$$\text{Let } y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$= \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right) = \frac{\pi}{2} - \frac{x}{2}.$$

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} = -\frac{1}{2}.$$

5. Solution:

The given differential equation is:

$(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$ Linear Equation

$\therefore$ I.F $= e^{\int \frac{1}{1+y^2} dx} = e^{\tan^{-1} y}$

6. Solution:

$$\text{We have : } y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$$

$$= \sin^{-1}\left(\frac{5}{13}x + \frac{12}{13}\sqrt{1-x^2}\right)$$

$$= \sin^{-1}\left(x\sqrt{1-\left(\frac{12}{13}\right)^2} + \sqrt{1-x^2}\cdot\frac{12}{13}\right)$$

(Note this step)

$$= \sin^{-1} x + \sin^{-1}\frac{12}{13}$$

$[\because \sin^{-1}A + \sin^{-1}B = \sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2})]$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + 0 = \frac{1}{\sqrt{1-x^2}}, |x| < 1.$$

7. Solution:

We have : $y = \sin^{-1}\left(\frac{6x - 4\sqrt{1-4x^2}}{5}\right)$

$$= \sin^{-1}\left(\frac{6x}{5} - \frac{4}{5}\sqrt{1-4x^2}\right)$$

$$= \sin^{-1}\left((2x).\frac{3}{5} - \frac{4}{5}\sqrt{1-(2x)^2}\right)$$

$$= \sin^{-1}\left((2x)\sqrt{1-\left(\frac{4}{5}\right)^2} - \left(\frac{4}{5}\right)\sqrt{1-(2x)^2}\right)$$

$$= \sin^{-1}(2x) - \sin^{-1}\frac{4}{5}.$$

Hence, $\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x^2)}}.(2) - 0 = \frac{2}{\sqrt{1-4x^2}}.$

8. Solution:

$y = \{x + \sqrt{x^2 + a^2}\}^n$ .... (1)

$$\therefore \frac{dy}{dx} = n\left\{x+\sqrt{x^2+a^2}\right\}^{n-1} \cdot \frac{d}{dx}\left\{x+\sqrt{x^2+a^2}\right\}$$

$$= n\left\{x+\sqrt{x^2+a^2}\right\}^{n-1} \cdot \left[1+\frac{1}{2\sqrt{x^2+a^2}}(2x+0)\right]$$

$$= n\left\{x+\sqrt{x^2+a^2}\right\}^{n-1} \left\{\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right\}$$

$$= \frac{n\left\{x+\sqrt{x^2+a^2}\right\}^{n}}{\sqrt{x^2+a^2}} = \frac{ny}{\sqrt{x^2+a^2}},$$

[Using (I)]

which is true.

Long Answer:

1. Solution:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} a\sin\frac{\pi}{2}(x+1)$$

$$= \lim_{h\to 0} a\sin\frac{\pi}{2}(0-h+1)$$

$$= a\sin\frac{\pi}{2}(0-0+1)$$

$$= a\sin\frac{\pi}{2} = a.1 = a$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{h \to 0} \frac{\tan(0+h) - \sin(0+h)}{(0+h)^3}$$

$$= \lim_{h \to 0} \frac{\tan h - \sin h}{h^3}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \; \frac{1-\cos h}{h^2} \cdot \frac{1}{\cos h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \cdot \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{h^2} \cdot \lim_{h \to 0} \frac{1}{\cos h}$$

$$= 1.\frac{1}{2} \lim_{h \to 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \cdot \frac{1}{\cos 0}$$

$$= 1.\frac{1}{2}(1)^2.\frac{1}{1} = \frac{1}{2}.$$

Also f(0) = a sin π/2 (0+1)

= a sin π/2 = a(1) = a

For continuity,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

⇒ a = 1/2 = a

Hence, a = ½

2. Solution:

$$\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} \frac{1-\sin^3 x}{3\cos^2 x}$$

$$= \lim_{h\to 0} \frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}$$

$$= \lim_{h\to 0} \frac{1-\cos^3 h}{3\sin^2 h}$$

$$= \lim_{h\to 0} \frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1-\cos h)(1+\cos h)}$$

$$= \lim_{h\to 0} \frac{1+\cos^2 h+\cos h}{3(1+\cos h)}$$

$$= \frac{1+1+1}{3(1+1)} = \frac{1}{2}.$$

$$\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi-2x)^2}$$

$$= \lim_{h\to 0} \frac{q\left[1-\sin\left(\frac{\pi}{2}+h\right)\right]}{\left[\pi-2\left(\frac{\pi}{2}+h\right)\right]^2}$$

$$= \lim_{h\to 0} \frac{q(1-\cos h)}{(\pi-\pi-2h)^2}$$

$$= \lim_{h\to 0} \frac{q(1-\cos h)}{4h^2}$$

$$= \lim_{h\to 0} \frac{q \cdot 2\sin^2\frac{h}{2}}{4h^2}$$

$$= \lim_{h\to 0} \frac{q}{8}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2$$

$$= \frac{q}{8}(1)^2 = \frac{q}{8}.$$

Also f($\frac{\pi}{2}$,) = p

For continuity $\lim_{x\to\frac{\pi^-}{2}} f(x) = \lim_{x\to\frac{\pi^+}{2}} f(x)$

= f($\frac{\pi}{2}$,)

$\Rightarrow \frac{1}{2} = \frac{q}{8}$ = p

Hence p = 1/2 and q = 4

3. Solution:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$$

$$= \lim_{x\to 0^-} \frac{(\sqrt{1+kx}-\sqrt{1-kx})(\sqrt{1+kx}+\sqrt{1-kx})}{x(\sqrt{1+kx}+\sqrt{1-kx})}$$

[*Rationalising Numerator*]

$$= \lim_{x\to 0^-} \frac{(1+kx)-(1-kx)}{x(\sqrt{1+kx}+\sqrt{1-kx})}$$

$$= \lim_{x\to 0^-} \frac{2kx}{x(\sqrt{1+kx}+\sqrt{1-kx})}$$

$$= \lim_{x\to 0^-} \frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}} \qquad [\because x \neq 0]$$

$$= \frac{2k}{1+1} = k$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{2x+1}{x-1} = \lim_{h\to 0} \frac{2(0+h)+1}{(0+h)-1}$$

$$= \frac{2(0)+1}{0-1} = \frac{1}{-1} = -1$$

Also $f(0) = \frac{2(0)+1}{0-1} = \frac{1}{-1} = -1$.

For continuity $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

⇒ k = -1 = -1

Hence k = -1

4. Solution:

$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} (3ax + b)$

$= \lim_{h\to0} (3a(1-h) + b]$

$= 3a(1 - 0) + b$

$= 3a + b$

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} (5ax - 2b)$

$= \lim_{h\to0} [5a(1+h) - 2b]$

$= 5a(1+0) - 2b$

$= 5a - 2b$

Also f(1) = 11

Since 'f' is continuous at x = 1,

$\therefore \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$

$\Rightarrow 3a + b = 5a - 2b = 11.$

From first and third,

3a + b = 11 …………… (1)

From last two,

5a – 2b = 11 …………… (2)

Multiplying (1) by 2,

6a + 2b = 22 ………….. (3)

Adding (2) and (3),

11a = 33

⇒ a = 3.

Putting in (1),

3(3) + b = 11

⇒ b = 11 – 9 = 2.

Hence, a = 3 and b = 2.

Case Study Answers-

1. Answer :

i. (b) $\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$

Solution:

$$x^3 + x^2y + xy^2 + y^3 = 81$$

$$\Rightarrow 3^2 + x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow (x^2 + 2xy + 3y^2)\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$$

ii. (c) $\frac{x-y}{x(1+\log x)}$

Solution:

$$x^y = e^{x-y} \Rightarrow y\log x = x - y$$

$$y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}[\log x + 1] = 1 - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x[1+\log x]}$$

iii. (d) $\frac{y}{x(y\cos y-1)}$

Solution:

$$e^{\sin y} = xy \Rightarrow \sin y = \log x + \log y$$

$$\Rightarrow \cos y\frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} \Rightarrow \frac{dy}{dx}\left[\cos y - \frac{1}{y}\right] = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x(y\cos y-1)}$$

iv. (d) $\frac{\sin 2x}{\sin 2y}$

Solution:

$$\sin^2 x + \cos^2 y = 1$$

$$\Rightarrow 2\sin x\cos x + 2\cos y\left(-\sin y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin 2x}{-\sin 2y} = \frac{\sin 2x}{\sin 2y}$$

v. (d) $\frac{y^2}{x(2-y\log x)}$

Solution:

$$y = (\sqrt{x})^{\sqrt{x}^{\sqrt{x}\ldots\infty}} \Rightarrow y = (\sqrt{x})^{y}$$

$$\Rightarrow y = y(\log\sqrt{x}) \Rightarrow \log y = \frac{1}{2}(y\log x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[y \times \frac{1}{x} + \log x\left(\frac{dy}{dx}\right)\right]$$

$$\Rightarrow \frac{dy}{dx}\left\{\frac{1}{y} - \frac{1}{2}\log x\right\} = \frac{1}{2}\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x} \times \frac{2y}{(2-y\log x)} = \frac{y^2}{x(2-y\log x)}$$

2. Answer :

i. (a) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

Solution:

Let $y = \cos\sqrt{x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\cos\sqrt{x}) = -\sin\sqrt{x} \cdot \frac{d}{dx}(\sqrt{x})$

$= -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

ii. (a) $\left(\frac{x^2-1}{x^2}\right) \cdot 7^{x+\frac{1}{x}} \cdot \log 7$

Solution:

Let $y = 7^{x+\frac{1}{x}} \therefore \frac{dy}{dx} = \frac{d}{dx}\left(7^{x+\frac{1}{x}}\right)$

$= 7^{x+\frac{1}{x}} \cdot \log 7 \cdot \frac{d}{dx}\left(x + \frac{1}{x}\right) = 7^{x+\frac{1}{x}} \cdot \log 7 \cdot \left(1 - \frac{1}{x^2}\right)$

$= \left(\frac{x^2-1}{x^2}\right) \cdot 7^{x+\frac{1}{x}} \cdot \log 7$

iii. (a) $\frac{1}{2}\sec^2 \frac{x}{2}$

Solution:

Let $y = \sqrt{\frac{1-\cos x}{1+\cos x}} = \sqrt{\frac{1-1+2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}-1+1}} = \tan\left(\frac{x}{2}\right)$

$\therefore \frac{dy}{dx} = \sec^2 \frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2}\sec^2 \frac{x}{2}$

iv. (b) $\frac{1}{x^2+b^2} + \frac{1}{x^2+a^2}$

Solution:

Let $y = \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$\therefore \frac{dy}{dx} = \frac{1}{b} \times \frac{1}{1+\frac{x^2}{b^2}} \times \frac{1}{b} + \frac{1}{a} \times \frac{1}{1+\frac{x^2}{a^2}} \times \frac{1}{a}$

$= \frac{1}{x^2+b^2} + \frac{1}{x^2+a^2}$

v. (d) $\frac{2}{|x|\sqrt{x^2-1}}$

Solution:

Let $y = \sec^{-1} x + \operatorname{cosec}^{-1} \frac{x}{\sqrt{x^2-1}}$

Put $x = \sec\theta \Rightarrow \theta = \sec^{-1} x$

$\therefore y = \sec^{-1}(\sec\theta) + \operatorname{cosec}^{-1}\left(\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}\right)$

$= \theta + \sin^{-1}\left[\sqrt{1-\cos^2\theta}\right]$

$= \theta + \sin^{-1}(\sin\theta) = \theta + \theta = 2\theta = 2\sec^{-1} x$

$\therefore \frac{dy}{dx} = 2\frac{d}{dx}(\sec^{-1} x) = 2 \times \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{2}{|x|\sqrt{x^2-1}}$