MATHEMATICS

Chapter 6: The Triangle and Its Properties

Multiple Choice Questions :

Question 1. A triangle has how many sides :

(a) three

(b) five

(c) four

(d) None of these

Question 2. A triangle has medians :

(a) 2

(b) 1

(c) 3

(d) None of these

Question 3. A triangle has altitudes :

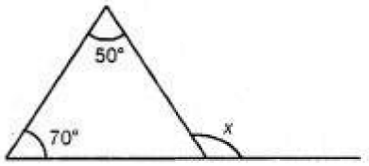
(a) 2

(b) 3

(c) 1

(d) None of these

Question 4. Find the value of x :



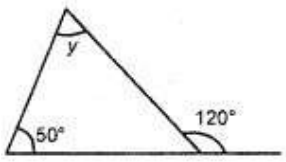
(a) 120°

(b) 110°

(c) 100°

(d) None of these

Question 5. Find the value of y :



(a) 50°

(b) 70°

(c) 40°

(d) None of these

Question 6. Sum of three angles of a triangle is :

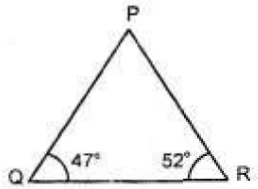
(a) 170°

(b) 90°

(c) 180°

(d) None of these

Question 7. Find the third angle of the given triangle



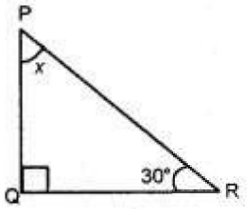
(a) 71°

(b) 61°

(c) 81°

(d) None of these

Question 8. Find the unknown x in the following diagram



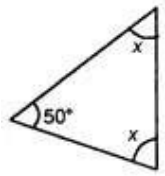
(a) 60°

(b) 30°

(c) 90°

(d) None of these

Question 9. Find the value of x in the given diagram :



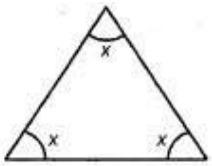
(a) 65°

(b) 50°

(c) 70°

(d) None of these

Question 10. Find the value of x in the given diagram :



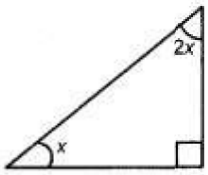
(a) 4°

(b) 60°

(c) 80°

(d) None of these

Question 11. Find the value of x in the given diagram :



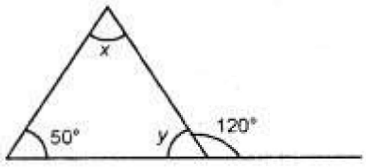
(a) 30°

(b) 45°

(c) 60°

(d) None of these

Question 12. Find the value of x and y in the following diagram



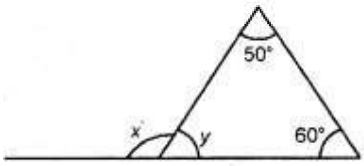
(a) (60, 70)

(b) (50, 70)

(c) (70, 60)

(d) None of these

Question 13. Find the value of x and y in the following diagram



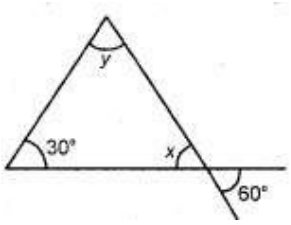
(a) (110, 70)

(b) 70, 110)

(c) (60, 120)

(d) none of these

Question 14. Find the value of x and y in the following diagram :



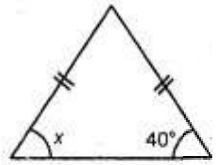
(a) (60,90)

(b) (90,60)

(c) (60,60)

(d) none of these

Question 15. Find the angle x in given diagram :



(a) 30°

(b) 60°

(c) 40°

(d) none of these

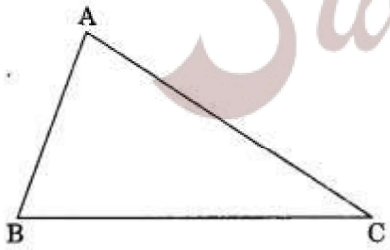
Very Short Questions :

1. In ΔABC, write the following:

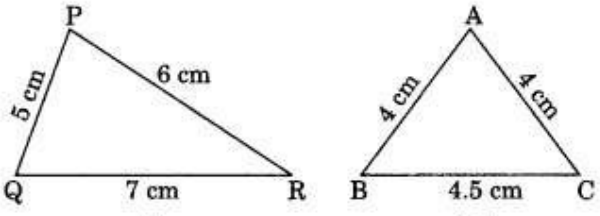
(a) Angle opposite to side BC.

(b) The side opposite to ∠ABC.

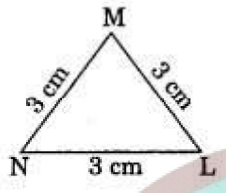
(c) Vertex opposite to side AC.



2. Classify the following triangle on the bases of sides



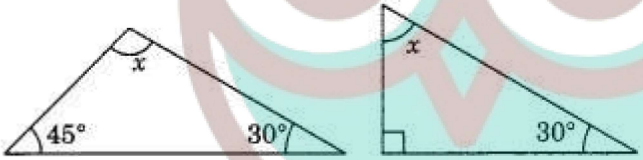
(i) (ii)



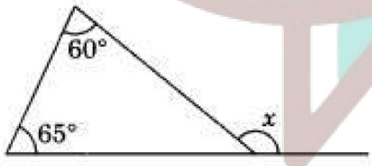
(iii)

3. In the given figure, name the median and the altitude. Here E is the midpoint of BC.

4. In the given diagrams, find the value of x in each case.



(i) (ii)



(iii)

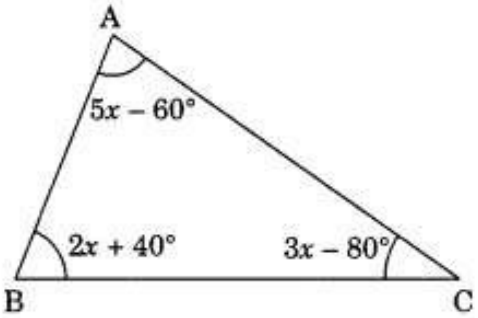
5. Which of the following cannot be the sides of a triangle?

(i) 4.5 cm, 3.5 cm, 6.4 cm

(ii) 2.5 cm, 3.5 cm, 6.0 cm

(iii) 2.5 cm, 4.2 cm, 8 cm

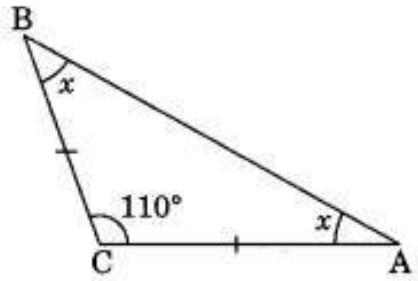
6. In the given figure, find x.



7. One of the equal angles of an isosceles triangle is 50°. Find all the angles

of this triangle.

8. In ΔABC, AC = BC and ∠C = 110°. Find ∠A and ∠B.



Short Questions :

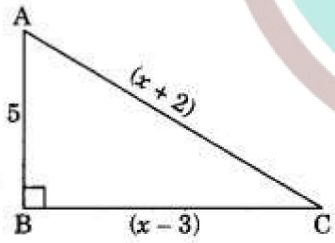
1. Two sides of a triangle are 4 cm and 7 cm. What can be the length of its third side to make the triangle possible?

2. Find whether the following triplets are Pythagorean or not?

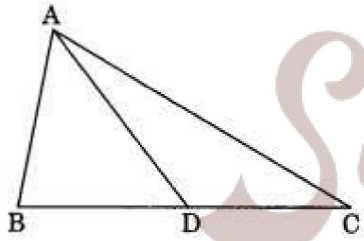
(a) (5, 8, 17)

(b) (8, 15, 17)

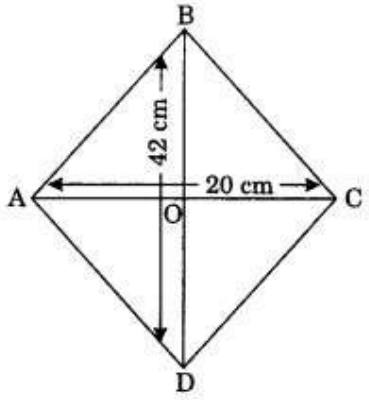
3. In the given right-angled triangle ABC, ∠B = 90°. Find the value of x.



4. AD is the median of a ΔABC, prove that AB + BC + CA > 2AD (HOTS)



5. The length of the diagonals of a rhombus is 42 cm and 40 cm. Find the perimeter of the rhombus.

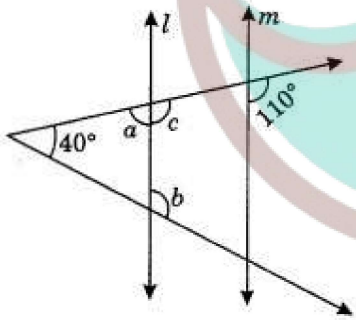


6. The sides of a triangle are in the ratio 3 : 4 : 5. State whether the triangle is right-angled or not.

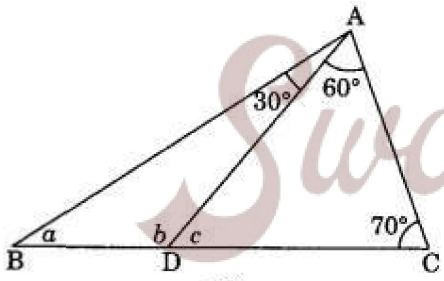
7. A plane flies 320 km due west and then 240 km due north. Find the shortest distance covered by the plane to reach its original position.

Long Questions :

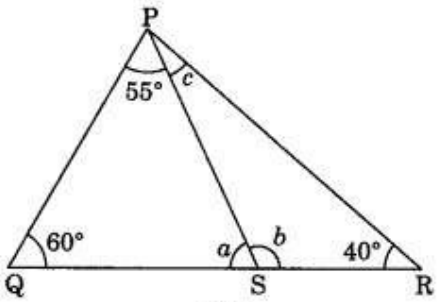
1. In the following figure, find the unknown angles a and b, if l || m.



2. In figure (i) and (ii), Find the values of a, b and c.



(i)

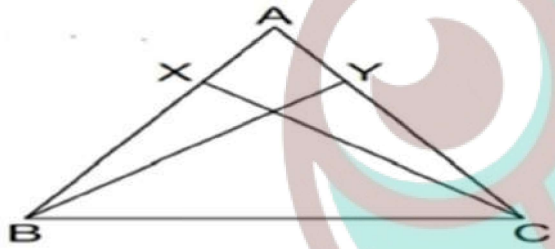


(ii)

3. I have three sides. One of my angle measure 15°. Another has a measure of 60°. What kind of a polygon am I? If I am a triangle, then what kind of triangle am I?
4. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Assertion and Reason Questions :

1) Assertion : In the adjoining figure, X and Y are respectively two points on equal sides AB and AC of ΔABC such that AX = AY then CX = BY



Reason: If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent

a) both Assertion and reason are correct and reason is correct explanation for Assertion.
b) both Assertion and reason are correct but reason is not correct explanation for Assertion
c) Assertion is true but reason is false.
d) both Assertion and reason are false

2) Assertion: Two angles measures a – 60° and 123º – 2a. If each one is opposite to equal sides of an isosceles triangle, then the value of a is 61°.
Reason: Sides opposite to equal angles of a triangle are equal.

a) both Assertion and reason are correct and reason is correct explanation for Assertion.
b) both Assertion and reason are correct but reason is not correct explanation for Assertion
c) Assertion is true but reason is false.
d) both Assertion and reason are false

ANSWER KEY -

Multiple Choice questions :

1. (a) three

2. (c) 3

3. (b) 3

4. (a) 120°

5. (b) 70°

6. (c) 180°

7. (c) 81°

8. (a) 60°

9. (a) 65°

10. (b) 60°

11. (a) 30°

12. (c) (70, 60)

13. (a) (110, 70)

14. (a) (60,90)

15. (c) 40°

Very Short Answer :

1. (a) In ΔABC, Angle opposite to BC is ∠BAC

(b) Side opposite to ∠ABC is AC

(c) Vertex opposite to side AC is B

2. (i) PQ = 5 cm, PR = 6 cm and QR = 7 cm

PQ ≠ PR ≠ QR

Thus, ΔPQR is a scalene triangle.

(ii) AB = 4 cm, AC = 4 cm

AB = AC

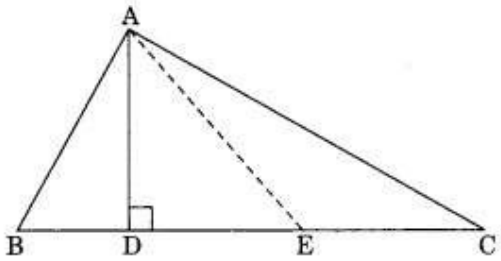
Thus, ΔABC is an isosceles triangle.

(iii) MN = 3 cm, ML = 3 cm and NL = 3 cm

MN = ML = NL

Thus, ΔMNL is an equilateral triangle.

3. In ΔABC, we have



AD is the altitude.

AE is the median.

4. (i) $x + 45° + 30° = 180°$ (Angle sum property of a triangle)

$\Rightarrow x + 75° - 180°$

$\Rightarrow x = 180° - 75°$

$x = 105°$

(ii) Here, the given triangle is right angled triangle.

$x + 30° = 90°$

$\Rightarrow x = 90° - 30° = 60°$

(iii) $x = 60° + 65°$ (Exterior angle of a triangle is equal to the sum of interior opposite angles)

$\Rightarrow x = 125°$

5. (i) Given sides are, 4.5 cm, 3.5 cm, 6.4 cm

Sum of any two sides = 4.5 cm + 3.5 cm = 8 cm

Since 8 cm > 6.4 cm (Triangle inequality)

The given sides form a triangle.

(ii) Given sides are 2.5 cm, 3.5 cm, 6.0 cm

Sum of any two sides = 2.5 cm + 3.5 cm = 6.0 cm

Since 6.0 cm = 6.0 cm

The given sides do not form a triangle.

(iii) 2.5 cm, 4.2 cm, 8 cm

Sum of any two sides = 2.5 cm + 4.2 cm = 6.7 cm

Since 6.7 cm < 8 cm

The given sides do not form a triangle.

6. In ΔABC, we have

$5x - 60° + 2x + 40° + 3x - 80° = 180°$ (Angle sum property of a triangle)

$\Rightarrow 5x + 2x + 3x - 60° + 40° - 80° = 180°$

$\Rightarrow 10x - 100° = 180°$

$\Rightarrow 10x = 180° + 100°$

$\Rightarrow 10x = 280°$

$\Rightarrow x = 28°$

Thus, $x = 28°$

7. Let the third angle be x°.

$x + 50° + 50° = 180°$

$\Rightarrow x° + 100° = 180°$

$\Rightarrow x° = 180° - 100° = 80°$

Thus $\angle x = 80°$

In given ΔABC, $\angle C = 110°$

8. Let $\angle A = \angle B = x°$ (Angle opposite to equal sides of a triangle are equal)

$x + x + 110° = 180°$

$\Rightarrow 2x + 110° = 180°$

$\Rightarrow 2x = 180° - 110°$

$\Rightarrow 2x = 70°$

$\Rightarrow x = 35°$

Thus, $\angle A = \angle B = 35°$

Short Answer :

1. Let the length of the third side be x cm.

Condition I: Sum of two sides > the third side

i.e. $4 + 7 > x \Rightarrow 11 > x \Rightarrow x < 11$

Condition II: The difference of two sides less than the third side.

i.e. $7 - 4 < x \Rightarrow 3 < x \Rightarrow x > 3$

Hence the possible value of x are $3 < x < 11$

i.e. $x < 3 < 11$

2. (a) Given triplet: (5, 8, 17)

$17^2 = 289$

$8^2 = 64$

$5^2 = 25$

$8^2 + 5^2 = 64 + 25 = 89$

Since $89 \neq 289$

$5^2 + 8^2 \neq 17^2$

Hence (5, 8, 17) is not Pythagorean triplet.

(b) Given triplet: (8, 15, 17)

$17^2 = 289$

$15^2 = 225$

$8^2 = 64$

$15^2 + 8^2 = 225 + 64 = 289$

$17^2 = 15^2 + 8^2$

Hence (8, 15, 17) is a Pythagorean triplet.

3. In ΔABC, $\angle B = 90°$

$AB^2 + BC^2 = AC^2$ (By Pythagoras property)

$(5)^2 + (x - 3)^2 = (x + 2)^2$

$\Rightarrow 25 + x^2 + 9 - 6x = x^2 + 4 + 4x$

$\Rightarrow -6x - 4x = 4 - 9 - 25$

$\Rightarrow -10x = -30$

$\Rightarrow x = 3$

Hence, the required value of $x = 3$

4. In ΔABD,

$AB + BD > AD$ ...(i)

(Sum of two sides of a triangle is greater than the third side)

Similarly, In ΔADC, we have

$AC + DC > AD$ ...(ii)

Adding (i) and (ii), we have

$AB + BD + AC + DC > 2AD$

$\Rightarrow AB + (BD + DC) + AC > 2AD$

$\Rightarrow AB + BC + AC > 2AD$

Hence, proved.

5. AC and BD are the diagonals of a rhombus ABCD.

Since the diagonals of a rhombus bisect at the right angle.

$AC = 40$ cm

$AO = \frac{40}{2} = 20$ cm

$BD = 42$ cm

$OB = \frac{42}{2} = 21$ cm

In right angled triangle AOB, we have

$AO^2 + OB^2 = AB^2$

$\Rightarrow 20^2 + 21^2 = AB^2$

$\Rightarrow 400 + 441 = AB^2$

$\Rightarrow 841 = AB^2$

$\Rightarrow AB = \sqrt{841} = 29$ cm.

Perimeter of the rhombus = 4 × side = 4 × 29 = 116 cm

Hence, the required perimeter = 116 cm

6. Let the sides of the given triangle are 3x, 4x and 5x units.

For right angled triangle, we have

Square of the longer side = Sum of the square of the other two sides

$(5x)^2 = (3x)^2 + (4x)^2$

$\Rightarrow 25x^2 = 9x^2 + 16x^2$

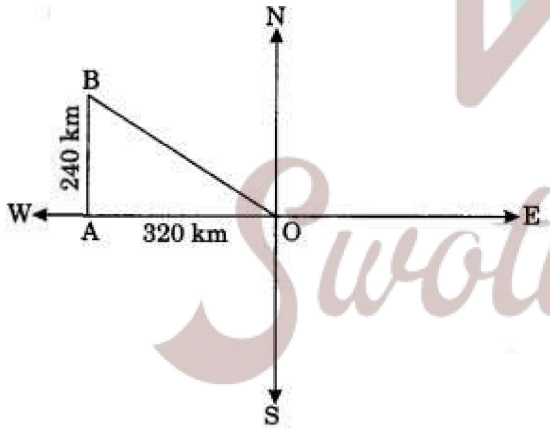
$\Rightarrow 25x^2 = 25x^2$

Hence, the given triangle is a right-angled.

Here, OA = 320 km

AB = 240 km

OB = ?



Clearly, ΔOBA is right angled triangle

$OB^2 = OA^2 + AB^2$ (By Pythagoras property)

$\Rightarrow OB^2 = 320^2 + 240^2$

$\Rightarrow OB^2 = 102400 + 57600$

$\Rightarrow OB^2 = 160000$

$\Rightarrow OB = \sqrt{160000} = 400$ km.

Hence the required shortest distance = 400 km.

Long Answer :

1. Here, l || m

$\angle c = 110°$ (Corresponding angles)

$\angle c + \angle a = 180°$ (Linear pair)

$\Rightarrow 110° + \angle a = 180°$

$\Rightarrow \angle a = 180° - 110° = 70°$

Now $\angle b = 40° + \angle a$ (Exterior angle of a triangle)

$\Rightarrow \angle b = 40° + 70° = 110°$

Hence, the values of unknown angles are a = 70° and b = 110°

2. (i) In ΔADC, we have

$\angle c + 60° + 70° = 180°$ (Angle sum property)

$\Rightarrow \angle c + 130° = 180°$

$\Rightarrow \angle c = 180° - 130° = 50°$

$\angle c + \angle b = 180°$ (Linear pair)

$\Rightarrow 50° + \angle b = 180°$

$\Rightarrow \angle b = 180° - 50° = 130°$

In ΔABD, we have

$\angle a + \angle b + 30° = 180°$ (Angle sum property)

$\Rightarrow \angle a + \angle 130° + 30° = 180°$

$\Rightarrow \angle a + 160° = 180°$

$\Rightarrow \angle a = 180° - 160° = 20°$

Hence, the required values are a = 20°, b = 130° and c = 50°

(ii) In ΔPQS, we have

$\angle a + 60° + 55° = 180°$(Angle sum property)

$\Rightarrow \angle a + 115° = 180°$

$\Rightarrow \angle a = 180° - 115°$

$\Rightarrow \angle a = 65°$

$\angle a + \angle b = 180°$ (Linear pair)

$\Rightarrow 65° + \angle b = 180°$

$\Rightarrow \angle b = 180° - 65° = 115°$

In ΔPSR, we have

$\angle b + \angle c + 40° = 180°$ (Angle sum property)

$\Rightarrow 115° + \angle c + 40° = 180°$

$\Rightarrow \angle c + 155° = 180°$

$\Rightarrow \angle c = 180° - 155° = 25°$

Hence, the required angles are a = 65°, b = 115° and c = 25°

3. Since I have three sides.

It is a triangle i.e. three-sided polygon.

Two angles are 15° and 60°.

Third angle = 180° – (15° + 60°)

= 180° – 75° (Angle sum property)

= 105°

which is greater than 90°.

Hence, it is an obtuse triangle.

4. By the rule of Pythagoras Theorem,

Pythagoras theorem states that for any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of square on the legs.

In the above figure RQ is the hypotenuse,

$15^2 = 12^2 + a^2$

$225 = 144 + a^2$

By transposing 144 from RHS to LHS it becomes – 144

$a^2 = 225 - 144$

$a^2 = 81$

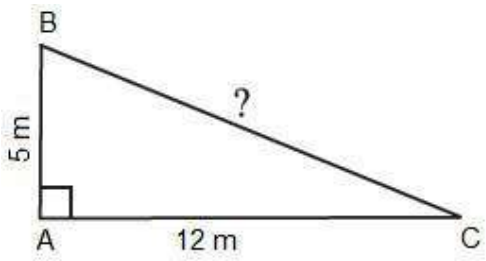
$a = \sqrt{81}$

a = 9 m

Hence, the length of a = 9 m.

5. Let ABC is the triangle and B is the point where tree is broken at the height 5 m from the ground.

Tree top touches the ground at a distance of AC = 12 m from the base of the tree,



By observing the figure we came to conclude that right angle triangle is formed at A.

From the rule of Pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$BC^2 = 52 + 122$

$BC^2 = 25 + 144$

$BC^2 = 169$

$BC = \sqrt{169}$

BC = 13 m

Then, the original height of the tree = AB + BC

= 5 + 13

= 18 m

Assertion and Reason Answers :

1) a) both Assertion and reason are correct and reason is correct explanation for Assertion
2) b) both Assertion and reason are correct but reason is not correct explanation for Assertion.