MATHEMATICS

Questions

Multiple Choice questions-

1. If in triangles ABC and DEF, $\frac{AB}{EF} = \frac{AC}{DE}$, then they will be similar when

(a) ∠A = ∠D

(b) ∠A = ∠E

(c) ∠B = ∠E

(d) ∠C = ∠F

2.A square and a rhombus are always

(a) similar

(b) congruent

(c) similar but not congruent

(d) neither similar nor congruent

3. If ΔABC ~ ΔDEF and EF = $\frac{1}{3}$ BC, then ar(ΔABC):(ΔDEF) is

(a) 3 : 1.

(b) 1 : 3.

(c) 1 : 9.

(d) 9 : 1.

4. If a triangle and a parallelogram are on the same base and between same parallels, then what is the ratio of the area of the triangle to the area of parallelogram?

(a) 1 : 2

(b) 3 : 2

(c) 1 : 3

(d) 4 : 1

5. D and E are respectively the points on the sides AB and AC of a triangle ABC such that

AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE || BC. Then, length of DE (in cm) is

(a) 2.5

(b) 3

(c) 5

(d) 6

6. Which geometric figures are always similar?

(a) Circles

(b) Circles and all regular polygons

(c) Circles and triangles

(d) Regular

7. ΔABC ~ ΔPQR, ∠B = 50° and ∠C = 70° then ∠P is equal to

(a) 50°

(b) 60°

(c) 40°

(d) 70°

8. In triangle DEF,GH is a line parallel to EF cutting DE in G and and DF in H. If DE = 16.5, DH = 5, HF = 6 then GE = ?

(a) 9

(b) 10

(c) 7.5

(d) 8

9. In a rectangle Length = 8 cm, Breadth = 6 cm. Then its diagonal = …

(a) 9 cm

(b) 14 cm

(c) 10 cm

(d) 12 cm

10. In triangle ABC ,DE || BC AD = 3 cm, DB = 8 cm AC = 22 cm. At what distance from A does the line DE cut AC?

(a) 6

(b) 4

(c) 10

(d) 5

Very Short Questions:

1. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

2. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm, and PB = 4 cm. Is AB || QR? Give reason.

3. If ΔABC ~ ΔQRP, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then find the length of PR.

4. If it is given that ΔABC ~ ΔPQR with $\frac{BC}{QR} = \frac{1}{3}$, then find $\frac{ar(\Delta PQR)}{ar(\Delta ABC)}$

5. ΔDEF ~ ΔABC, if DE : AB = 2 : 3 and ar(ΔDEF) is equal to 44 square units. Find the area (ΔABC).

6. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle? Give reason.

7. In triangles PQR and TSM, ∠P = 55°, ∠Q = 25°, ∠M = 100°, and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

8. If ABC and DEF are similar triangles such that ∠A = 47° and ∠E = 63°, then the measures of ∠C = 70°. Is it true? Give reason.

9. Let ΔABC ~ ΔDEF and their areas be respectively 64 cm^2 and 121 cm2. If EF = 15.4 cm, find BC.

10. ABC is an isosceles triangle right-angled at C. Prove that $AB^2 = 2AC^2$.

Short Questions :

1. In Fig. 7.10, DE || BC. If AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, find the value of x.

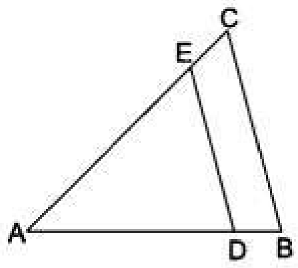


Fig. 7.10

2. E and F are points on the sides PQ and PR respectively of a ΔPQR. Show that EF ||QR if PQ = 1.28 cm, PR= 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.

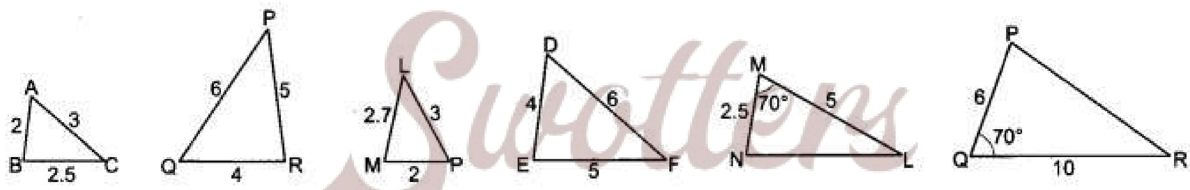
3. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

4. In Fig. 7.13, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$

5. In Fig. 7.14, DE || OQ and DF || OR Show that EF || QR.

6. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side.

7. State which pairs of triangles in the following figures are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form.



8. In Fig. 7.17, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and AB = 5cm. Find the value of DC.

9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

10. S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Long Questions :

1. Using Basic Proportionality Theorem, prove that a line drawn through the mid-

point of one side of a triangle parallel to another side bisects the third side.

2. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

3. If AD and PM are medians of triangles ABC and PQR respectively, where ΔABC ~ ΔPQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

4. In Fig. 7.37, ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC.

5. Prove that the area of an equilateral triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.

6. If the areas of two similar triangles are equal, prove that they are congruent.

7. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

8. In Fig. 7.41,0 is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

9. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD (see Fig. 7.42). Prove that $2AB^2 = 2AC^2 + BC^2$

10. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Case Study Questions:

1. Rahul is studying in X Standard. He is making a kite to fly it on a Sunday. Few questions came to his mind while making the kite. Give answers to his questions by looking at the figure.

i. Rahul tied the sticks at what angles to each other?

   a. 30º
   b. 60º
   c. 90º
   d. 60º

ii. Which is the correct similarity criteria applicable for smaller triangles at the upper part of this kite?

   a. RHS
   b. SAS
   c. SSA
   d. AAS

iii. Sides of two similar triangles are in the ratio 4:9. Corresponding medians of these triangles are in the ratio:

   a. 2 : 3
   b. 4 : 9
   c. 81 : 16
   d. 16 : 81

iv. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle. This theorem is called.

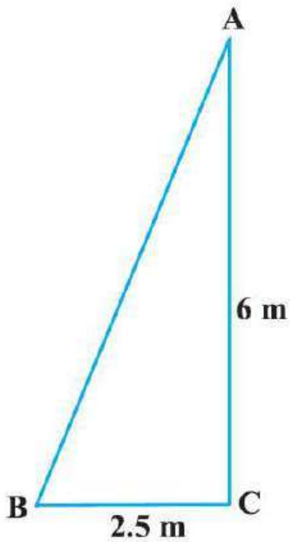
   a. Pythagoras theorem
   b. Thales theorem

c. The converse of Thales theorem
d. The converse of Pythagoras theorem

v. What is the area of the kite, formed by two perpendicular sticks of length 6cm and 8cm?

a. 48 cm^2
b. 14 cm^2
c. 24 cm^2
d. 96 cm^2

2. There is some fire incident in the house. The fireman is trying to enter the house from the window as the main door is locked. The window is 6m above the ground. He places a ladder against the wall such that its foot is at a distance of 2.5m from the wall and its top reaches the window.



i. Here, ________ be the ladder and ________ be the wall with the window.

a. CA, AB
b. AB, AC
c. AC, BC
d. AB, BC

ii. We will apply Pythagoras Theorem to find length of the ladder. It is:

a. $AB^2 = BC^2 - CA^2$
b. $CA^2 = BC^2 + AB^2$
c. $BC^2 = AB^2 + CA^2$

d. $AB^2 = BC^2 + CA^2$

iii. The length of the ladder is ________.

a. 4.5m
b. 2.5m
c. 6.5m
d. 5.5m

iv. What would be the length of the ladder if it is placed 6m away from the wall and the window is 8m above the ground?

a. 12m
b. 10m
c. 14m
d. 8m

v. How far should the ladder be placed if the fireman gets a 9m long ladder?

a. 6.7m (approx.)
b. 7.7m (approx.)
c. 5.7m (approx.)
d. 4.7m (approx.)

Assertion Reason Questions-

1. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

Reason: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. **Directions:** In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

a. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

b. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

c. Assertion (A) is true but reason (R) is false.

d. Assertion (A) is false but reason (R) is true.

Assertion: If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles.

Reason: All congruent triangles are similar but the similar triangles need not be congruent.

Answer Key-

Multiple Choice questions-

1. (b) ∠A = ∠E
2. (d) neither similar nor congruent
3. (c) 1 : 9.
4. (a) 1 : 2
5. (b) 3
6. (b) Circles and all regular polygons
7. (b) 60°
8. (a) 9
9. (c) 10cm
10. (a) 6

Very Short Answer :

1. Since the perimeters and two sides are proportional

∴ The third side is proportional to the corresponding third side.

i.e., The two triangles will be similar by SSS criterion.

2.

Yes, $\frac{PA}{AQ} = \frac{5}{12.5-5} = \frac{5}{7.5} = \frac{2}{3}$

$\frac{PB}{BR} = \frac{4}{6} = \frac{2}{3}$

Since $\frac{PA}{AQ} = \frac{PB}{BR} = \frac{2}{3}$

∴ $AB \parallel QR$

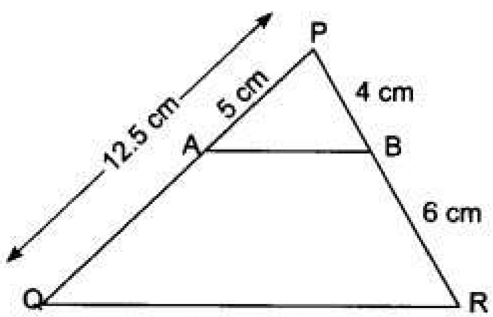


Fig. 7.4

3.

$$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta QRP} = \frac{BC^2}{RP^2} \Rightarrow \frac{9}{4} = \frac{(15)^2}{RP^2}$$

$$\therefore \quad RP^2 = \frac{225\times 4}{9} = \frac{900}{9} = 100 \Rightarrow RP = 10\text{ cm}$$

4.

$$\frac{BC}{QR} = \frac{1}{3} \quad \text{(Given)}$$

$$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{(QR)^2}{(BC)^2}$$

[∵ Ratio of area of similar triangles is equal to the ratio of square of its corresponding sides]

$$= \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9:1$$

5.

Since $\Delta DEF \sim \Delta ABC$

$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{(DE)^2}{(AB)^2}$$

[∵ Ratio of area of similar triangles is equal to the ratio of square of its corresponding sides]

$$\frac{44}{ar(\Delta ABC)} = \left(\frac{2}{3}\right)^2 \Rightarrow ar(\Delta ABC) = \frac{44\times 9}{4}$$

So, $ar(\Delta ABC) = 99\text{ cm}^2$

6. Here, $12^2 + 16^2 = 144 + 256 = 400 \neq 182$

∴ The given triangle is not a right triangle.

7. Şince, ∠R = 180° – (∠P + ∠Q)

= 180° – (55° + 25°) = 100° = ∠M

∠Q = ∠S = 25° (Given)

ΔQPR ~ ΔSTM

i.e., . ΔQPR is not similar to ΔTSM.

8. Since ΔABC ~ ΔDEF

∴ ∠A = ∠D = 47°

∠B = ∠E = 63°

∴ ∠C = 180° – (∠A + ∠B) = 180° – (47° + 63°) = 70°

∴ Given statement is true.

9.

We have, $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \frac{BC^2}{EF^2}$ = (as $\Delta ABC \sim \Delta DEF$)

$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{EF^2} \quad \Rightarrow \quad \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \quad \frac{BC}{15.4} = \frac{8}{11}$

$\therefore \quad BC = \frac{8}{11} \times 15.4 = 11.2 \text{ cm}$

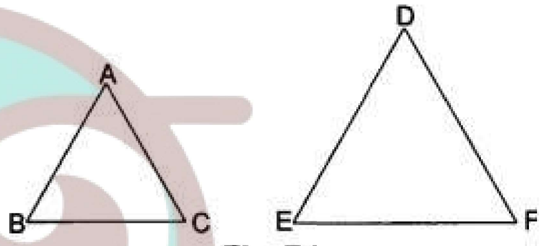


Fig. 7.5

10. ΔABC is right-angled at C.

∴ $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

⇒ $AB^2 = AC^2 + AC^2$

[∵ AC = BC]

⇒ $AB^2 = 2AC^2$

Short Answer :

1. In ΔABC, we have

DE || BC,

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

⇒ x(x – 1) = (x – 2) (x + 2)

⇒ $x^2 - x = x^2 - 4$

⇒ x = 4

2.

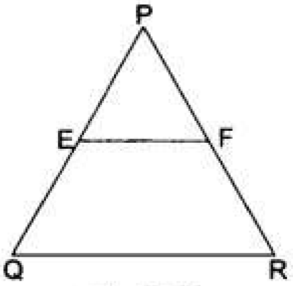


Fig. 7.11

We have, PQ = 1.28 cm, PR = 2.56 cm

PE = 0.18 cm, PF = 0.36 cm

Now, EQ = PQ-PE = 1.28 – 0.18 = 1.10 cm and

FR = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$

and, $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ $\quad \therefore \quad \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF || QR [By the converse of Basic Proportionality Theorem]

3. Let AB be a vertical pole of length 6m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

Now, in ΔABC and ΔDEF, we have

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the Sun)

$\therefore$ $\Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{6}{h} = \frac{4}{28}$ (Let $DE = h$)

$\Rightarrow \frac{6}{h} = \frac{1}{7}$ $\Rightarrow h = 42$

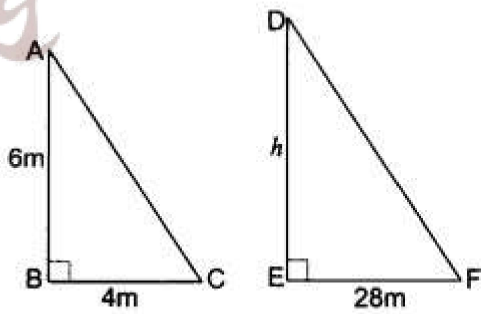


Fig. 7.12

h = 42 Hence, height of tower, DE = 42m

4.

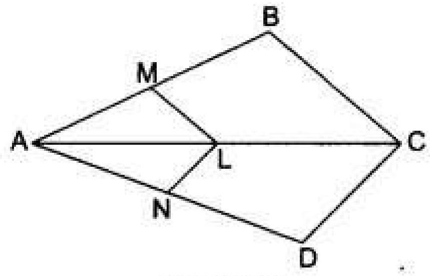


Fig. 7.13

Firstly, in ΔABC, we have

LM || CB (Given)

Therefore, by Basic Proportionality Theorem, we have

$$\frac{AM}{AB} = \frac{AL}{AC} \quad ...(i)$$

Again, in ΔACD, we have

$LN \parallel CD$ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AN}{AD} = \frac{AL}{AC} \quad ...(ii)$$

Now, from (i) and (ii), we have $\frac{AM}{AB} = \frac{AN}{AD}$.

5. In ΔPOQ, we have

DE || OQ (Given)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{PE}{EQ} = \frac{PD}{DO} \quad ...(i)$$

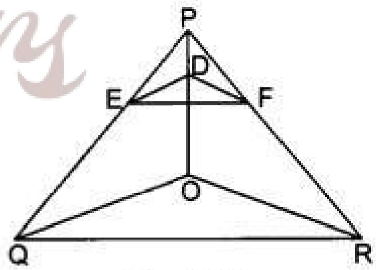


Fig. 7.14

Similarly, in ΔPOR, we have

$DF \parallel OR$ (Given)

$$\frac{PD}{DO} = \frac{PF}{FR} \quad ...(ii)$$

Now, from (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad \Rightarrow \quad EF \parallel QR$$

[Applying the converse of Basic Proportionality Theorem in ΔPQR]

6.

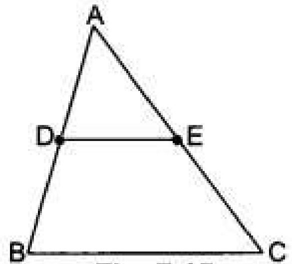


Fig. 7.15

Given: ΔABC in which D and E are the mid-points of sides AB and AC respectively.

To prove: DE || BC

Proof: Since D and E are the mid-points of AB and AC respectively

∴ AD = DB and AE = EC

$$\Rightarrow \quad \frac{AD}{DB} = 1 \quad \text{and} \quad \frac{AE}{EC} = 1$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

DB EC Therefore, DE || BC (By the converse of Basic Proportionality Theorem)

7. (i) In ΔABC and ΔQRP, we have

$$\frac{BC}{RP} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$$

Hence, $\frac{AB}{QR} = \frac{AC}{QP} = \frac{BC}{RP}$

∴ $\Delta ABC \sim \Delta QRP$, by SSS criterion of similarity.

(ii) In ΔLMP and ΔFED, we have

$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}, \quad \frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}, \quad \frac{LM}{FE} = \frac{2.7}{5}$$

Hence, $\frac{LP}{FD} = \frac{MP}{ED} \neq \frac{LM}{FE}$

∴ ΔLMP is not similar to ΔFED.

(iii) In ΔNML and ΔPQR, we have

$$\angle M = \angle Q = 70°$$

Now, $\frac{MN}{QP} = \frac{2.5}{6} = \frac{5}{12}$ and $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

Hence, $\frac{MN}{QP} \neq \frac{ML}{QR}$

ΔNML is not similar to ΔPQR.

8.

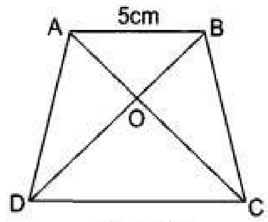


Fig. 7.17

In ΔAOB and ΔCOD, we have

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD}$ [Given]

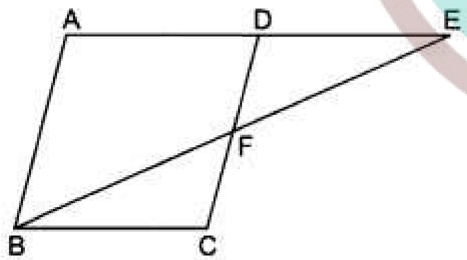
So, by SAS criterion of similarity, we have

$\Delta AOB \sim \Delta COD$

$\Rightarrow \quad \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC} \quad \Rightarrow \quad \frac{1}{2} = \frac{5}{DC}$ [$\because AB = 5$ cm]

⇒ DC = 10 cm.

9.



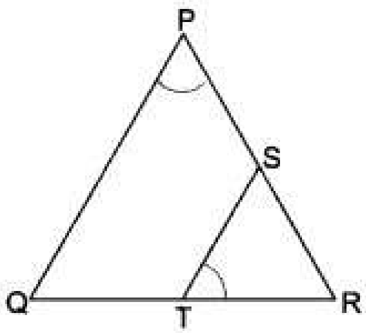
In ΔABE and ΔCFB, we have

∠AEB = ∠CBF (Alternate angles)

∠A = ∠C (Opposite angles of a parallelogram)

∴ ΔABE ~ ΔCFB (By AA criterion of similarity)

10.



In ΔRPQ and ΔRTS, we have

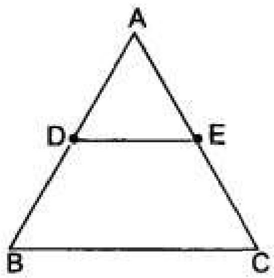
∠RPQ = ∠RTS (Given)

∠PRQ = ∠TRS = ∠R (Common)

∴ ΔRPQ ~ ΔRTS (By AA criterion of similarity)

Long Answer :

1.



Given: A ΔABC in which D is the mid-point of AB and DE is drawn parallel to BC, which meets AC at E.

To prove: AE = EC

Proof: In ΔABC, DE || BC

∴ By Basic Proportionality Theorem, we have

$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AAND}}{\mathrm{ANDC}}$ … (i)

Now, since D is the mid-point of AB

⇒ AD = BD … (ii)

From (i) and (ii), we have

$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AAND}}{\mathrm{ANDC}}$

$$\Rightarrow\ 1\ = \frac{AAND}{ANDC}$$

Hence, E is the mid-point of AC.

2. Given: ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at point O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw $OE \parallel AB$ *i.e.*, $OE \parallel DC$.

Proof: In ΔADC, we have $OE \parallel DC$ (Construction)

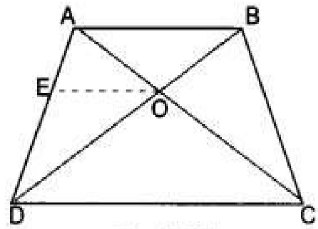


Fig. 7.35

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{AE}{ED} = \frac{AO}{CO} \quad ...(i)$$

Now, in ΔABD, we have $OE \parallel AB$ (Construction)

$\therefore$ By Basic Proportionality Theorem, we have

$$\frac{ED}{AE} = \frac{DO}{BO} \Rightarrow \frac{AE}{ED} = \frac{BO}{DO} \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$\frac{AO}{CO} = \frac{BO}{DO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

3. In ΔABD and ΔPQM we have

∠B = ∠Q (∵ ΔABC ~ ΔPQR) … (i)

$$\frac{AB}{PQ} = \frac{BC}{QR} \quad (\because \Delta ABC \sim \Delta PQR)$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad ...(ii)$$

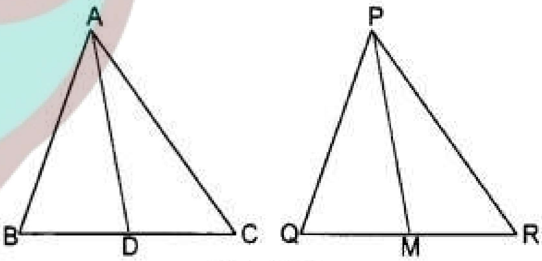


Fig. 7.36

[Since AD and PM are the medians of ΔABC and ΔPQR respectively]

From (*i*) and (*ii*), it is proved that

$\Delta ABD \sim \Delta PQM$ (By SAS criterion of similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

4. In ΔEDC and ΔEBA we have

∠1 = ∠2 [Alternate angles]

∠3 = ∠4 [Alternate angles]

∠CED = ∠AEB [Vertically opposite angles]

∴ ΔEDC ~ ΔEBA [By AA criterion of similarity]

$$\Rightarrow \quad \frac{ED}{EB}=\frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC}=\frac{EB}{EA} \quad \ldots(i)$$

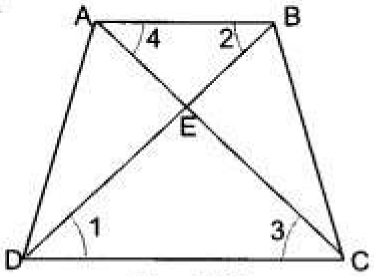


Fig. 7.37

It is given that $\Delta AED \sim \Delta BEC$

$$\therefore \quad \frac{ED}{EC}=\frac{EA}{EB}=\frac{AD}{BC} \quad \ldots(ii)$$

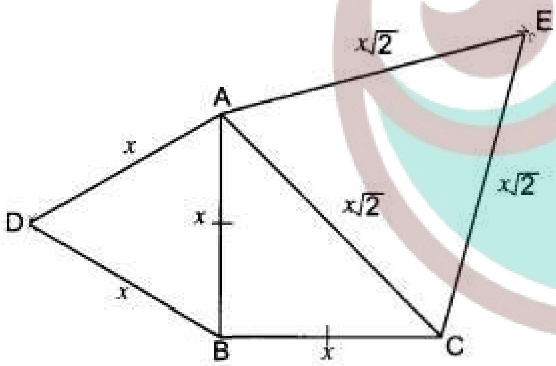
From (i) and (ii), we get

$$\frac{EB}{EA}=\frac{EA}{EB} \quad \Rightarrow \quad (EB)^2=(EA)^2 \quad \Rightarrow \quad EB=EA$$

Substituting $EB = EA$ in (ii), we get

$$\frac{EA}{EA}=\frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC}=1 \quad \Rightarrow \quad AD=BC$$

5.



Given: A ΔABC in which ∠ABC = 90° and AB = BC.

ΔABD and ΔCAE are equilateral triangles.

To Prove: ar(ΔABD) = $\frac{1}{2}$ × ar(ΔCAE)

Proof: Let AB = BC = x units.

∴ hyp. CA = $\sqrt{x^2} + \sqrt{x^2} = x\sqrt{2}$ units.

Each of the ABD and ΔCAE being equilateral has each angle equal to 60°.

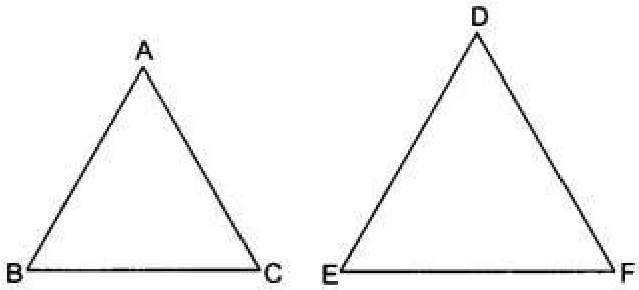
∴ ΔABD ~ ΔCAE

But, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABD)}{ar(\Delta CAE)}=\frac{AB^2}{CA^2}=\frac{x^2}{(x\sqrt{2})^2}=\frac{x^2}{2x^2}=\frac{1}{2}$$

Hence, $ar(\Delta ABD)=\frac{1}{2}\times ar(\Delta CAE)$

6.



Given: Two triangles ABC and DEF, such that

ΔABC ~ ΔDEF and area (ΔABC) = area (ΔDEF)

To prove: ΔABC ≅ ΔDEF

Proof: ΔABC ~ ΔDEF

⇒ ∠A = ∠D, ∠B = ∠E, ∠C = ∠F

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Now, $ar(\Delta ABC) = ar(\Delta DEF)$ (Given)

∴ $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$

and $\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)}$ ($\because \Delta ABC \sim \Delta DEF$)

From (*i*) and (*ii*), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

AB = DE, BC = EF, AC = DF

ΔABC ≅ ΔDEF (By SSS criterion of congruency)

7. Let ΔABC and ΔPQR be two similar triangles. AD and PM are the medians of ΔABC and ΔPQR respectively.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...(*i*)

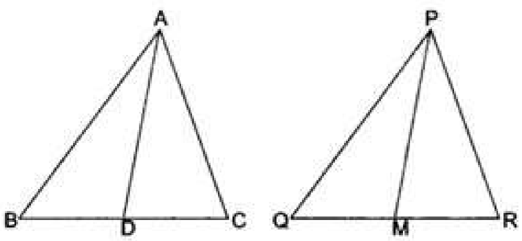


Fig. 7.40

In ΔABD and ΔPQM

$$\frac{AB}{PQ}=\frac{BD}{QM} \qquad \left(\because \quad \frac{AB}{PQ}=\frac{BC}{QR}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}\right)$$

and $\angle B = \angle Q$ $\quad (\because \quad \Delta ABC \sim \Delta PQR)$

Hence, $\Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$$\frac{AB}{PQ}=\frac{AD}{PM} \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\frac{AD^2}{PM^2}$$

8.

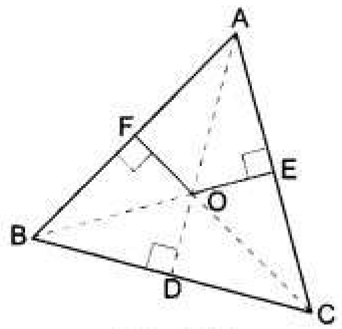


Fig. 7.41

Join OA, OB and OC.

(i) In right Δ's OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$ ... (i)

WHETHER2 = BD^2 + $FROM^2$ ... (ii)

and $C^2 = EC^2 + OE^2$

Adding (i), (ii) and (iii), we have

⇒ $0A^2 + OB^2 + OC^2 = AF^2 + BD^2 + EC^2 + OF^2 + FROM^2 + OE^2$

⇒ $0A^2 + OB^2 + OC^2 – IP^2 – OE^2 – OF^2 = AF^2 + BD^2 + EC^2$

(ii) We have, $OA^2 + OB^2 + OC^2$ – IP2 – $OE^2 – OF^2$ = AF2 + $BD^2 + EC^2$

⇒ $(OA^2 – OE^2) + (OB^2 – OF^2) – (OC^2 – IP^2) = AF^2 + BD^2 + EC^2$

⇒ $AE^2 + CD^2 + BF^2 = AP^2 + BD^2 + EC^2$

[Using Pythagoras Theorem in ΔAOE, ΔBOF and ΔCOD]

9.

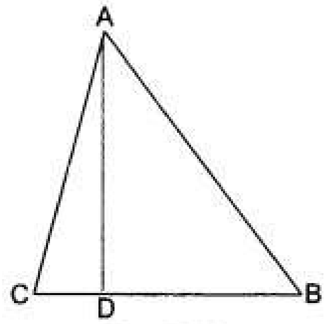


Fig. 7.42

We have, DB = 3CD

Now,

BC = BD + CD

⇒ BC = 3CD + CD = 4CD (Given DB = 3CD)

∴ CD = $\frac{1}{4}$ BC

and DB = 3CD = $\frac{1}{4}$BC

Now, in right-angled triangle ABD using Pythagoras Theorem we have

$AB^2 = AD^2 + DB^2$ … (i)

Again, in right-angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ … (ii)

Subtracting (ii) from (i), we have

$AB^2 – AM^2 = DB^2 – CD^2$

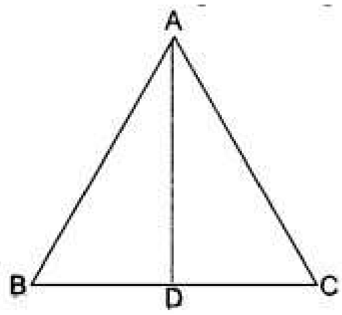
$$\Rightarrow \quad AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2$$

$$\Rightarrow \quad AB^2 - AC^2 = \frac{1}{2}BC^2$$

∴ $2AB^2 – 2AM^2 = BC^2$

⇒ $2AB^2 = 2AM^2 + BC^2$

10.



Let ABC be an equilateral triangle and let AD ⊥ BC.

∴ BD = DC

Now, in right-angled triangle ADB, we have

$AB^2 = AD^2 + BD^2$ [Using Pythagoras Theorem]

$$\Rightarrow \quad AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 \quad \Rightarrow \quad AB^2 = AD^2 + \frac{1}{4}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{AB^2}{4} \qquad [\because AB = BC]$$

$$\Rightarrow \quad AB^2 - \frac{AB^2}{4} = AD^2 \quad \Rightarrow \quad \frac{3AB^2}{4} = AD^2 \quad \Rightarrow \quad 3AB^2 = 4AD^2$$

Case Study Answers:

1. Answer :

i	c	90º
ii	b	SAS
iii	b	4 : 9
iv	d	The converse of Pythagoras theorem
v	a	48 cm^2

2. Answer :

i	b	AB, AC
ii	d	$AB^2 = BC^2 + CA^2$
iii	c	6.5m
iv	b	10m
v	a	6.7m (approx)

Assertion Reason Answer-

1. (d) Assertion (A) is false but reason (R) is true
2. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).