MATHEMATICS

Chapter 7: Congruence of Triangle



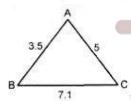
Important Questions

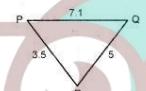
Multiple Choice Questions:

Question 1. An angle is of 50° then its congruent angle is of:

- (a) 40°
- (b) 60°
- (c) 50°
- (d) None of these

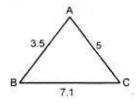
Question 2. Given two triangles are congruent then we can write:

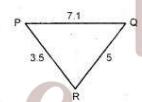




- (a) $\triangle ABC \equiv \triangle PQR$
- (b) $\triangle ABC \equiv \triangle RPQ$
- (c) $\triangle ABC \equiv \triangle QRP$
- (d) none of these

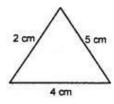
Question 3. In the given figure, lengths of the sides of the triangles are given. Which pair of triangle are congruent?

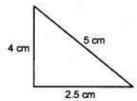




- (a) $\triangle ABC \equiv \triangle PQR$
- (b) $\triangle BCA \equiv \triangle PQR$
- (c) $\triangle ABC \equiv \triangle QRP$
- (d) none of these

Question 4. Are the following triangles congruent?

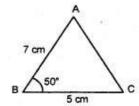


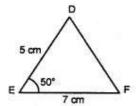


- (a) yes
- (b) no

(c) none of these

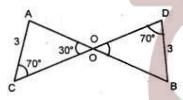
Question 5. Are the following triangles congruent?





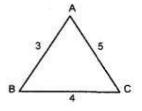
- (a) yes
- (b) no
- (c) none of these

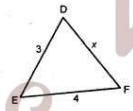
Question 6. In the given figure, say congruency of two triangles.



- (a) ΔAOC U ΔBOD
- (b) ΔAOC ≠ ΔBOD
- (c) ΔΑΟC U ΔΟΒD
- (d) none of these

Question 7. Given triangles are congruent, then what is the measurement of x?





- (a) 3
- (b) 4
- (c)5
- (d) none of these

Question 8. Given below are measurements of some parts of two triangles. Write the result in symbolic form.

In \triangle ABC, \angle B = 90°, AC 8 cm AB = 3 cm and

 Δ PQR, \angle P = 90°, PR = 3 cm QR = 8 cm

- (a) $\triangle ABC \equiv \triangle RPQ$
- (b) $\triangle ABC \equiv \triangle PQR$
- (c) $\triangle ABC \equiv \triangle RPQ$

(d) none of these

Question 9. Given below are measurements of some parts of two triangles. Write the result in symbolic form if they are congruent.

In ΔABC,

$$\angle A = 90^{\circ}$$
, AC = 5 cm, BC = 9 cm

In APQR,

$$\angle P = 90^{\circ}$$
, PR = 3 cm QR = 8 cm

- (a) are congruent
- (b) are not congruent

Question 10. \triangle ABC and \triangle PQR are congruent under the correspondence: ABC \leftrightarrow RPQ, then the part of \triangle ABC that correspond to PQ is

- (a) AC
- (b) AB
- (c) BC
- (d) None of These

Question 11. \triangle ABC is right triangle in which \angle A = 90° and AB = AC. The values of \angle B and \angle C will be

(a)
$$\angle B = \angle C = 30^{\circ}$$

(b)
$$\angle B = \angle C = 50^{\circ}$$

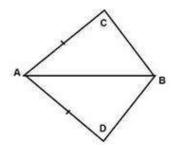
(c)
$$\angle B = \angle C = 45^{\circ}$$

(d)
$$\angle B = \angle C = 60^{\circ}$$

Question 12. Two students drew a line segment each. What is the condition for them to be congruent?

- (a) They should be drawn with a scale.
- (b) They should be drawn on the same sheet of paper.
- (c) They should have different lengths.
- (d) They should have the same length.

Question 13. In the quadrilateral ABCD, AC = AD and AB bisect $\angle A$ and $\triangle ABC \cong \triangle ABD$. The relation between BC and BD is



- (a) BC < BD
- (b) BC > BD
- (c) BC = BD
- (d) None of these

Question 14. A triangle in which all three sides are of equal lengths is called

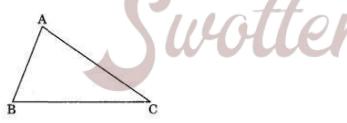
- (a) Isosceles
- (b) Equilateral
- (c) Scalene
- (d) None of these

Question 15. In \triangle ABC and \triangle PQR, AB = 4 cm, BC = 5 cm, AC = 6 cm and PQ = 4 cm. QR = 5 cm. PR = 6 cm. then which of the following is true?

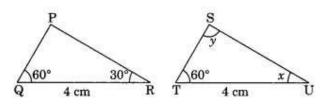
- (a) $\triangle ABC \cong \triangle QRP$
- (b) $\triangle ABC \cong \triangle PQR$
- (c) $\triangle ABC \cong \triangle RQP$
- (d) None of these

Very Short Questions:

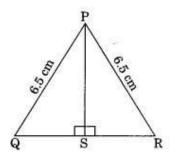
- 1. In the given figure, name
 - (a) the side opposite to vertex A
 - (b) the vertex opposite A to side AB
 - (c) the angle opposite to side AC
 - (d) the angle made by the sides CB and CA



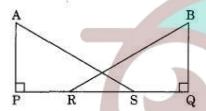
- **2.** Examine whether the given triangles are congruent or not.
- 3. In the given congruent triangles under ASA, find the value of x and y, $\Delta PQR = \Delta STU$.



4. In the following figure, show that $\triangle PSQ = \triangle PSR$.



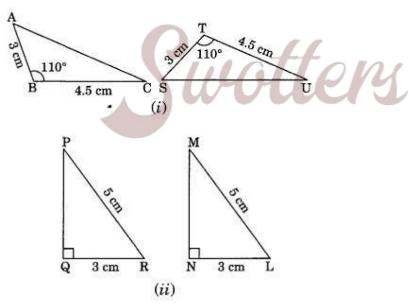
- **5.** Can two equilateral triangles always be congruent? Give reasons.
- **6.** In the given figure, AP = BQ, PR = QS. Show that \triangle APS = \triangle BQR



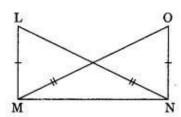
- 7. Without drawing the figures of the triangles, write all six pairs of equal measures in each of the following pairs of congruent triangles.
 - (i) $\triangle ABC = ADEF$
 - (ii) $\Delta XYZ = \Delta MLN$
- **8.** Lengths of two sides of an isosceles triangle are 5 cm and 8 cm, find the perimeter of the triangle.

Short Questions:

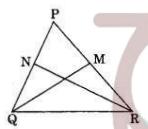
1. Write the rule of congruence in the following pairs of congruent triangles.



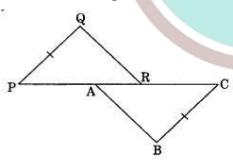
2. In the given figure, state the rule of congruence followed by congruent triangles LMN and ONM.



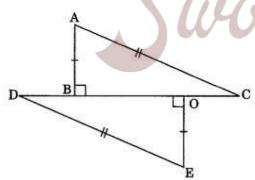
- **3.** In the given figure, PQR is a triangle in which PQ = PR. QM and RN are the medians of the triangle. Prove that
 - (i) $\triangle NQR = \triangle MRQ$
 - (ii) QM = RN
 - (iii) $\Delta PMQ = \Delta PNR$



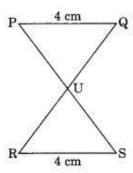
4. In the given figure, PQ = CB, PA = CR, \angle P = \angle C. Is \triangle QPR = \triangle BCA? If yes, state the criterion of congruence.



5. In the given figure, state whether $\triangle ABC = \triangle EOD$ or not. If yes, state the criterion of congruence.

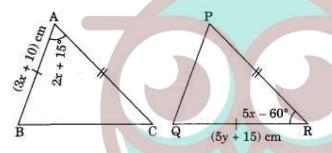


6. In the given figure, PQ | | RS and PQ = RS. Prove that Δ PUQ = Δ SUR.

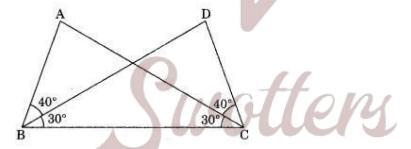


Long Questions:

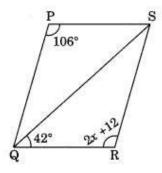
1. In the given figure $\triangle BAC = \triangle QRP$ by SAS criterion of congruence. Find the value of x and y.



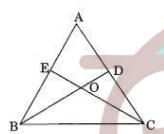
- 2. Observe the figure and state the three pairs of equal parts in triangles ABC and DCB.
 - (i) Is $\triangle ABC = \triangle DCB$? Why?
 - (ii) Is AB = DC? Why?
 - (iii) Is AC = DB? Why?



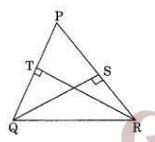
- 3. In the given figure, $\triangle QPS = \triangle SRQ$. Find each value.
 - (a) x
 - (b) ∠PQS
 - (c) ∠PSR



4. In \triangle ABC, medians BD and CE are equal and intersect each other at O. Prove that \triangle ABC is an isosceles triangle.



- **5.** Prove that the lengths of altitudes drawn to equal sides of an isosceles triangle are also equal.
 - (i) $\angle TRQ = \angle SQR$?
 - (ii) If \angle TRQ = 30°, find the base angles of the \triangle PQR.
 - (iii) Is ΔPQR an equilateral triangle?



ANSWER KEY

Multiple Choice questions:

- **1.** (c) 50°
- 2. (b) $\triangle ABC \equiv \triangle RPQ$
- 3. (a) $\triangle ABC \equiv \triangle PQR$
- **4.** (b) no
- **5.** (b) no
- **6.** (a) ΔΑΟC U ΔΒΟD
- **7.** (c) 5
- 8. (a) $\triangle ABC \equiv \triangle RPQ$

- 9. (b) are not congruent
- **10.** (c) BC
- **11.** (c) $\angle B = \angle C = 45^{\circ}$
- 12. (d) They should have the same length.
- **13.** (c) BC = BD
- **14.** (b) Equilateral
- **15.** (b) $\triangle ABC \cong \triangle PQR$

Very Short Answer:

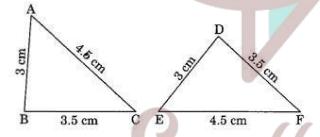
- 1. (a) The side opposite to vertex A is BC.
 - (b) The vertex opposite to side AB is C.
 - (c) The angle opposite to side AB is ∠ACB.
 - (d) The angle made by the sides CB and CA is ∠ACB.
- 2. Here,

$$AB = DE = 3 \text{ cm}$$

$$BC = DF = 3.5 \text{ cm}$$

$$AC = EF = 4.5 \text{ cm}$$

 $\triangle ABC = \triangle EDF$ (By SSS rule)



So, \triangle ABC and \triangle EDF are congruent.

3. Given: $\triangle PQR = \triangle STU$ (By ASA rule)

$$\angle Q = \angle T = 60^{\circ}$$
 (given)

$$\overline{QR} = \overline{TU} = 4 \text{ cm (given)}$$

$$\angle x = 30^{\circ}$$
 (for ASA rule)

Now in ΔSTU,

$$\angle$$
S + \angle T + \angle U = 180° (Angle sum property)

$$\angle$$
y + 60° + \angle x = 180°

$$\angle$$
y + 60° + 30° = 180°

$$\angle$$
y + 90° = 180°

$$\angle y = 180^{\circ} - 90^{\circ} = 90^{\circ}$$

Hence, $x = 30^{\circ}$ and $y = 90^{\circ}$.

4. In $\triangle PSQ$ and $\triangle PSR$

$$\overline{PQ} = \overline{PR} = 6.5 \text{ cm (Given)}$$

$$\overline{PS} = \overline{PS}$$
 (Common)

$$\angle PSQ = \angle PSR = 90^{\circ}$$
 (Given)

$$\Delta PSQ = \Delta PSR$$
 (By RHS rule)

5. No, any two equilateral triangles are not always congruent.

Reason: Each angle of an equilateral triangle is 60° but their corresponding sides cannot always be the same.

6. In $\triangle APS$ and $\triangle BQR$

$$PR + RS = QS + RS$$
 (Adding RS to both sides)

$$PS = QR$$

$$\angle APS = \angle BQR = 90^{\circ}$$
 (Given)

$$\triangle APS = \triangle BQR$$
 (by SAS rule)

7. (i) Given: $\triangle ABC = \triangle DEF$

$$AC = DF$$

$$\angle A = \angle D$$
, $\angle B = \angle E$ and $\angle C = \angle F$

(ii) Given
$$\Delta XYZ = \Delta MLN$$

$$YZ = LN$$

$$XZ = MN$$

$$\angle X = \angle M$$
, $\angle Y = \angle L$ and $\angle Z = \angle N$

8. Since the lengths of any two sides of an isosceles triangle are equal, then

Case I: The three sides of the triangle are 5 cm, 5 cm and 8 cm.

Perimeter of the triangle = 5 cm + 5 cm + 8 cm = 18 cm

Case II: The three sides of the triangle are 5 cm, 8 cm and 8 cm.

Perimeter of the triangle = 5 cm + 8 cm + 8 cm = 21 cm

Hence, the required perimeter is 18 cm or 21 cm.

Short Answer:

$$BC = TU = 4.5 \text{ cm}$$

$$\triangle ABC = \triangle STU$$
 (By SAS rule)

(ii) Here
$$\angle PQR = \angle MNL = 90^{\circ}$$

$$QR = NL = 3 cm$$

$$\Delta PQR = \Delta MNL$$
 (By RHS rule)

2. In \triangle LMN and \triangle ONM

$$LM = ON$$

$$LN = OM$$

$$MN = NM$$

$$\Delta$$
LMN = Δ ONM

3. $\triangle PQR$ is an isosceles triangle. [: PQ = PR]

$$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}PR$$

$$\Rightarrow$$
 NQ = MR and PN = PM

$$\angle$$
NQR = \angle MRQ (Angles opposite to equal sides)

$$\Delta$$
NQR = Δ MRQ (By SAS rule)

$$\angle P = \angle P$$
 (Common)

$$\Delta$$
PMQ = Δ PNR (By SAS rule)

4.
$$PQ = CB, PA = CR$$

and
$$\angle P = \angle C$$

$$\angle QPR = \angle BCA \text{ (Given)}$$

$$PA + AR = CR + AR$$
 (Adding AR to both sides)

or
$$PR = CA$$

$$\Delta QPR = \Delta BCA$$
 (By SAS rule)

5. In \triangle ABC and \triangle EOD

$$AB = OE$$

$$\angle ABC = \angle EOD = 90^{\circ}$$

$$\triangle ABC = \triangle EOD$$

Hence,
$$\triangle ABC = \triangle EOD$$

RHS is the criterion of congruence.

6. In $\triangle PUQ$ and $\triangle SUR$

$$PQ = SR = 4 cm$$

$$\angle$$
UPQ = \angle USR (Alternate interior angles)

$$\Delta PUQ = \Delta SUR$$
 (By ASA rule)

Long Answer:

1. Given:
$$\triangle BAC = \triangle QRP$$
 (By SAS rule)

So,
$$BA = QR$$

$$\Rightarrow$$
 3x + 10 = 5y + 15(i)

$$\angle BAC = \angle QRP$$

$$\Rightarrow$$
 2x + 15° = 5x - 60°(ii)

From eq. (ii), we have

$$2x + 15 = 5x - 60$$

$$\Rightarrow$$
 2x - 5x = -15 - 60

$$\Rightarrow$$
 -3x = -75

$$\Rightarrow$$
 x = 25

From eq. (i), we have

$$3x + 10 = 5y + 15$$

$$\Rightarrow$$
 3 × 25 + 10 = 5y + 15

$$\Rightarrow$$
 75 + 10 = 5y + 15

$$\Rightarrow$$
 85 = 5y + 15

$$\Rightarrow$$
 85 – 15 = 5y

$$\Rightarrow$$
 70 = 5y

$$\Rightarrow$$
 y = 14

Hence, the required values of x and y are 25 and 14 respectively.

2. (i) In \triangle ABC and \triangle DCB

$$\angle ABC = \angle DCB = 70^{\circ} (40^{\circ} + 30^{\circ} = 70^{\circ}) (Given)$$

$$\angle ACB = \angle DCB = 30^{\circ}$$
 (Given)

$$\triangle ABC = \triangle DCB$$
 (By ASA rule)

3. (a) $\triangle QPS = \triangle SRQ$

$$\angle QPS = \angle SRQ$$
 (Congruent part of congruent triangles)

$$106 = 2x + 12$$

$$\Rightarrow$$
 106 – 12 = 2x

$$\Rightarrow$$
 94 = 2x

$$\Rightarrow$$
 x = 47

$$\angle QRS = 2 \times 47 + 12 = 94 + 12 = 106^{\circ}$$

So, PQRS is a parallelogram.

$$\angle$$
QSR = 180° - (42° + 106°) = 180° - 148° = 32°

(b) ∠PQS = 32° (alternate interior angles)

(c)
$$\angle PSQ = 180^{\circ} - (\angle QPS + \angle PQS) = 180^{\circ} - (106^{\circ} + 32^{\circ}) = 180^{\circ} - 138^{\circ} = 42^{\circ}$$

$$\angle PSR = 32^{\circ} + 42^{\circ} = 74^{\circ}$$

4. We know that the medians of a triangle intersect each other in the ratio 2 : 1.

$$\frac{2}{3}$$
BD = $\frac{2}{3}$ CE

$$\Rightarrow$$
 OB = OC

$$\frac{1}{3}BD = \frac{1}{3}CE$$

$$\Rightarrow$$
 OE = OD

In $\triangle BOE$ and $\triangle COD$,

OB = OC

OE = OD

 $\angle BOE = \angle COD$ (Vertically opposite angles)

 $\Delta BOE = \Delta COD$ (By SAS rule)

BE = CD (Congruent parts of congruent triangles)

2BE = 2CD

 \Rightarrow AB = AC

Hence ΔABC is an isosceles triangle.

5. In \triangle QTR and \triangle RSQ

$$\angle QTR = \angle RSQ = 90^{\circ}$$
 (Given)

 $\angle TQR = \angle SRQ$ (Base angle of an isosceles triangle)

 $\angle QRT = \angle RQS$ (Remaining third angles)

QR = QR (Common)

 $\Delta QTR = \Delta RSQ$ (By ASA rule)

QS = RT (Congruent parts of congruent triangles)

Hence proved.

(i)
$$\angle$$
TRQ = \angle SQR (Congruent parts of congruent triangles)

(ii) In ΔQTR,

$$\angle$$
TRQ = 30° (Given)

$$\angle QTR + \angle TQR + \angle QRT = 180^{\circ}$$
 (Angle sum property)

$$\Rightarrow$$
 90° + \angle TQR + 30° = 180°

$$\Rightarrow$$
 120° + \angle TQR = 180°

$$\Rightarrow$$
 \angle TQR = 180° - 120° = 60°

$$\Rightarrow \angle TQR = \angle SRQ = 60^{\circ}$$

Each base angle = 60°

(iii) In ΔPQR,

$$\angle P + \angle Q + \angle R = 180^{\circ}$$
 (Angle sum property)

$$\Rightarrow$$
 $\angle P + 60^{\circ} + 60^{\circ} = 180^{\circ}$ (From ii)

$$\Rightarrow \angle P + 120^{\circ} = 180^{\circ}$$

$$\Rightarrow$$
 \angle P = 180° - 120° = 60°

Hence, ΔPQR is an equilateral triangle.