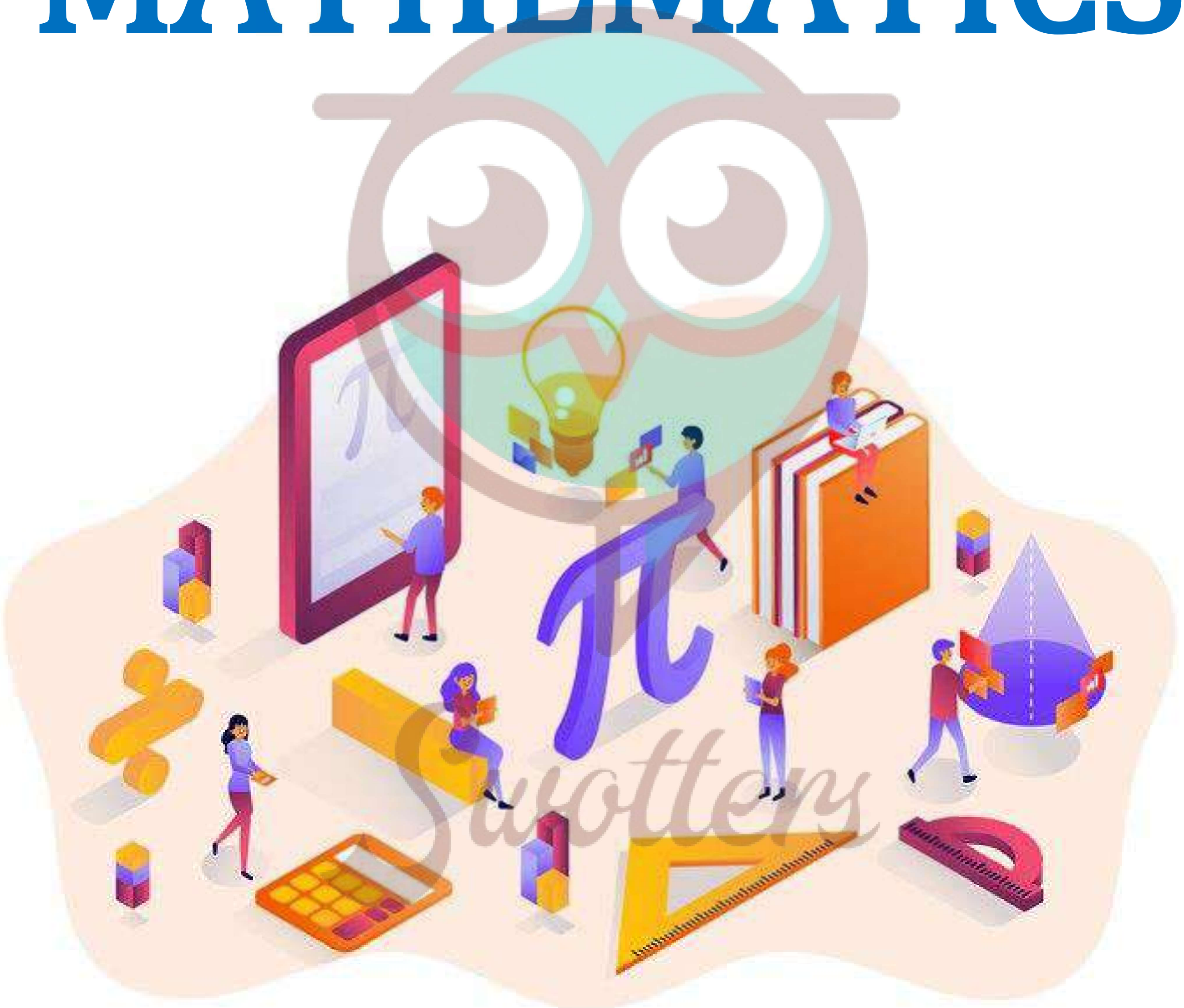
MATHEMATICS



Important Questions

Multiple Choice questions-

1. The anti-derivative of $(\sqrt{x} + \frac{1}{\sqrt{x}})$ equals

(a) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + c$

(b) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^{2} + c$

(c) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

(d) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + c.$

2. If $\frac{1}{dx}(f(x)) = 4x^3 - \frac{3}{x^4}$ such that f(2) = 0 then f(x) is ....

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

(b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$

(d) $x^3 + \frac{1}{x^4} - \frac{129}{8}.$

3. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}$ dx equals

(a) $10^x - x^{10} + c$

(b) $10^x + x^{10} + c$

(c) $(10^x - x^{10})^{-1} + c$

(d) $\log(10^x + x^{10}) + c.$

4. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

(a) tan x + cot x + c

(b) tan x - cot x + c

(c) tan x cot x + c

(d) tan x - cot 2x + c.

5. $\int \frac{sin^2x - cos^2x}{sin^2x cos^2x}$ dx is equals to

(a) tan x + cot x + c

(b) tan x + cosec x + c

(c) -tan x + cot x + c

(d) tan x + sec x + c.

6. $\int \frac{e^x(1+x)}{cos^2(xe^2)}$ dx is equals to

(a) -cot (xe^x) + c

(b) tan (xe^x) + c

(c) tan (e^x) + c

(d) cot (e^x) + c

7. $\int \frac{dx}{x^2+2x+2}$ equals

(a) x $\tan^{-1}$ (x + 1) + c

(b) $\tan^{-1}$ (x + 1) + c

(c) (x + 1) $\tan^{-1}$ x + c

(d) $\tan^{-1}$ x + c.

8. $\int \frac{dx}{\sqrt{9-25x^2}}$ equals

(a) $\sin^{-1}\left(\frac{5x}{3}\right)+c$

(b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+c$

(c) $\frac{1}{6}\log\left(\frac{3+5x}{3-5x}\right)+c$

(d) $\frac{1}{30}\log\left(\frac{3+5x}{3-5x}\right)+c.$

$\int \frac{xdx}{(x-1)(x-2)}$ equals

9.

(a) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + c$

(b) $\log\left|\frac{(x-2)^2}{x-1}\right| + c$

(c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + c$

(d) $\log|(x-1)(x-2)| + c$.

10. $\int \frac{dx}{x(x^2+1)}$ equals

(a) $\log|x| - \frac{1}{2}\log(x^2+1) + c$

(b) $\frac{1}{2}\log|x| + \frac{1}{2}\log(x^2+1) + c$

(c) $-\log|x| + \frac{1}{2}\log(x^2+1) + c$

(d) $\log|x| + \log(x^2+1) + c$

Very Short Questions:

1. Find $\int \frac{3+3\quad x}{x+\sin x}$ dx (C.B.S.E. Sample Paper 2019-20)
2. Find: $\int (\cos^2 2x - \sin^2 2x)dx$. (C.B.S.E. Sample Paper 2019-20)
3. Find: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$ (C.B.S.E. Outside Delhi 2019)
4. Evaluate $\int \frac{x^3-1}{x^2}$ dx (N.C.E.R.T. C.B.S.E. 2010C)
5. Find: $\int \frac{\sin^2 x - c\quad ^2 x}{\sin x \cos x}$ dx (A.I.C.B.S.E. 2017)
6. Write the value of $\int \frac{dx}{x^2+16}$
7. Evaluate: $\int (x^3 + 1)\, dx$. (C.B.S.E. Sample Paper 2019-20)
8. Evaluate: $\int_0^{\pi/2} e^x(\sin x - \cos x)\, dx$. (C.B.S.E. 2014)
9. Evaluate: $\int_0^2 \sqrt{4 - x^2}\, dx$ (A.I.C.B.S.E. 2014)

10. Evaluate: If f(x) = $\int_0^x t \sin t\, dt$, then write the value of f' (x). (A.I. C.B.S.E. 2014)

Short Questions:

1. Evaluate:

$\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x} dx$ (C.B.S.E)

2. Find: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$ dx
3. Find: $\int \sqrt{1 - \sin 2x dx}, \frac{\pi}{4} < x < \frac{\pi}{2}$
4. Find ∫ sinx . log cos x dx (C.B.S.E 2019 C)
5. Find: $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2}$ dx (CBSE Sample Paper 2018-19)
6. Evaluate $\int \frac{e^x(x-3)}{(x-1)^3}$ dx (CBSE Sample Paper 2018-19)
7. Find $\int \sin^{-1}(2x)$ dx
8. Evaluate: $\int_{-\pi}^{\pi}(1 - x^2) \sin x \cos 2x\, dx$.

Long Questions:

1. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}$ dx (C.B.S.E. 2019 (Delhi))
2. Integrate the function $\frac{\cos(x+a)}{\sin(x+b)}$ w.r.t. x. (C.B.S.E. 2019 (Delhi))
3. Evaluate: $\int x^2 \tan^{-1} x$ dx. (C.B.S.E. (F) 2012)
4. Find: $\int [\log(\log x) + \frac{1}{(\log x)^2}]$ dx (N.C.E.R.T.; A.I.C.B.S.E. 2010 C)

Case Study Questions-

1. Integration is the process of finding the antiderivative of a function. In this process, we are provided with the derivative of a function and asked to find out the function (i.e., Primitive) Integration is the inverse process of differentiation.

Let f (x) be a function of x. If there is a function g(x), such that d/dx (g(x)) = f (x), then g(x) is called an integral of f (x) w.r.t x and is denoted by ∫f (x)dx = g(x) + c, where c is constant of integration.

(i) $\int(3x+4)^3\,dx$ is equal to:

(a) $\frac{(3x+4)^4}{12}+c$ (b) $\frac{3(3x+4)^4}{4}+c$

(c) $\frac{3(3x+4)^2}{2}+c$ (d) $\frac{3(3x+4)^2}{4}+c$

(ii) $\int\frac{(x+1)^2}{x(x^2+1)}dx$ is equal to

(a) $\log|x|+c$ (b) $\log|x|+2\tan^{-1}x+c$

(c) $-\log|x^2+1|+c$ (d) $\log|x(x^2+1)|+c$

(iii) $\int\sin^2(x)\,dx$ is equal to:

(a) $\frac{x}{2}+\frac{\sin 2x}{4}+c$ (b) $\frac{x}{2}-\frac{\sin 2x}{4}+c$

(c) $x+\frac{\sin 2x}{2}+c$ (d) $x-\frac{\sin 2x}{2}+c$

(iv) $\int\tan^2(x)\,dx$ is equal to:

(a) $\tan x + x + c$ (b) $-\tan x - x + c$

(c) $-\tan x + x + c$ (d) $\tan x - x + c$

(v) $\int\frac{dx}{\sin^2 x\cos^2 x}$ is equal to

(a) $2\tan 2x + c$ (b) $-2\tan 2x + c$

(c) $-2\cot 2x + c$ (d) $2\cot 2x + c$

2. When the integrated can be expressed as a product of two functions, one of which can be differentiated and the other can be integrated, then we apply integration by parts. If f(x) = first function (that can be differentiated) and g(x) = second function (that can be integrated), then the preference of this order can be decided by the word "ILATE", where

I stands for Inverse Trigonometric Function
L stands for Logarithmic Function
A stands for Algebraic Function
T stands for Trigonometric Function
E stands for Exponential Function, then

$$\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left\{\frac{d}{dx}f(x)\int g(x)dx\right\}dx$$

(i) $\int x.\sin 3x\, dx$ =

(a) $\frac{x\cos 3x}{3} - \frac{\sin 3x}{a} + c$

(b) $-\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$

(c) $\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + c$

(d) $-\frac{x\cos 3x}{3} - \frac{\sin 3x}{9} + c$

(ii) $\int \log(x + 1)\, dx$ =

(a) log (x + 1) – x + c

(b) x log(x + 1) – x + c

(c) x log(x + 1) – log (x + 1) + x + c

(d) x log(x + 1) + log (x + 1) – x + c

(iii) $\int x^2 e^{3x}\, dx =$

(a) $\frac{e^{3x}}{9}(9x^2 + 6x + 2) + c$

(b) $\frac{e^{3x}}{9}(9x^2 - 6x + 2) + c$

(c) $\frac{e^{3x}}{27}(9x^2 + 6x + 2) + c$

(d) $\frac{e^{3x}}{27}(9x^2 - 6x + 2) + c$

(iv) $\int (f(x)g''(x) - f''(x)g(x))\, dx =$

(a) $f(x)g'(x) - f'(x)g(x) + c$
(b) $f(x)g'(x) + f'(x)g(x) + c$
(c) $f'(x)g(x) - f(x)g'(x) + c$
(d) $\frac{f(x)}{g'(x)} + c$

Answer Key-

Multiple Choice questions-

1. Answer: (c) $\frac{2}{3}x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + c$

2. Answer: (a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

3. Answer: (d) log $(10^x + x^{10})$ + c.

4. Answer: (b) tan x – cot x + c

5. Answer: (a) tan x + cot x + c

6. Answer: (b) tan (xe^x) + c

7. Answer: (b) $\tan^{-1}(x + 1) + c$

8. Answer: (b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + c$

9. Answer: (b) $\log\left|\frac{(x-2)^2}{x-1}\right| + c$

10. Answer: (a) $\log|x| - \frac{1}{2}\log(x^2 + 1) + c$

Very Short Answer:

1. Solution:

$I = \int \frac{3+3\cos x}{x+\sin x}dx = 3\log|x + \sin x| + c.$

[∵ Num. = $\frac{d}{dx}$ denom.]

2. Solution:

$I = \int \cos 4x\, dx = \frac{\sin 4x}{4} + c$

3. Solution:

$$I = \int \frac{dx}{\sqrt{5-4x-2x^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\frac{5}{2}-2x-x^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2-(x+1)^2}}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left[\frac{\sqrt{2}}{\sqrt{7}}(x+1)\right] + C.$$

4. Solution:

$$\int \frac{x^3 - 1}{x^2} dx = \int \left(x - \frac{1}{x^2}\right) dx$$

$$= \int x\, dx - \int x^{-2}\, dx$$

$$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c,\ x \neq 0.$$

5. Solution:

$$I = \int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

$$= \int \frac{\sin^2 x}{\sin x \cos x} dx - \int \frac{\cos^2 x}{\sin x \cos x} dx$$

$$= \int \tan x\, dx - \int \cot x\, dx$$

$$= \log |\sec x| - \log |\sin x| + c.$$

6. Solution:

$$\int \frac{dx}{x^2 + 16} = \int \frac{dx}{4^2 + x^2}$$

| "**Form :** $\int \frac{dx}{a^2 + x^2}$"

$$= \frac{1}{4} \tan^{-1} \frac{x}{4} + c.$$

7. Solution:

$I = \int_{-2}^{2} x^3\, dx + \int_{-2}^{2} 1 \cdot dx = I_1$

$\Rightarrow 0 + [x]_{-2}^{2}$ [$\because I_1$ is an odd function] = 2 – (-2) = 4.

⇒ 2 – (-2) = 4.

8. Solution:

$\int_0^{\pi/2} e^x$ (sin x – cos x)dx

$\int_0^{\pi/2} e^x$ (-cos x + sin x)dx

I" Form: ∫e^x (f(x) + f'(x) dx"

= $[e^x(-\cos x)]_0^{\pi/2}$

= -$e^{\pi/2}$ cos $\frac{\pi}{2}$ + e^0 cos 0

= -$e^{\pi/2}$ (0) + (1) (1)

= -0 + 1 = 1

9. Solution:

$$\int_0^2 \sqrt{4-x^2}\,dx = \int_0^2 \sqrt{2^2-x^2}\,dx$$

I"Form : $\int \sqrt{a^2-x^2}\,dx$"

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

= [0 + 2 $\sin^{-1}$(1)] – [0 + 0]

= 2$\sin^{-1}$(1) = 2(π/2) = π

10.Solution:

We have: f(x) = $\int_0^x t$ sin t dt.

f'(x) = x sin x. $\frac{d}{dx}$ (x) – 0

[Property XII; Leibnitz's Rule]

= x sin x . (1)

= x sin x.

Short Answer:

1. Solution:

$$I = \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{(1 - 2\sin^2 x) + 2\sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx = \int \sec^2 x \, dx$$

$= \tan x + c.$

2. Solution:

I = $\frac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

Put tan x = t so that $\sec^2$ x dx = dt.

∴ I = $\int \frac{dt}{\sqrt{t^2+2^2}}$

= log $|t + \sqrt{t^2 + 4}|$ + C

= log $|\tan x + \sqrt{t^2 + 4}|$ + C

3. Solution:

$$I = \int \sqrt{1 - \sin 2x} \, dx$$

$$= \int \sqrt{(\sin^2 x + \cos^2 x) - 2\sin x \cos x} \, dx$$

$$= \int \sqrt{(\sin x - \cos x)^2} \, dx = \int (\sin x - \cos x) dx$$

$= -\cos x - \sin x + C.$

4. Solution:

∫sinx . log cos x dx

Put cox x = t

so that – sin x dx = dt

i.e., sin x dx = – dt.

∴ I = -∫log t.1dt

= -[log t.t – ∫ 1/t. t dt]

[Integrating by parts]

= – [t log t – t] + C = f(1 – log t) + C

= cos x (1 – log (cos x)) + C.

5. Solution:

$$I = \int \frac{(x^2 + \sin^2 x)\sec^2 x}{1+x^2} dx$$

$$= \int \left\{\frac{(1+x^2)+(\sin^2 x - 1)}{1+x^2}\right\} \sec^2 x \, dx$$

$$= \int \left(1 - \frac{\cos^2 x}{1+x^2}\right) \sec^2 x \, dx$$

$$= \int \sec^2 x \, dx - \int \frac{1}{1+x^2} dx$$

$$= \tan x - \tan^{-1} x + c.$$

6. Solution:

$$I = \int \frac{e^x (x-3)}{(x-1)^3} dx$$

$$= \int e^x \left[\frac{(x-1)-2}{(x-1)^3}\right] dx$$

$$= \int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right]$$

| "**Form :** $\int e^x [f(x) + f'(x)] dx$"

$$= \frac{e^x}{(x-1)^2} + c.$$

7. Solution:

$$I = \int \sin^{-1}(2x)dx = \int \sin^{-1}(2x) \cdot 1\, dx$$

$$= \sin^{-1}(2x) \cdot x - \int \frac{1}{\sqrt{1-4x^2}}(2)(x)\, dx$$

[*Integrating by Parts*]

$$= x \sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}}\, dx$$

$$= x \sin^{-1}(2x) + \frac{1}{4} \int (1-4x^2)^{-1/2}(-8x)dx$$

$$= x \sin^{-1}(2x) + \frac{1}{4} \frac{(1-4x^2)^{1/2}}{1/2} + C$$

$$= x \sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C.$$

8. Solution:

Here, $f(x) = (1 - x^2) \sin x \cos^2 x$.

$f(x) = (1 - x^2) \sin(-x) \cos^2(-x)$

$= -(1 - x^2) \sin x \cos^2 x$

$= -f(x)$

$\Rightarrow$ f is an odd function.

Hence, I = 0.

Long Answer:

1. Solution:

$$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x} dx$$

$$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$$

$$= \int \frac{(1)^3 - 3\sin^2 x \cos^2 x (1)}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1 - 3\sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \left(\frac{1}{\sin^2 x \cos^2 x} - 3 \right) dx$$

$$= \int \left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} - 3 \right) dx$$

(Note this step)

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} - 3 \right) dx$$

$$= \int \sec^2 x \, dx + \int \operatorname{cosec}^2 x \, dx - 3\int 1 \, dx$$

$$= \tan x - \cot x - 3x + c.$$

2. Solution:

$$I = \int \frac{\cos(x+a)}{\sin(x+b)} dx$$

$$= \int \frac{\cos(\overline{x+b} + \overline{a-b})}{\sin(x+b)} dx$$

$$= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)} dx$$

$$= \cos(a-b)\int \frac{\cos(x+a)}{\sin(x+b)}dx - \sin(a-b)\int 1\cdot dx$$

$$= \cos(a-b)\log|\sin(x+b)| - \sin(a-b)\cdot x + c.$$

3. Solution:

$$\int x^2 \tan^{-1}x\, dx = \int \tan^{-1}x. x^2\, dx$$

$$= \tan^{-1}x.\frac{x^3}{3} - \int \frac{1}{1+x^2}.\frac{x^3}{3}dx$$

(Integrating by parts)

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \frac{x^3}{1+x^2}dx$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int \left(x - \frac{x}{1+x^2}\right)dx$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\int x\, dx + \frac{1}{6}\int \frac{2x}{1+x^2}dx$$

$$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}.\frac{x^2}{2} + \frac{1}{6}\log|1+x^2| + c$$

$$\left[\because \frac{d}{dx}(1+x^2) = 2x\right]$$

$$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log(1+x^2) + c.$$

$$\left[\because x^2 \geq 0 \Rightarrow 1+x^2 > 0 \therefore |1+x^2| = 1+x^2\right]$$

4. Solution:

Let $\int [\log(\log x) + \frac{1}{(\log x)^2}]\, dx$

$= \int \log(\log x)\, dx + \int \frac{1}{(\log x)^2}]\, dx$ …… (1)

Let $I = I_1 + I_2$

Now $I_1 = \int \log(\log x)\, dx$

$= \int \log(\log x)\, 1\, dx$

$= \log(\log x).x - \int \frac{1}{\log x \cdot x} x.dx$

(Integrating by parts)

$= x\log(\log x) - \int \frac{1}{\log x} dx$ ……….. (2)

Let $I_1 = I_3 + I_4$

And $I_4 = \int \frac{1}{\log x} dx$

$= \int \frac{1}{\log x}.1\, dx$

$= \frac{1}{\log x}.x - \int -\left(\frac{1}{\log x}\right)^2 .\frac{1}{x}.x\, dx$

(Integrating by parts)

$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} dx.$

Putting in (2),

$I_1 = x\log(x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx$

Putting in (1),

I = x log (log x)

$- \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx$

$= x\log(\log x) - \frac{x}{\log x} + c$

$= x\left(\log(\log x) - \frac{1}{\log x}\right) + c.$

Case Study Answers-

1.

(i) (a) $\frac{(3x+4)^4}{12} + c$

(ii) (b) $\log|x| + 2\tan^{-1}x + c$

(iii) (b) $\frac{x}{2}-\frac{\sin 2x}{4}+c$

(iv) (d) tan x – x + c

(v) (c) $-2 \cot 2x + c$

2.

(i) (b) $-\frac{x\cos 3x}{3}+\frac{\sin 3x}{9}+c$

(ii) (d) x log(x + 1) + log (x + 1) – x + c

(iii) (d) $\frac{e^{3x}}{27}(9x^2 - 6x + 2) + c$

(iv) (a) $f(x)g'(x) - f'(x)g(x) + c$