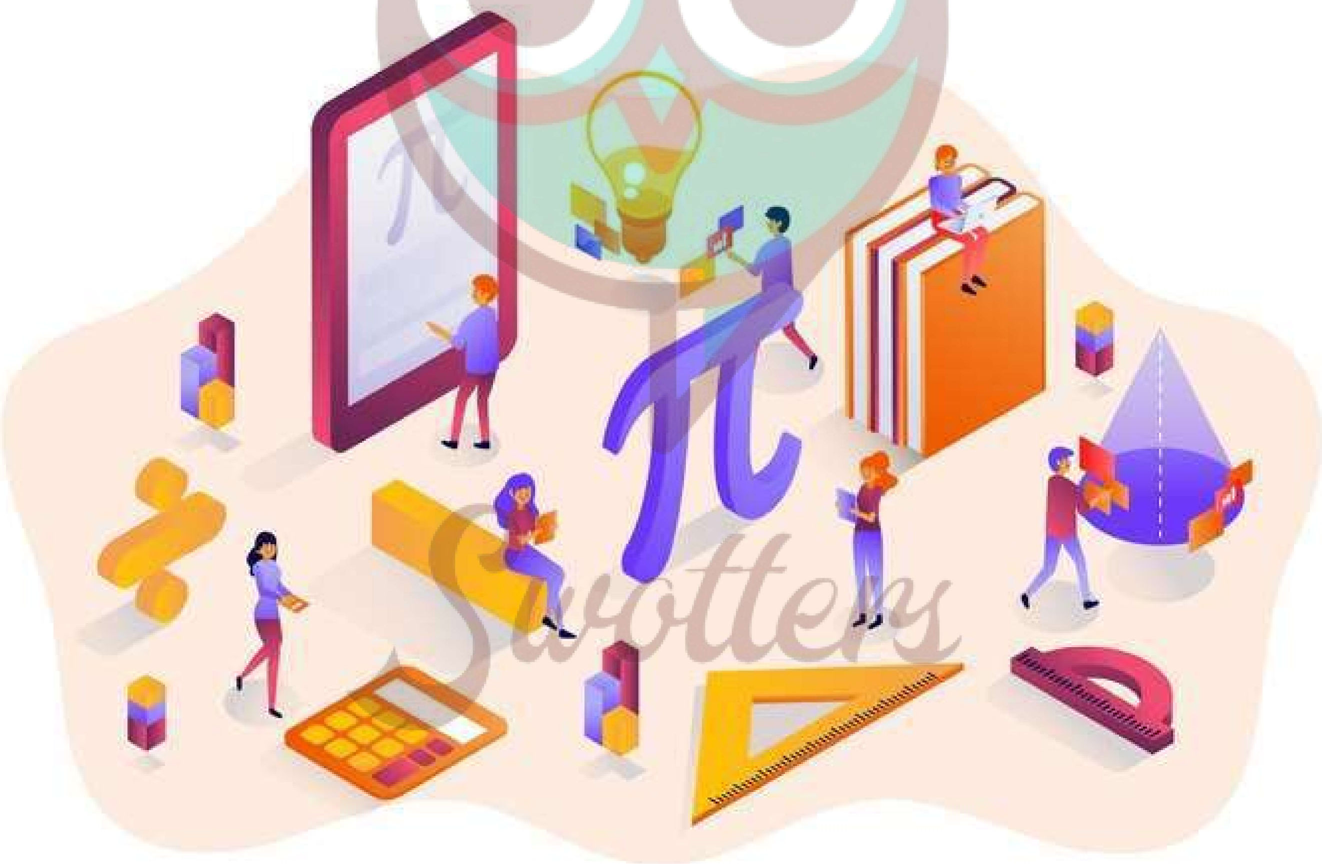
MATHEMATICS

Chapter 7: PERMUTATION AND COMBINATION



Important Questions

Multiple Choice questions-

Question 1. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. The number of ways such arrangements are possible are

(a) 8820

(b) 2880

(c) 2088

(d) 2808

Question 2. Six boys and six girls sit along a line alternately in x ways and along a circle (again alternatively in y ways), then

(a) x = y

(b) y = 12x

(c) x = 10y

(d) x = 12y

Question 3. How many 3-letter words with or without meaning, can be formed out of the letters of the word, LOGARITHMS, if repetition of letters is not allowed

(a) 720

(b) 420

(c) none of these

(d) 5040

Question 4. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of at least 3 girls

(a) 588

(b) 885

(c) 858

(d) None of these

Question 5. In how many ways can 12 people be divided into 3 groups where 4 persons must be there in each group?

(a) none of these

(b) $12!/(4!)^3$

(c) Insufficient data

(d) $12!/\{3! \times (4!)^3\}$

Question 6. How many factors are $2^5 \times 3^6 \times 5^2$ are perfect squares

(a) 24

(b) 12

(c) 16

(d) 22

Question 7. If ${}^nC_{15} = {}^nC_6$ then the value of ${}^nC_{21}$ is

(a) 0

(b) 1

(c) 21

(d) None of these

Question 8. If ${}^{n+1}C_3 = 2\ {}^nC_2$, then the value of n is

(a) 3

(b) 4

(c) 5

(d) 6

Question 9. There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a straight line. The number of triangle that can be formed by using these 15 points is

(a) 15C3

(b) 490

(c) 451

(d) 415

Question 10. In how many ways in which 8 students can be sated in a circle is

(a) 40302

(b) 40320

(c) 5040

(d) 50040

Very Short:

1. Evaluate 4! – 3!

2. If ${}^nc_a = {}^nc_b$ find n
3. The value of 0! Is?
4. Given 5 flags of different colours here many different signals can be generated if each signal requires the use of 2 flags. One below the other
5. How many 4 letter code can be formed using the first 10 letter of the English alphabet, if no letter can be repeated?
6. A coin is tossed 3 times and the outcomes are recorded. How many possible out comes are there?
7. Compute $\frac{8!}{6!\times 2!}$
8. If ${}^nC_3 = {}^nC_2$ find ${}^nC_2$
9. In how many ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colours.
10. Find r, if $5.4\, P_r = 6.5P_{r-1}$

Short Questions:

1. How many words, with or without meaning can be made from the letters of the word MONDAY. Assuming that no. letter is repeated, it

   (i) 4 letters are used at a time

   (ii) All letters are used but first letter is a vowel?

2. Prove that:

   $${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

3. A bag contains 5 black and 6 red balls determine the number of ways in which 2 black and 3 red balls can be selected.
4. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?
5. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE.

Long Questions:

1. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has :

   (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

2. Find the number of words with or without meaning which can be made using all the letters of the word. AGAIN. If these words are writer as in a dictionary, what will be the 50th word?

3. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of there

   (i) Four cards one of the same suit

   (ii) Four cards belong to four different suits

   (iii) Are face cards.

   (iv) Two are red cards & two are black cards.

   (v) Cards are of the same colour?

4. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nc_r = {}^nc_{r-1}$ find the value of n and r.

5. Find the value of such n that.

$(i)\ {}^nP_5 = 42\,{}^nP_3,\ n > 4$ $\qquad (ii)\ \frac{{}^nP_4}{{}^{n-1}P_4} = \frac{5}{3},\ n > 4$

Answer Key:

MCQ:

1. (b) 2880
2. (d) x = 12y
3. (a) 720
4. (a) 588
5. (d) $12!/\{3! \times (4!)^3\}$
6. (a) 24
7. (b) 1
8. (d) 6
9. (c) 451
10. (c) 5040

Very Short Answer:

1.

$4!-3! = 4.3!-3!$

$=(4-1).3!$

$=3.3! = 3\times3\times2\times1$

$=18$

2.

$^nC_a = {^nC_b} \Rightarrow {^nC_a} = {^nC_{n-b}}$

$a = n - b$

$n = a + b$

3.

$0! = 1$

4.

First flag can be chosen is 5 ways

Second flag can be chosen is 4 ways

By F.P.C. total number of ways = 5 × 4 = 20

5. First letter can be used in 10 ways

Second letter can be used in 9 ways

Third letter can be used in 8 ways

Forth letter can be used in 7 ways

By F.P.C total no. of ways = 10.9.8.7

= 5040

6. Total no. of possible out comes = 2 × 2 × 2 = 8

7.

$$\frac{8!}{6!2!} = \frac{8.7.\cancel{6!}}{\cancel{6!}.2.1}$$

$$= 4 \times 7 = 28$$

8. Given

$$^nC_8 = {^nC_2} \Rightarrow {^nC_{n-8}} = {^nC_2}$$

$$n - 8 = 2$$

$$n = 10$$

$$\therefore {^nC_2} = {^{10}C_2} = \frac{\lfloor 10}{\lfloor 10-2 \lfloor 2}$$

$$= \frac{10.9.\cancel{\lfloor 8}}{\cancel{\lfloor 8} \times 2.1} = 5 \times 9 = 45$$

9. No. of ways of selecting 9 balls

$$= {^6C_3} \times {^5C_3} \times {^5C_3}$$

$$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$$

$$= \frac{6.5.4\underline{|3}}{6.\underline{|3}} \times \frac{5.4.\underline{|3}}{2.\underline{|3}} \times \frac{5.4.\underline{|3}}{2.\underline{|3}}$$

$$= 20 \times 10 \times 10 = 2000$$

10.

$$5.\,{}^{4}p_{r} = 6.\,{}^{5}p_{r-1}$$

$$\Rightarrow 5.\frac{\underline{|4}}{\underline{|4-r}} = 6.\frac{\underline{|5}}{\underline{|5-r+1}}$$

$$\Rightarrow \frac{5\underline{|4}}{\underline{|(4-r)}} = \frac{6.5\underline{|4}}{\underline{|6-r}}$$

$$\Rightarrow \frac{1}{\cancel{\underline{|4-r}}} = \frac{6}{(6-r)(5-r)\cancel{\underline{|4-r}}}$$

$$\Rightarrow (6-r)(5-r) = 6$$

$$\Rightarrow 30 - 6r - 5r + r^2 = 6$$

$$\Rightarrow r^2 - 11r - 5r + r^2 = 6$$

$$\Rightarrow r^2 - 8r - 3r + 24 = 0$$

$$\Rightarrow r(r-8) - 3(r-8) = 0$$

$$\Rightarrow (r-3)(r-8) = 0$$

$r = 3$ or $r = 8$

$\therefore$ r = 3

$r = 8$ Rejected. Because if we put $r = 8$ the no. in the factorial is –ve.

Short Answer:

1.

Part-I In the word MONDAY there are 6 letters

$\therefore n = 6$

4 letters are used at a time

$\therefore r = 4$

Total number of words = ${}^{n}P_{r}$

$$= {}^{6}P_{4} = \frac{\underline{|6}}{\underline{|6-4}}$$

$$= \frac{\underline{|6}}{\underline{|2}} = \frac{6.5.4.3.\cancel{\underline{|2}}}{\cancel{\underline{|2}}} = 360$$

Part-II All letters are used at a time but first letter is a vowel then OAMNDY

2 vowels can be arranged in 2! Ways

4 consonants can be arranged in 4! Ways

∴ Total number of words = 2! × 4!

$= 2 \times 4.3.2.1 = 48$

2. Proof L.H.S.

$${}^{n}C_{r} + {}^{n}C_{r-1} = \frac{\lfloor n}{\lfloor n-r \lfloor r} + \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor (n-r)\, r \lfloor r-1} + \frac{\lfloor n}{(n-r+1) \lfloor n-r \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$= \frac{\lfloor n (n+1)}{\lfloor n-r (n-r+1) \lfloor r-1\, r}$$

$$= \frac{\lfloor n+1}{\lfloor n+1-r \lfloor n-r} = {}^{n+1}C_{r}$$

3. No. of black balls =5

No. of red balls = 6

No. of selecting black balls = 2

No. of selecting red balls = 3

Total no. of selection = ${}^{5}C_{2} \times {}^{6}C_{3}$

$$= \frac{\lfloor 5}{\lfloor 5-2 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 6-3\ \lfloor 3}$$

$$\frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2} = 200$$

4. Let us first seat 0 the 5 girls. This can be done in 5! Ways

X G X G X G X G X G X

There are 6 cross marked placer and the three boys can be seated ${}^{6}P_{3}$ in ways

Hence by multiplication principle

The total number of ways

$$= 5! \times {}^{6}P_{3} = 5! \times \frac{6!}{3!}$$

$= 4\times5\times2\times3\times4\times5\times6$

$= 14400$

5. In the INVOLUTE there are 4 vowels, namely I.O.E.U and 4 consonants namely M.V.L and T

The number of ways of selecting 3 vowels

Out of 4 = $^4C_3 = 4$

The number of ways of selecting 2 consonants

Out of 4 = $^4C_2 = 6$

∴ No of combinations of 3 vowels and 2 consonants = 4 × 6 = 24

5 letters 2 vowel and 3 consonants can be arranged in 5! Ways

Therefore required no. of different words = 24 × 5! = 2880

Long Answer:

1.

Number of girls = 4

Number of boys = 7

Number of selection of members = 5

(i) If team has no girl

We select 5 boys

∴ Number of selection of 5 members

$$= {}^7C_5 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} = 21$$

(ii) At least one boy and one girl the team consist of

Boy	Girls
1	4
2	3
3	2
4	1

The required number of ways

$$= {}^7C_1\times{}^4C_4 + {}^7C_2\times{}^4C_3 + {}^7C_3\times{}^4C_2 + {}^7C_4\times{}^4C_1$$

$$= 7 + 84 + 210 + 140$$

$$= 441$$

(iii) At least 3 girls

Girls	Boys
3	2
4	1

The required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7$

$= 91$

2.

In the word 'AGAIN' there are 5 letters in which 2 letters (A) are repeated

Therefore total no. of words $\frac{5!}{2!} = 60$

If these words are written as in a dictionary the number of words starting with Letter A. AAGIN = 4! = 24

The no. of wards starting with G GAAIN $= \frac{4!}{2!} = 12$

The no. of words starting with I IAAGN = $\frac{4!}{2!} = 12$

Now

Total words = 24 + 12 + 12 = 48

49^{th} words = NAAGI

50^{th} word = NAAIG

3. The no. of ways of choosing 4 cards form 52 playing cards.

$${}^{52}C_4 = \frac{52!}{4!48!} = 270725$$

(i) If 4 cards are of the same suit there are 4type of suits. Diamond club, spade and heart 4 cards of each suit can be selected in ${}^{13}C_4$ ways

∴ Required no. of selection = ${}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4 = 2860$

(ii) If 4 cards belong to four different suits then each suit can be selected in ${}^{13}C_1$ ways required no. of selection

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) If all 4 cards are face cards. Out of 12 face cards 4 cards can be selected in ${}^{12}C_4$ ways.

∴ required no. of selection ${}^{12}C_4$ $= \frac{\lfloor 12}{\lfloor 8 \lfloor 4} = 495$

(iv) If 2 cards are red and 2 are black then. Out of 26 red card 2 cards can be selected in ways similarly 2 black card can be selected in $^{26}C_2$ ways

$\therefore$ required no. of selection $= {}^{26}C_2 \times {}^{26}C_2$

$= \frac{26!}{2!4!} \times \frac{26!}{2!4!} = (325)^2$

$= 105625$

(v) If 4 cards are of the same colour each colour can be selected in $^{26}C_4$ ways

Then required no. of selection

$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!22!}$

$= 29900$

4.

Given that

${}^nP_r = {}^nP_{r+1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r-1}$

$\Rightarrow \frac{1}{(n-r)\lfloor n-r-1} = \frac{1}{\lfloor n-r-1}$

$\Rightarrow n-r=1 \ldots\ldots\ldots (i)$

also ${}^nC_r = {}^nC_{r-1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r \lfloor r} = \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$

$\Rightarrow \frac{1}{\lfloor n-r\ r\lfloor r-1} = \frac{1}{(n-r+1)\lfloor n-r \lfloor r-1}$

$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$

$\Rightarrow n-2r=-1 \ldots\ldots\ldots (ii)$

Solving eq (i) and eq (ii) we get $n=3$ and $r=2$

5.

(i) ${}^nP_5 = 42\,{}^nP_3$

$\Rightarrow \frac{\lfloor n}{\lfloor n-5} = 42\frac{\lfloor n}{\lfloor n-3}$

$\Rightarrow \frac{1}{\lfloor n-5} = \frac{42}{(n-3)(n-4)\lfloor n-5}$

$\Rightarrow (n-3)(n-4)=42$

$\Rightarrow n^2-4n-3n+12=42$

$\Rightarrow n^2-7n-30=0$

$\Rightarrow n^2-10n+3n-30=0$

$\Rightarrow n(n-10)+3(n-10)=0$

$\Rightarrow (n+3)(n-10)=0$

$n=-3$ or $n=10$

$n=-3$ Is rejected

Because negative factorial is not defined $\therefore n=10$

(ii)

$\frac{{}^{n}P_4}{{}^{n-1}P_4}=\frac{5}{3} \qquad n>4$

$\Rightarrow \frac{\frac{\lfloor n}{\lfloor n-4}}{\frac{\lfloor n-1}{\lfloor n-5}}=\frac{5}{3}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-4}\times\frac{\lfloor n-5}{\lfloor n-1}=\frac{5}{3}$

$\Rightarrow \frac{n\lfloor n-1}{(n-4)\lfloor n-5}\times\frac{\lfloor n-5}{\lfloor n-1}=\frac{5}{3}$

$\Rightarrow 3n=5n-20$

$\Rightarrow -2n=-20 \quad \Rightarrow n=10$