MATHEMATICS

Important Questions

Multiple Choice questions-

1. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{4}$

2. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(a) 2

(b) $\frac{9}{4}$

(c) $\frac{9}{3}$

(d) $\frac{9}{2}$

3. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

(a) 2 (π – 2)

(b) π – 2

(c) 2π – 1

(d) 2 (π + 2).

4. Area lying between the curves $y^2 = 4x$ and $y = 2$ is:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{4}$

5. Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(a) -9

(b) $-\frac{15}{4}$

(c) $\frac{15}{4}$

(d) $\frac{17}{4}$

6. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

(a) 0

(b) $-\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{4}{3}$

7. The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

(a) $\frac{4}{3}(4\pi - \sqrt{3})$

(b) $\frac{1}{3}(4\pi + \sqrt{3})$

(c) $\frac{2}{3}(8\pi - \sqrt{3})$

(d) $\frac{4}{3}(8\pi + \sqrt{3})$

8. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to

(a) 4π sq. units

(b) $2\sqrt{2}\pi$ sq. units

(c) $4\pi^2$ sq. units

(d) 2π sq. units.

9. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

(a) $\pi^2 ab$

(b) πab

(c) $\pi a^2 b$

(d) πab^2.

10. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(a) $\frac{32}{3}$

(b) $\frac{256}{3}$

(c) $\frac{64}{3}$

(d) $\frac{128}{3}$

Very Short Questions:

1. Find the area of region bounded by the curve $y = x^2$ and the line $y = 4$.
2. Find the area bounded by the curve $y = x^3$, $x = 0$ and the ordinates $x = -2$ and $x = 1$.
3. Find the area bounded between parabolas $y^2 = 4x$ and $x^2 = 4y$.
4. Find the area enclosed between the curve $y = \cos x$, $0 \le x \le \frac{\pi}{4}$ and the co-ordinate axes.
5. Find the area between the x-axis curve $y = \cos x$ when $0 \le x < 2$.
6. Find the ratio of the areas between the center $y = \cos x$ and $y = \cos 2x$ and x-axis for $x = 0$ to

   $x = \frac{\pi}{3}$
7. Find the areas of the region:

   $\{(x,y): x^2 + y^2 \le 1 \le x + 4\}$

Long Questions:

1. Find the area enclosed by the circle:

   $x^2 + y^2 = a^2$. (N.C.E.R.T.)
2. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. (C.B.S.E. 2018)

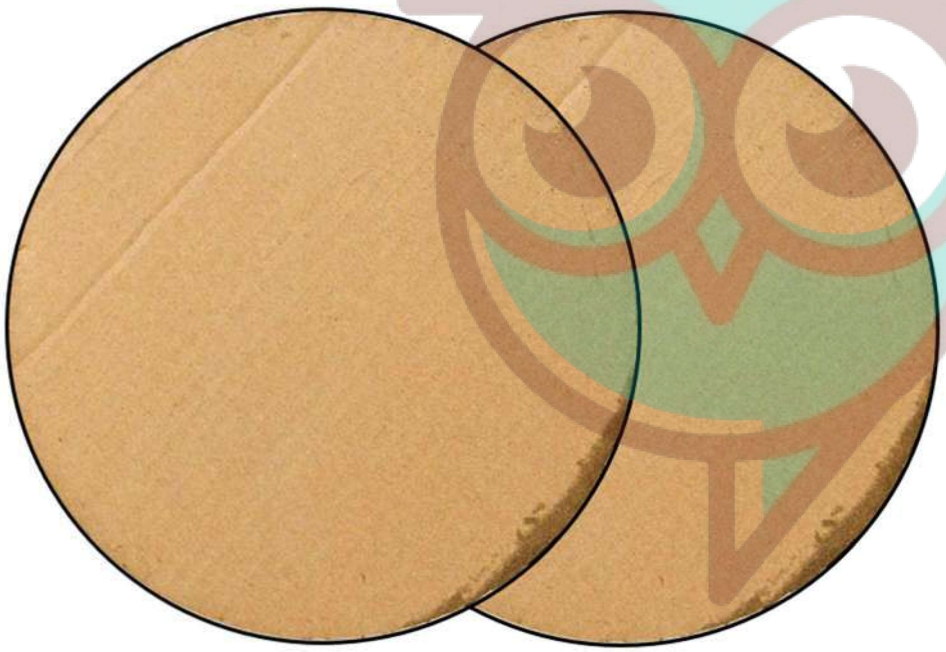
3. Find the area bounded by the curves $y = \sqrt{x}$, $2y + 3 = Y$ and Y-axis. (C.B.S.E. Sample Paper 2018-19)

4. Find the area of region:

   $\{(x,y): x^2 + y^2 < 8, x^2 < 2y\}$. (C.B.S.E. Sample Paper 2018-19)

Case Study Questions:

1. Ajay cut two circular pieces of cardboard and placed one upon other as shown in figure. One of the circle represents the equation $(x - 1)^2 + y^2 = 1$, while other circle represents the equation $x^2 + y^2 = 1$.



Based on the above information, answer the following questions.

i. Both the circular pieces of cardboard meet each other at

a. $x = 1$

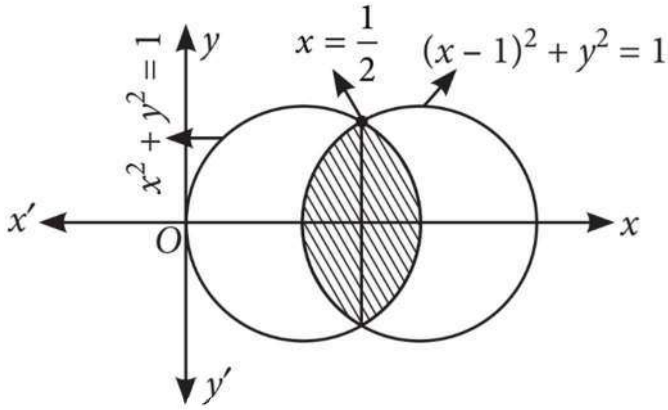
b. $x = \frac{1}{2}$

c. $x = \frac{1}{3}$

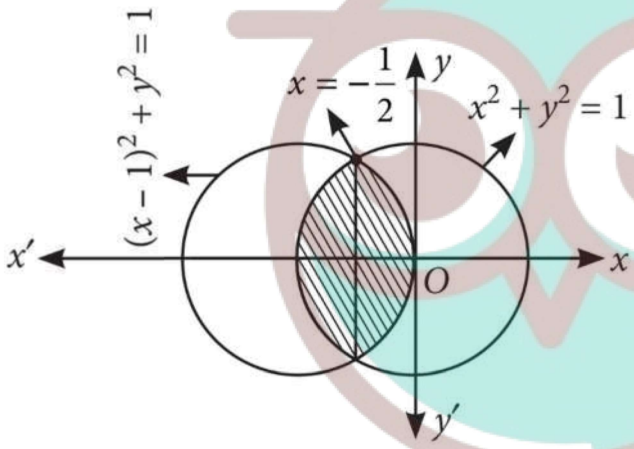
d. $x = \frac{1}{4}$

ii. Graph of given two curves can be drawn as.

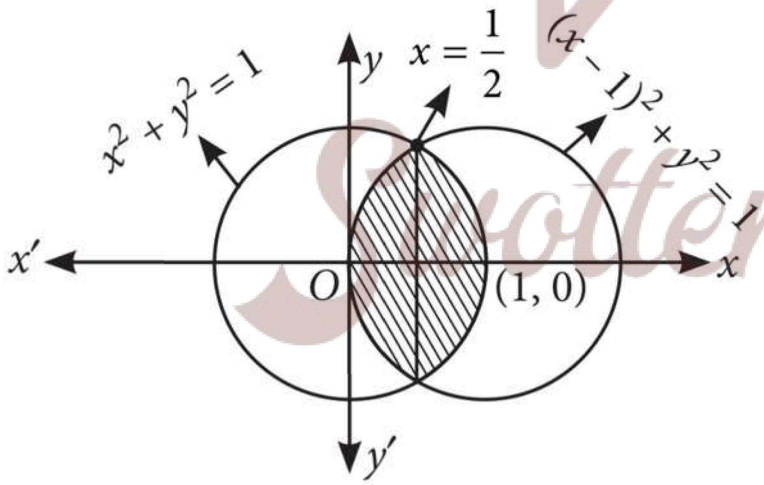
a.



b.



c.



d. None of these

iii. Value of $\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}dx$ is.

a. $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

b. $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

c. $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$

d. $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

iv. Value of $\int_{\frac{1}{2}}^{1} \sqrt{1-x^2}dx$ is.

a. $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$

b. $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$

c. $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

d. $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

v. Area of hidden portion of lower circle is.

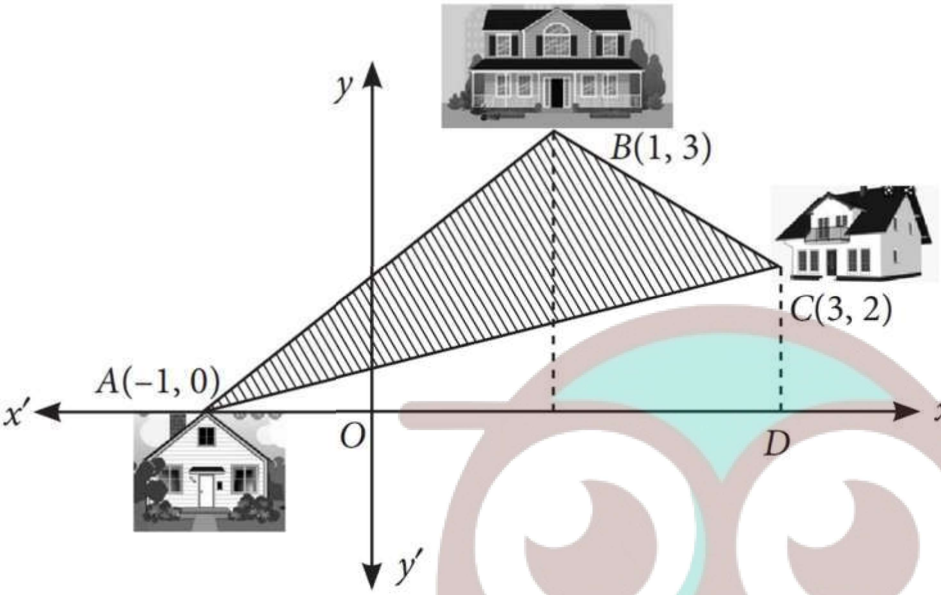
a. $\left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right)$ sq.units

b. $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{8}\right)$ sq.units

c. $\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$ sq.units

d. $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq.units

2. Location of three houses of a society is represented by the points A(-1, 0), B(1, 3) and C(3, 2) as shown in figure.



Based on the above information, answer the following questions

(i) Equation of line AB is.

a. $y = \frac{3}{2}(x+1)$

b. $y = \frac{3}{2}(x-1)$

c. $y = \frac{1}{2}(x+1)$

d. $y = \frac{1}{2}(x-1)$

(ii) Equation of line BC is.

a. $y = \frac{1}{2}x - \frac{7}{2}$

b. $y = \frac{3}{2}x - \frac{7}{2}$

c. $y = \frac{-1}{2}x + \frac{7}{2}$

d. $y = \frac{3}{2}x + \frac{7}{2}$

(iii) Area of region ABCD is.

a. 2 sq. units
b. 4 sq. units
c. 6 sq. units
d. 8 sq. units

(iv) Area of △ADC is,

a. 4 sq. units
b. 8 sq. units
c. 16 sq. units
d. 32 sq. units

(v) Area of △ABC is.

a. 3 sq. units
b. 4 sq. units
c. 5 sq. units
d. 6 sq. units

Answer Key-

Multiple Choice questions-

1. Answer: (a) π
2. Answer: (a) 2
3. Answer: (b) π – 2
4. Answer: (b) $\frac{1}{3}$
5. Answer: (b) - $\frac{15}{4}$
6. Answer: (c) $\frac{2}{3}$
7. Answer: (c) $\frac{2}{3}(8\pi - \sqrt{3})$
8. Answer: (d) 2π sq. units.
9. Answer: (b) πab
10. Answer: (b) $\frac{256}{3}$

Very Short Answer:

1. Solution: $\frac{32}{2}$ sq. units.
2. Solution: $\frac{17}{4}$ sq. units.
3. Solution: $\frac{16}{3}$ sq. units.
4. Solution: $\frac{1}{2}$ sq. units.
5. Solution: 4 sq. units
6. Solution: 2 : 1.
7. Solution: $\frac{1}{2}$ (π – 1) sq. units.

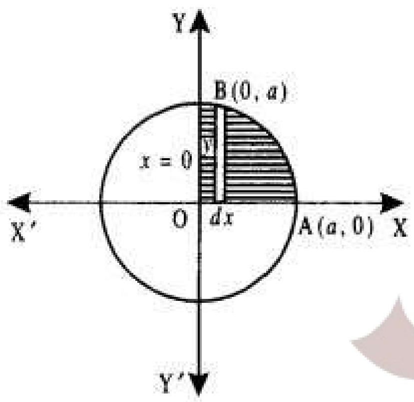
Long Answer:

1. Solution:

The given circle is

$x^2 + y^2 = a^2$ ………… (1)

This is a circle whose center is (0,0) and radius ‘a’.



Area of the circle=4 x (area of the region OABO, bounded by the curve, x-axis and ordinates x = 0, x = a)

[∵ Circle is symmetrical about both the axes]

= 4 $\int_0^a ydx$ [Taking vertical strips] o

= 4 $\int_0^a \sqrt{a^2 - x^2}dx$

[∵ (1) ⇒ $y = \pm\sqrt{a^2 - x^2}$

But region OABO lies in 1st quadrant, ∴ y is + ve]

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= 4\left[\left\{\frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}(1)\right\} - \{0 + 0\}\right]$$

$$= 4\left(\frac{a^2}{2}\cdot\frac{\pi}{2}\right) = \pi a^2 \text{ sq. units.}$$

2. Solution:

We have:

y = x ...(I)

and $x^2 + y^2 = 32$ ...(2)

(1) is a st. line, passing through (0,0) and (2) is a circle with centre (0,0) and radius 4√2 units. Solving (1) and (2) :

Putting the value of y from (1) in (2), we get:

$x^2 + x^2 = 32$

$2x^2 = 32$

$x^2 = 16$

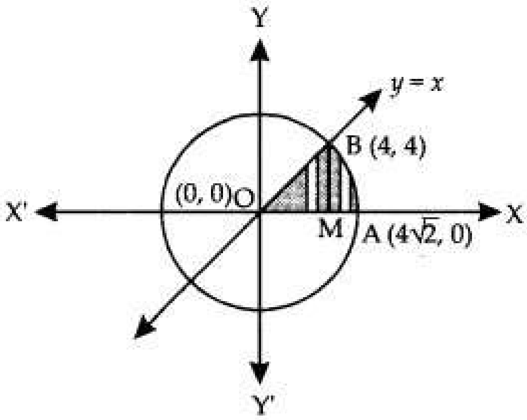
x = 4.

[∵ region lies in first quadrant]

Also, y = 4

Thus, the line (1) and the circle (2) meet each other at B (4,4), in the first quadrant.

Draw BM perp. to x – axis.



∴ Reqd. area = area of the region OMBO + area of the region BMAB ...(3)

Now, area of the region OMBO

$= \int_0^4 y\,dx$ [*Taking vertical strips*]

$= \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{1}{2}(16-0) = 8.$

Again, area of the region BMAB

$= \int_4^{4\sqrt{2}} y\,dx$ [*Taking vertical strips*]

$= \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx$

[$\because$ $y^2 = 32 - x^2 \Rightarrow y = \sqrt{32-x^2}$, *taking +ve sign, as it lies in Ist quadrant*]

$= \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$

$= \left[\frac{x\sqrt{32-x^2}}{2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$

$= \left\{\frac{1}{2}4\sqrt{2}\times 0 + \frac{32}{2}\sin^{-1}(1)\right\}$

$- \left\{\frac{4}{2}\sqrt{32-16} + \frac{32}{2}\sin^{-1}\frac{1}{\sqrt{2}}\right\}$

$= 0 + 16\left(\frac{\pi}{2}\right) - \left(2\times 4 + 16\times\frac{\pi}{4}\right)$

$= 8\pi - (8 + 4\pi) = 4\pi - 8$

∴ From (3),

Required area = $8 + (4\pi - 8) = 4\pi$ sq. units.

3. Solution:

The given curves are

$y = \sqrt{x}$ ………….(1)

and $2y + 3 = x$ …(2)

Solving (1) and (2), we get;

$\sqrt{2y+3} = y$

Squaring, $2y + 3 = y2$

$\Rightarrow y^2 2 - 2y - 3 = 0$

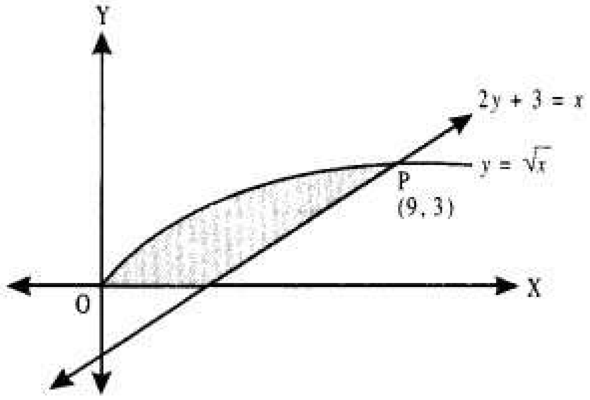
$\Rightarrow (y + 1)(y-3) = 0 \Rightarrow y = -1, 3$

$\Rightarrow y = 3$ [∵ $y > 0$]

Putting in (2),

$x = 2(3) + 3 = 9$.

Thus, (1) and (2) intersects at (9, 3).



$$\therefore \text{ Reqd. Area } = \int_0^3 (2y+3)dy - \int_0^3 y^2 dy$$

$$= [y^2+3y]_0^3 - \left[\frac{y^3}{3}\right]_0^3$$

$$= (9+9) - \left(\frac{27}{3}\right)$$

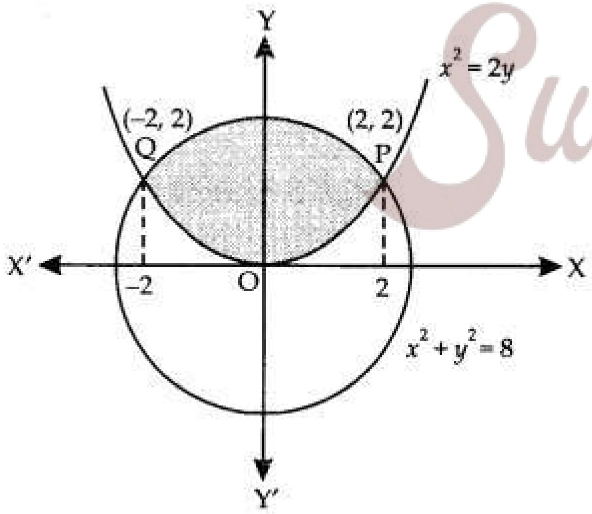
$$= 9+9-9 = 9 \text{ sq. units.}$$

4. Solution:

The given curves are;

$x^2 + y^2 = 8$ ………… (1)

$x^2 = 2y$ ………… (2)



Solving (1) and (2):

$8 - y^2 = 2y$

$\Rightarrow y^2 + 2y - 8 = 0$

$\Rightarrow (y + 4)(y - 2) = 0$

$= y = -4, 2$

$\Rightarrow y = 2.\ [\because y > 0]$

Putting in (2), $x^2 = 4$

$\Rightarrow x = -2$ or 2.

Thus, (1) and (2) intersect at P(2, 2) and Q(-2, 2).

$$\therefore \text{Required area} = \int_{-2}^{2} \sqrt{8 - x^2}\,dx - \int_{-2}^{2} \frac{x^2}{2}\,dx$$

$$= 2\left[\int_0^2 \sqrt{(2\sqrt{2})^2 - x^2}\,dx - \int_0^2 \frac{x^2}{2}\,dx\right]$$

$$= 2\left[\frac{x\sqrt{8 - x^2}}{2} + \frac{8}{2}\sin^{-1}\left(\frac{x}{2\sqrt{2}}\right)\right]_0^2 - \frac{1}{3}[x^3]_0^2$$

$$= 2\left[2 + 4\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 0\right] - \frac{1}{3}[8 - 0]$$

$$= 4 + 8\left(\frac{\pi}{4}\right) - \frac{8}{3} = \left(2\pi + \frac{4}{3}\right)\text{sq. units}.$$

Case Study Answers:

1. Answer :

i. (b) $x = \frac{1}{2}$

Solution:

We have, $(x - 1)^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-(x-1)^2}$

Also $x^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-x^2}$

From (i) and (ii), we get

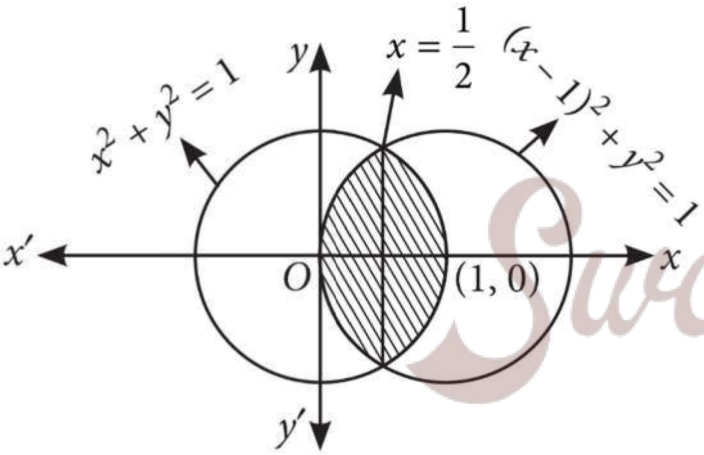
$\sqrt{1-(x-1)^2} = \sqrt{1-x^2}$

$\Rightarrow (x-1)^2 = x^2$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

ii. (c)



iii. (a) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

Solution:

$$\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-1}{1}\right)\right]$$

$$= \frac{1}{2}\left(\frac{1}{2}-1\right)\sqrt{1-\frac{1}{4}} + \frac{1}{2} + \sin^{-1}\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)$$

$$-\frac{1}{2}\sin^{-1}$$

$$= \left[\frac{-1}{4}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\pi}{6} + 0 + \frac{1}{2}\cdot\frac{\pi}{2}\right] = \frac{\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4}$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

iv. (c) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$

$$\int_{\frac{1}{2}}^{1} \sqrt{1-x^2}dx = \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$= 0 + \frac{1}{2}\sin^{-1}(1) - \frac{1}{4}\sqrt{1-\frac{1}{4}} - \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

v. (d) $\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ sq.units

Solution:

$$=2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}dx\right]$$

$$=2\left[\frac{\pi}{6}-\frac{\sqrt{3}}{8}+\frac{\pi}{6}-\frac{\sqrt{3}}{8}\right]$$

$$=2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)\text{ sq.units}$$

2. Answer :

i. (a) $y=\frac{3}{2}(x+1)$

Solution:

Equation of line AB is. $y-0=\frac{3-0}{1+1}(x+1)\Rightarrow y=\frac{3}{2}(x+1)$

ii. (c) $y=\frac{-1}{2}x+\frac{7}{2}$

Solution:

Equation of line BC is $y-3=\frac{2-3}{3-1}(x+1)$

$$\Rightarrow y=-\frac{1}{2}x+\frac{1}{2}+3\Rightarrow y=\frac{-1}{2}x+\frac{7}{2}$$

iii. (d) 8 sq. units

Solution:

Area of region ABCD = Area of $\triangle \mathrm{ABE}$ + Area of region BCDE

$$= \int_{-1}^{1} \frac{3}{2}(x+1)dx + \int_{1}^{3}\left(\frac{-1}{2}x+\frac{7}{2}\right)dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} + \left[\frac{-x^2}{4}+\frac{7}{2}x\right]_{1}^{3}$$

$$\frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right]+\left[\frac{-9}{4}+\frac{21}{2}+\frac{1}{4}-\frac{7}{2}\right]$$

$$= 3+5 = 8 \text{ sq.units}$$

iv. (a) 4 sq. units

Solution:

Equation of line AC is $y - 0 = \frac{2-0}{3+1}(x+1)$

$$\Rightarrow y = \frac{1}{2}(x+1)$$

$$\therefore \text{Area of } \triangle \mathrm{ADC} = \int_{-1}^{3} \frac{1}{2}(x+1)dx = \left[\frac{x^2}{4}+\frac{1}{2}x\right]_{-1}^{3}$$

$$= \frac{9}{4}+\frac{3}{2}-\frac{1}{4}+\frac{1}{2} = 4 \text{ sq.units}$$

v. (b) 4 sq. units

Solution:

Area of $\triangle \mathrm{ABC}$ = Area of region ABCD - Area of $\triangle \mathrm{ACD} = 8 - 4 = 4$ sq.units