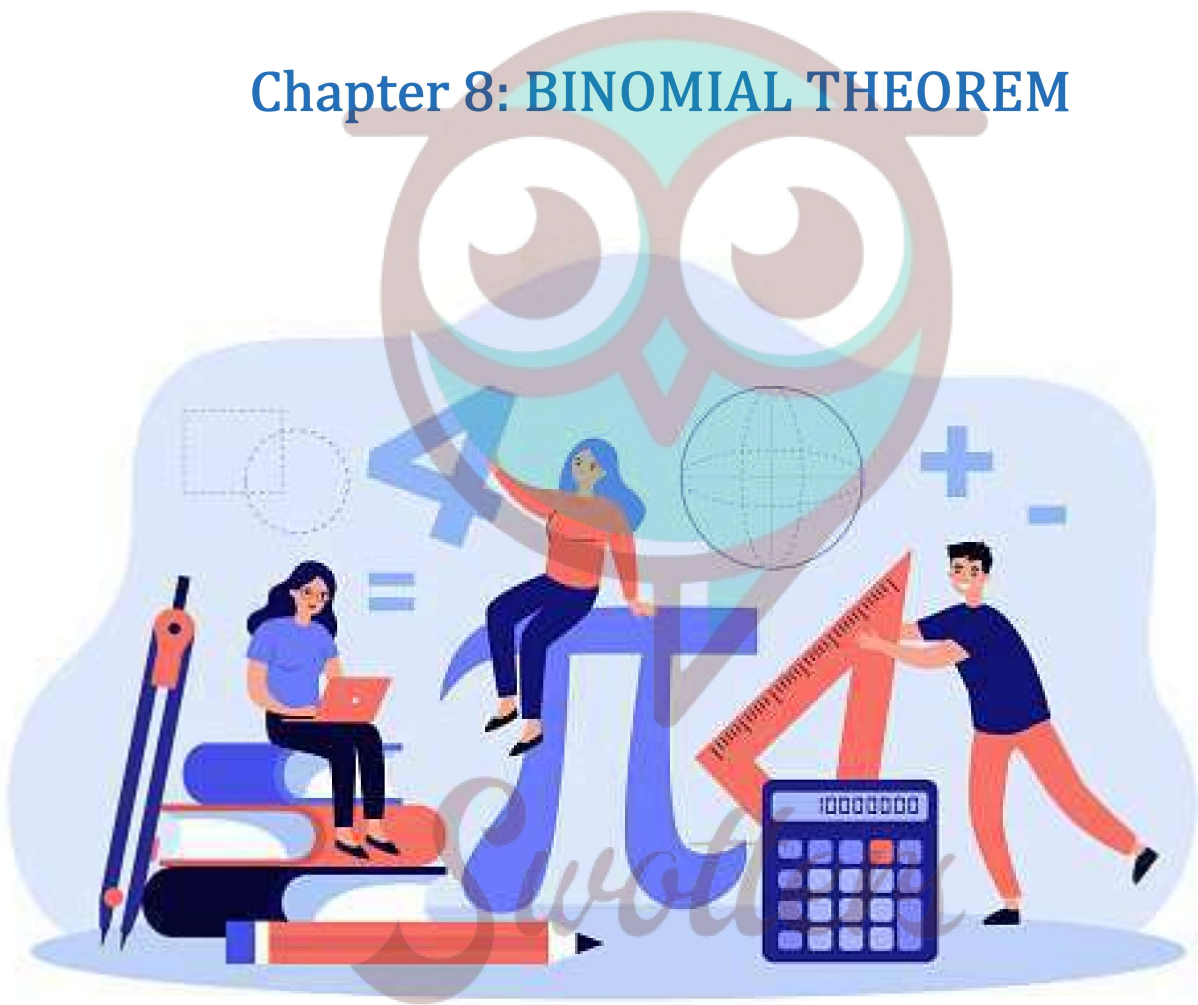


# MATHEMATICS

## Chapter 8: BINOMIAL THEOREM



## Important Questions

### Multiple Choice questions-

Question 1. The number  $(101)^{100} - 1$  is divisible by

- (a) 100
- (b) 1000
- (c) 10000
- (d) All the above

Question 2. The value of  $-1^\circ$  is

- (a) 1
- (b) -1
- (c) 0
- (d) None of these

Question 3. If the fourth term in the expansion  $(ax + 1/x)^n$  is  $5/2$ , then the value of x is

- (a) 4
- (b) 6
- (c) 8
- (d) 5

Question 4. The number 111111 ..... 1 (91 times) is

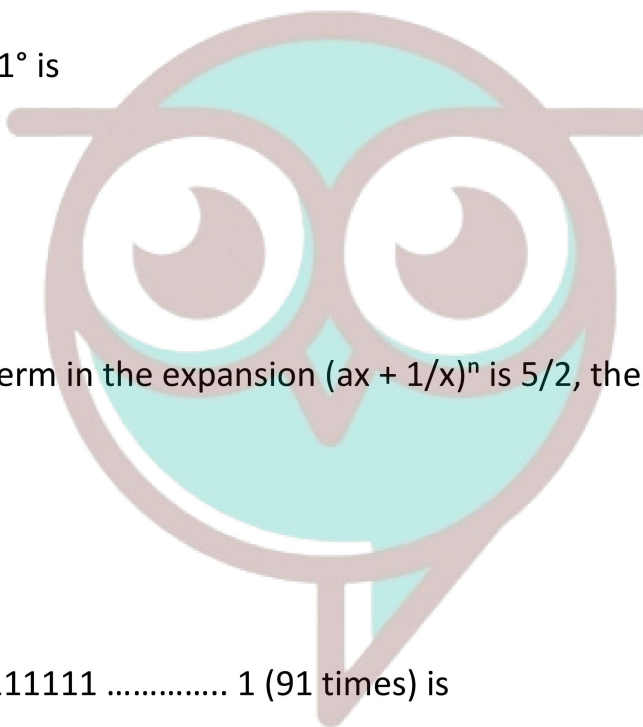
- (a) not an odd number
- (b) none of these
- (c) not a prime
- (d) an even number

Question 5. In the expansion of  $(a + b)^n$ , if n is even then the middle term is

- (a)  $(n/2 + 1)^{\text{th}}$  term
- (b)  $(n/2)^{\text{th}}$  term
- (c)  $n^{\text{th}}$  term
- (d)  $(n/2 - 1)^{\text{th}}$  term

Question 6. The number of terms in the expansion  $(2x + 3y - 4z)^n$  is

- (a)  $n + 1$
- (b)  $n + 3$



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(c)  $\{(n + 1) \times (n + 2)\}/2$

(d) None of these

Question 7. If A and B are the coefficient of  $x^n$  in the expansion  $(1 + x)^{2n}$  and  $(1 + x)^{2n-1}$  respectively, then A/B equals

(a) 1

(b) 2

(c)  $1/2$

(d)  $1/n$

Question 8. The coefficient of y in the expansion of  $(y^2 + c/y)^5$  is

(a)  $29c$

(b)  $10c$

(c)  $10c^3$

(d)  $20c^2$

Question 9. The coefficient of  $x^{-4}$  in  $(3/2 - 3/x^2)^{10}$  is

(a)  $405/226$

(b)  $504/289$

(c)  $450/263$

(d) None of these

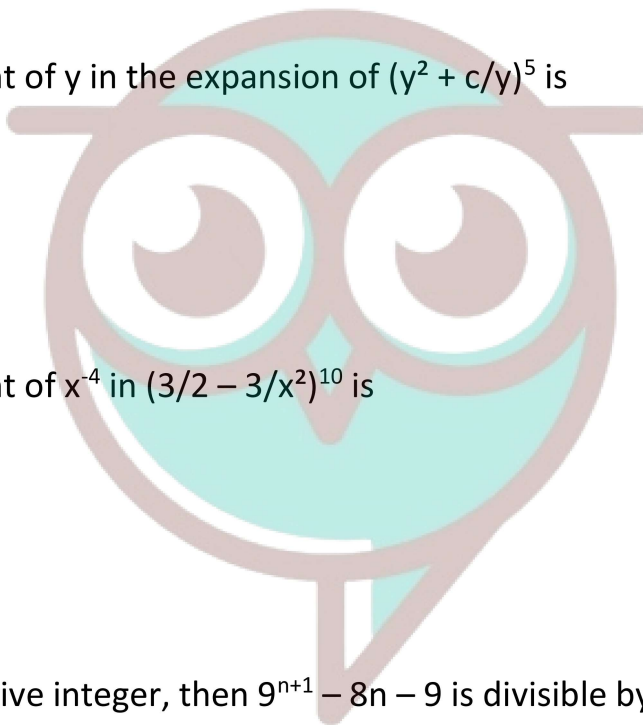
Question 10. If n is a positive integer, then  $9^{n+1} - 8n - 9$  is divisible by

(a) 8

(b) 16

(c) 32

(d) 64



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**Very Short:**

1. What is The middle term in the expansion of  $(1 + x)^{2n+1}$
2. When is n a positive integer, the no. of terms in the expansion  $(x + a)^n$  of is
3. Write the general term  $(x^2 - y)^6$
4. In the expansion of  $(x + \frac{1}{x})^6$ , find the 3<sup>rd</sup> term from the end.
5. Expand  $(1 + x)^n$
6. The middle term in the expansion of  $(1 + x)^{2n}$  is.

7. Find the no. of terms in the expansions of  $(1 - 2x + x^2)^7$
8. Find the coeff of  $x^5$  in  $(x + 3)^9$
9. Find the term independent of  $x$   $\left(x + \frac{1}{x}\right)^{10}$
10. Expand  $(a + b)^n$

**Short Questions:**

1. Which is larger  $(1.01)^{10,000,000}$  or 10,000.
2. Prove that:

$$\sum_{r=0}^n 3^r \binom{n}{r}$$

3. Using binomial theorem, prove that  $6^n - 5n$  always leaves remainder 1 when divided by 25.
4. Find the 13th term in the expansion of  $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$
5. Find the term independent of  $x$  in the expansion of  $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$

**Long Questions:**

1. Find , if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of  $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$  is  $\sqrt{6}: 1$
2. The coefficients of three consecutive terms in the expansion of  $(1 + a)^n$  are in the ratio 1 : 7 : 42. Find n.
3. The second, third and fourth terms in the binomial expansion  $(x + a)^n$  are 240, 720 and 1080 respectively. Find x , a and n.
4. If a and b are distinct integers, prove that a-b is a factor  $a^n - b^n$ , of whenever n is positive.
5. The sum of the coeff. Of the first three terms in the expansion of  $\left(x - \frac{3}{x^2}\right)^m$  m being natural no. is 559. Find the term of expansion containing  $x^3$ .

**Answer Key:**

**MCQ:**

1. (d) All the above
2. (b) -1
3. (b) 6
4. (c) not a prime
5. (a)  $(n/2 + 1)^{\text{th}}$  term
6. (c)  $\{(n + 1) \times (n + 2)\}/2$
7. (b) 2
8. (c)  $10c^3$
9. (d) None of these
10. (d) 64

**Very Short Answer:**

1. Since  $(2n + 1)$  is odd there is two middle term

$$i.e. \binom{2n+1}{n} x^{n+1} \quad \text{and} \quad \binom{2n+1}{n+1} x^n$$

2. The no. of terms in the expansion of  $(x + a)$  is one more than the index  $n$ . i.e.  $(n + 1)$ .

3.

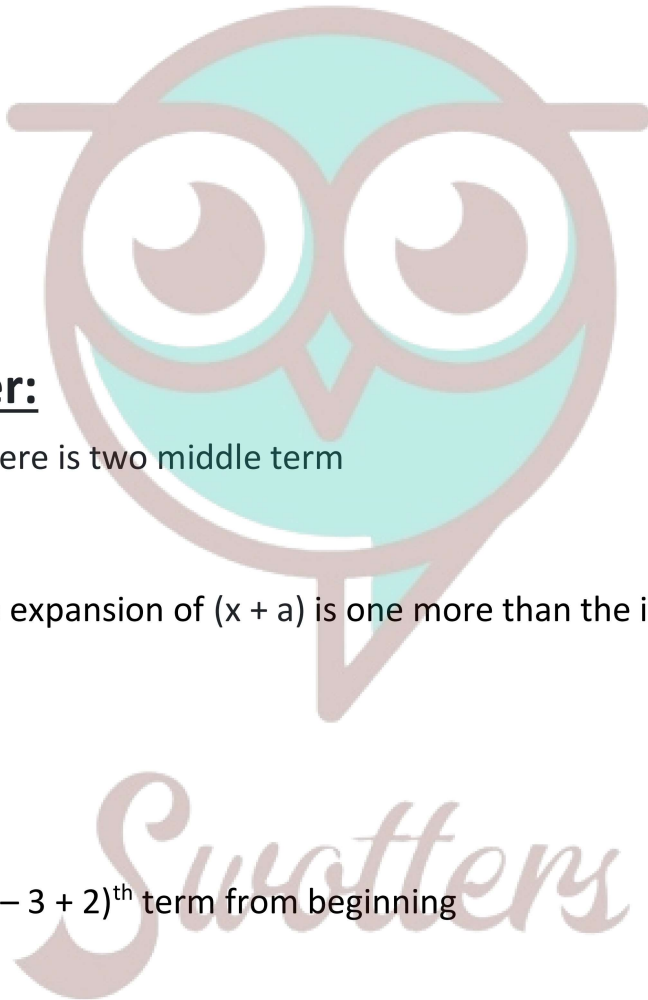
$$\begin{aligned} T_{r+1} &= {}^6C_r (x^2)^{6-r} \cdot (-y)^r \\ &= {}^6C_r (x)^{12-2r} \cdot (-1)^r \cdot (y)^r \end{aligned}$$

4. 3<sup>rd</sup> term from end =  $(6 - 3 + 2)^{\text{th}}$  term from beginning

$$\begin{aligned} T_5 &= {}^6C_4 (x)^{6-4} \cdot \left(\frac{1}{x}\right)^4 \\ &= {}^6C_4 x^2 \cdot x^{-4} \\ &= 15x^{-2} \\ &= \frac{15}{x^2} \end{aligned}$$

5.

$$(1+x)^n = 1 + {}^nC_1(x)^1 + {}^nC_2(x)^2 + {}^nC_3(x)^3 + \dots + x^n$$



6.

$${}^{2n}C_n \cdot x^n$$

7.

$$(1 - 2x + x^2)^7$$

$$= (x^2 - 2x + 1)^7$$

$$= [(x-1)^2]^7$$

$$= (x-1)^{14}$$

No. of term is 15

8.

$$T_{r+1} = {}^9C_r (x)^{9-r} \cdot (3)^r$$

Put  $9 - r = 5$

$$r = 4$$

$$T_5 = {}^9C_4 (x)^5 \cdot (3)^4$$

Coeff of  $x^5$  is  ${}^9C_4 (3)^4$

9.

$$T_{r+1} = {}^{10}C_r (x)^{10-r} \cdot \left(\frac{1}{x}\right)^r$$

$$= {}^{10}C_r (x)^{10-r} \cdot (x)^{-r}$$

$$= {}^{10}C_r (x)^{10-2r}$$

Put  $10 - 2r = 0$

$$r = 5$$

Independent term is  ${}^{10}C_5$

10.

$$(a + b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_n b^n$$

**Short Answer:**

1.



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$$\begin{aligned}
 (1.01)^{10,00000} &= (1+0.01)^{10,00000} \\
 &= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other positive term} \\
 &= 1 + 10,00000 \times 0.01 + \text{other positive term} \\
 &= 1 + 10,000 \\
 &= 10,001
 \end{aligned}$$

Hence  $(1.01)^{10,00000} > 10,000$

2.

$$\begin{aligned}
 \sum_{r=0}^n 3^r {}^n C_r &= \sum_{r=0}^n {}^n C_r \cdot 3^r \\
 &= {}^n C_0 + {}^n C_1 \cdot 3 + {}^n C_2 \cdot 3^2 + \dots + {}^n C_n \cdot 3^n \\
 &[\because (1+a)^n = 1 + {}^n C_1 a + {}^n C_2 a^2 + {}^n C_3 a^3 + \dots + a^n] \\
 &= (1+3)^n \\
 &= (4)^n \\
 &H.P
 \end{aligned}$$

3.

$$\begin{aligned}
 \text{Let } 6^n &= (1+5)^n \\
 &= 1 + {}^n C_1 5^1 + {}^n C_2 5^2 + {}^n C_3 5^3 + \dots + 5^n \\
 &= 1 + 5n + 5^2 \left( {}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right) \\
 6^n - 5n &= 1 + 25 \left( {}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right) \\
 &= 1 + 25k \left[ \text{where } k = {}^n C_2 + {}^n C_3 \cdot 5 + \dots + 5^{n-2} \right] \\
 &= 25k + 1 \\
 &H.P
 \end{aligned}$$

4.

The general term in the expansion of

$$\left( 9x - \frac{1}{3\sqrt{x}} \right)^{18} \text{ is}$$

$$T_{r+1} = {}^{18}C_r (9x)^{18-r} \left( -\frac{1}{3\sqrt{x}} \right)^r$$



For 13th term,  $r + 1 = 13$

$$r = 12$$

$$= {}^{13}C_{12} (9x)^6 \left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{13}C_{12} (3)^{12} \cdot x^6 \left(-\frac{1}{3}\right)^{12} \cdot (x)^{-6}$$

$$= {}^{13}C_{12} (3)^{12} \cdot (-1)^{12} \cdot (3)^{-12}$$

$$= {}^{13}C_{12}$$

$$= 18564$$

5.

$$T_{r+1} = {}^{18}C_r (\sqrt[3]{x})^{18-r} \left(\frac{1}{2\sqrt[3]{x}}\right)^r$$

$$= {}^{18}C_r (x)^{\frac{18-r}{3}} \cdot \left(\frac{1}{2}\right)^r \cdot x^{-\frac{r}{3}}$$

$$= {}^{18}C_r (x)^{\frac{18-r-r}{3}} \cdot \left(\frac{1}{2}\right)^r$$

For independent term  $\frac{18-2r}{3} = 0$

$$r = 9$$

The req. term is  ${}^{18}C_9 \left(\frac{1}{2}\right)^9$



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**Long Answer:**

1.

Fifth term from the beginning in the expansion of  $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$  is

$$T_{4+1} = {}^n C_4 (\sqrt[4]{2})^{n-4} \cdot \left(\frac{1}{\sqrt[4]{3}}\right)^4$$

$$T_5 = {}^n C_4 (2)^{\frac{n-4}{4}} \cdot (3)^{-1} \dots (i)$$

How fifth term from the end would be equal to  $(n-5+2)$  in term from the beginning



$$T_{(n-4)+1} = {}^n C_{n-4} (\sqrt[4]{2})^{n-(n-4)} \cdot \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$= {}^n C_{n-4} (2)^1 (3)^{\frac{n-4}{4}} \dots\dots (ii)$$

$$\frac{{}^n C_{n-4} (2)^{\frac{n-4}{4}} (3)^{-1}}{{}^n C_{n-4} (2)^1 (3)^{\frac{n-4}{4}}} = \frac{\sqrt{6}}{1}$$

ATQ

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}} = (6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}} = (6)^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2}$$

$$\Rightarrow 2n-16 = 4$$

$$n = 10$$

2.

Let three consecutive terms in the expansion of  $(1+a)^n$  are  $(r-1)^{th}$ ,  $r^{th}$  and  $(r+1)^{th}$  term

$$T_{r-1} = {}^n C_{r-1} (1)^{n-r} \cdot (a)^r$$

$$T_{r+1} = {}^n C_r (a)^r$$

Coefficients are

$${}^n C_{r-2}, {}^n C_{r-1} \text{ and } {}^n C_r \text{ respectively}$$

$$\text{ATQ } \frac{{}^n C_{r-2}}{{}^n C_{r-1}} = \frac{1}{7}$$

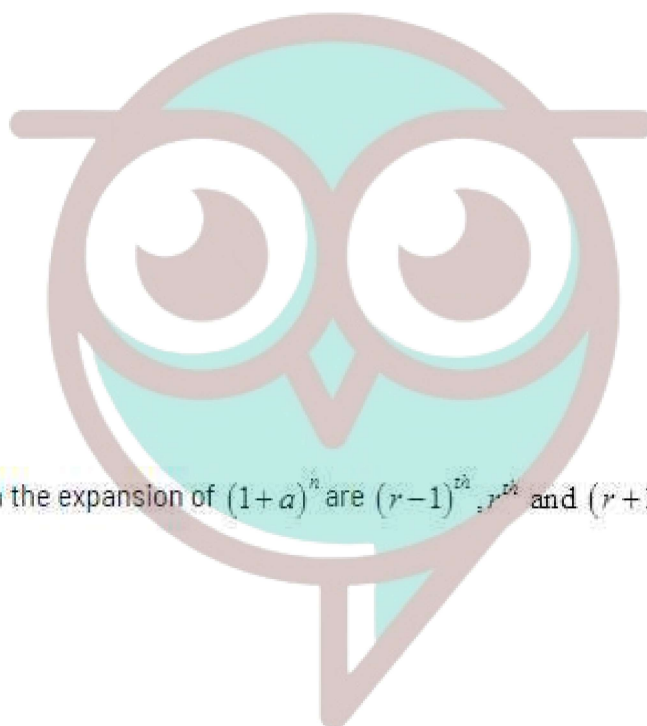
$$\Rightarrow n - 8r + 9 = 0 \dots\dots (i)$$

$$\frac{{}^n C_{r-1}}{{}^n C_r} = \frac{7}{42}$$

$$\Rightarrow n - 7r + 1 = 0 \dots\dots (ii)$$

On solving eq. (i) and (ii) we get  $n = 55$

3.



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$$T_2 = 240$$

$${}^n C_1 x^{n-1} \cdot a = 240 \dots\dots (i)$$

$${}^n C_2 x^{n-2} \cdot a^2 = 720 \dots\dots (ii)$$

$${}^n C_3 x^{n-3} \cdot a^3 = 1080 \dots\dots (iii)$$

Divide (ii) by (i) and (iii) by (ii)

We get

$$\frac{a}{x} = \frac{6}{n-1} \text{ and } \frac{a}{x} = \frac{9}{2(n-2)}$$

$$\Rightarrow n = 5$$

On solving we get

$$x = 2$$

$$a = 3$$

4.

$$\text{Let } a^n = (a-b+b)^n$$

$$a^n = (b+a-b)^n$$

$$= {}^n C_0 b^n + {}^n C_1 b^{n-1} (a-b) + {}^n C_2 b^{n-2} \cdot (a-b)^2 + {}^n C_3 b^{n-3} \cdot (a-b)^3 + \dots + {}^n C_n (a-b)^n$$

$$a^n = b^n + (a-b) \left[ {}^n C_0 b^n + {}^n C_1 b^{n-1} (a-b) + {}^n C_2 b^{n-2} \cdot (a-b)^2 + {}^n C_3 b^{n-3} \cdot (a-b)^3 + \dots + {}^n C_n (a-b)^n \right]$$

$$a^n - b^n = (a-b)k$$

Where

$${}^n C_1 b^{n-1} + {}^n C_2 b^{n-2} (a-b) + \dots + (a-b)^{n-1} = k$$

HP

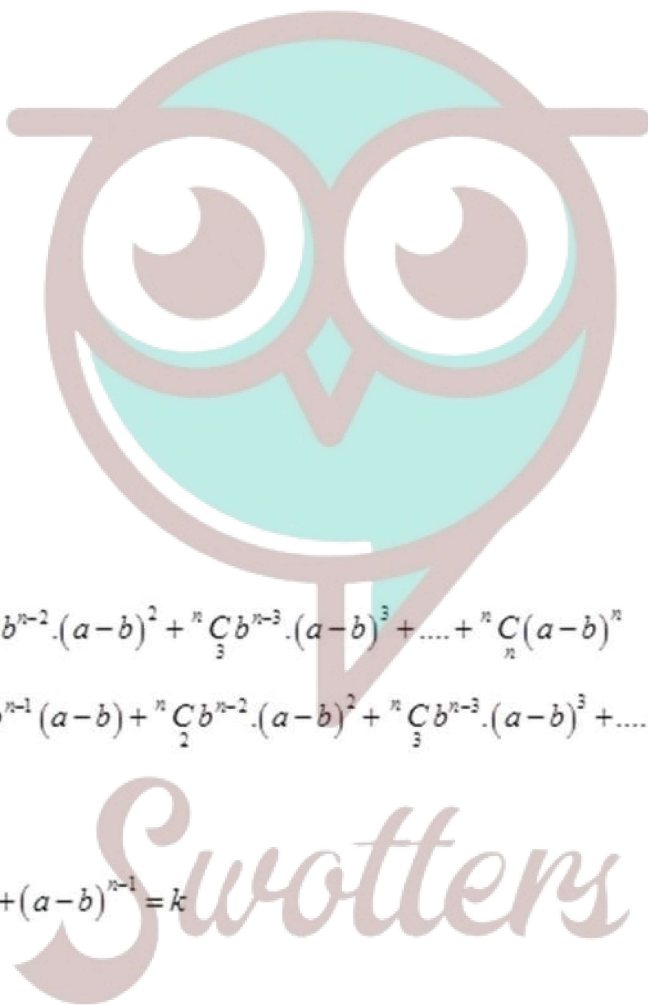
5. The coeff. Of the first three terms of  $\left(x - \frac{3}{x^2}\right)^m$  are  ${}^m C_0$ ,  $(-3) {}^m C_1$  and  $9 {}^m C_2$ .

Therefore, by the given condition

$${}^m C_0 - 3 {}^m C_1 + 9 {}^m C_2 = 559$$

$$1 - 3m + \frac{9m(m-1)}{2} = 559$$

On solving we get  $m = 12$



$$\begin{aligned}T_{r+1} &= {}^{12}C_r (x)^{12-r} \left(\frac{-3}{x^2}\right)^r \\&= {}^{12}C_r (x)^{12-r} \cdot (-3)^r \cdot (x)^{-2r} \\&= {}^{12}C_r (x)^{12-3r} \cdot (-3)^r\end{aligned}$$

$$12 - 3r = 3 \Rightarrow r = 3, \text{ req. term is } -5940 x^3$$



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