MATHEMATICS

Chapter 8: BINOMIAL THEOREM

Important Questions

Multiple Choice questions-

Question 1. The number $(101)^{100} - 1$ is divisible by

(a) 100

(b) 1000

(c) 10000

(d) All the above

Question 2. The value of -1° is

(a) 1

(b) -1

(c) 0

(d) None of these

Question 3. If the fourth term in the expansion $(ax + 1/x)^n$ is 5/2, then the value of x is

(a) 4

(b) 6

(c) 8

(d) 5

Question 4. The number 111111 …………. 1 (91 times) is

(a) not an odd number

(b) none of these

(c) not a prime

(d) an even number

Question 5. In the expansion of $(a + b)^n$, if n is even then the middle term is

(a) $(n/2 + 1)^{th}$ term

(b) $(n/2)^{th}$ term

(c) n^{th} term

(d) $(n/2 – 1)^{th}$ term

Question 6. The number of terms in the expansion $(2x + 3y – 4z)^n$ is

(a) n + 1

(b) n + 3

(c) {(n + 1) × (n + 2)}/2

(d) None of these

Question 7. If A and B are the coefficient of x^n in the expansion $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then A/B equals

(a) 1

(b) 2

(c) 1/2

(d) 1/n

Question 8. The coefficient of y in the expansion of $(y^2 + c/y)^5$ is

(a) 29c

(b) 10c

(c) $10c^3$

(d) $20c^2$

Question 9. The coefficient of x^{-4} in $(3/2 – 3/x^2)^{10}$ is

(a) 405/226

(b) 504/289

(c) 450/263

(d) None of these

Question 10. If n is a positive integer, then $9^{n+1} – 8n – 9$ is divisible by

(a) 8

(b) 16

(c) 32

(d) 64

Very Short:

1. What is The middle term in the expansion of $(1 + x)^{2n+1}$
2. When is n a positive integer, the no. of terms in the expansion $(x + a)^n$ of is
3. Write the general term $(x^2 - y)^6$
4. In the expansion of $\left(x + \frac{1}{x}\right)^6$, find the 3rd term from the end.
5. Expand $(1 + x)^n$
6. The middle term in the expansion of $(1 + x)^{2n}$ is.

7. Find the no. of terms in the expansions of $(1 - 2x + x^2)^7$
8. Find the coeff of x^5 in $(x + 3)^9$
9. Find the term independent of x $\left(x + \frac{1}{x}\right)^{10}$
10. Expand $(a + b)^n$

Short Questions:

1. Which is larger $(1.01)^{10,000000}$ or 10,000.
2. Prove that:

$$\sum_{r=0}^{n} 3^r \ {}^nC_r$$

3. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.
4. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$
5. Find the term independent of x in the expansion of $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$

Long Questions:

1. Find , if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}: 1$
2. The coefficients of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio 1 : 7 : 42. Find n.
3. The second, third and fourth terms in the binomial expansion $(x + a)^n$ are 240, 720 and 1080 respectively. Find x , a and n.
4. If a and b are distinct integers, prove that a-b is a factor $a^n - b^n$, of whenever n is positive.
5. The sum of the coeff. Of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^m$ m being natural no. is 559. Find the term of expansion containing x^3.

Answer Key:

MCQ:

1. (d) All the above
2. (b) -1
3. (b) 6
4. (c) not a prime
5. (a) $(n/2 + 1)^{th}$ term
6. (c) {(n + 1) × (n + 2)}/2
7. (b) 2
8. (c) $10c^3$
9. (d) None of these
10. (d) 64

Very Short Answer:

1. Since (2n + 1) is odd there is two middle term

$i.e\ {}^{2n+1}C_{n}\, x^{n+1}$ and ${}^{2n+1}C_{n+1}\, x^{n}$

2. The no. of terms in the expansion of (x + a) is one more than the index n. i.e. (n + 1).

3.

$$T^{r+1} = {}^{6}C_{r}(x^2)^{6-r}.(-y)^r$$

$$= {}^{6}C_{r}(x)^{12-2r}.(-1)^r.(y)^r$$

4. 3^{rd} term form end = $(6 - 3 + 2)^{th}$ term from beginning

$$T_5 = {}^{6}C_{4}(x)^{6-4}.\left(\frac{1}{x}\right)^4$$

$$= {}^{6}C_{4}x^2.x^{-4}$$

$$=15^{x-2}$$

$$=\frac{15}{x^2}$$

5.

$$(1+x)^n = 1 + {}^{n}C_{1}(x)^1 + {}^{n}C_{2}(x)^2 + {}^{n}C_{3}(x)^3 + ...x^n$$

6.

${}^{2n}C_n . x^n$

7.

$(1-2x+x^2)^7$

$=(x^2-2x+1)^7$

$=\left[(x-1)^2\right]^7$

$=(x-1)^{14}$

No. of term is 15

8.

$T_{r+1} = {}^9C_r (x)^{9-r} . (3)^r$

Put $9-r=5$

$r=4$

$T_5 = {}^9C_4 (x)^5 . (3)^4$

Coeff of x^5 is ${}^9C_4 (3)^4$

9.

$T_{r+1} = {}^{10}C_r (x)^{10-r} . \left(\frac{1}{x}\right)^r$

$= {}^{10}C_r (x)^{10-r} . (x)^{-r}$

$= {}^{10}C_r (x)^{10-2r}$

Put $10-2r=0$

$r=5$

Independent term is ${}^{10}C_5$

10.

$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + ... + {}^nC_n b^n$

Short Answer:

1.

$(1.01)^{10,00000} = (1+0.01)^{10,00000}$

$= {}^{10,00000}C_0 + {}^{10,00000}C_1(0.01) + \text{other positive term}$

$= 1 + 10,00000 \times 0.01 + \text{other positive term}$

$= 1 + 10,000$

$= 10,001$

Hence $(1.01)^{10,00000} > 10,000$

2.

$\sum_{r=0}^{n} 3^r \ {}^nC_r = \sum_{r=0}^{n} {}^nC_r . 3^r$

$= {}^nC_0 + {}^nC_1 .3 + {}^nC_2 .3^2 + ... + {}^nC_n 3^n$

$\left[\because (1+a)^n = 1 + {}^nC_1 .a + {}^nC_2 a^2 + {}^nC_3 a^3 + ... + a^n\right]$

$= (1+3)^n$

$= (4)^n$

$H.P$

3.

Let $6^n = (1+5)^n$

$= 1 + {}^nC_1 5^1 + {}^nC_2 5^2 + {}^nC_3 5^3 + ... + 5^n$

$= 1 + 5n + 5^2\left({}^nC_2 + {}^nC_3 .5 + ... + 5^{n-2}\right)$

$6^n - 5n = 1 + 25\left({}^nc_2 + {}^nc_3 .5 + ... + 5^{n-2}\right)$

$= 1 + 25k\left[\text{where } k = {}^nc_2 + {}^nc_3 .5 + ...5^{n-2}\right]$

$= 25k + 1$

$H.P$

4.

The general term in the expansion of

$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ is

$T_{r+1} = {}^{18}C_r (9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

For 13th term, $r+1=13$

$r=12$

$= {}^{18}C_{12}(9x)^{6}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$= {}^{18}C_{12}(3)^{12}.x^{6}\left(-\frac{1}{3}\right)^{12}.(x)^{-6}$

$= {}^{18}C_{12}(3)^{12}.(-1)^{12}.(3)^{-12}$

$= {}^{18}C_{12}$

$=18564$

5.

$T_{r+1} = {}^{18}C_{r}\left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^{r}$

$= {}^{18}C_{r}(x)^{\frac{18-r}{3}}.\left(\frac{1}{2}\right)^{r}.x^{\frac{-r}{3}}$

$= {}^{18}C_{r}(x)^{\frac{18-r-r}{3}}.\left(\frac{1}{2}\right)^{r}$

For independent term $\frac{18-2r}{3}=0$

$r=9$

The req. term is ${}^{18}C_{9}\left(\frac{1}{2}\right)^{9}$

Long Answer:

1.

Fifth term from the beginning in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^{n}$ is

$T_{4+1} = {}^{n}C_{4}\left(\sqrt[4]{2}\right)^{n-4}.\left(\frac{1}{\sqrt[4]{3}}\right)^{4}$

$T_{5} = {}^{n}C_{4}(2)^{\frac{n-4}{4}}.(3)^{-1}......(i)$

How fifth term from the end would be equal to $(n-5+2)$ in term from the beginning

$$T_{(n-4)+1} = {}^{n}C_{n-4}\left(\sqrt[4]{2}\right)^{n-(n-4)}\cdot\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$= {}^{n}C_{n-4}(2)^{1}(3)^{\frac{n-4}{4}} \text{.......}(ii)$$

ATQ $$\frac{{}^{n}C_{4}.(2)^{\frac{n-4}{4}}(3)^{-1}}{{}^{n}C_{n-4}(2)^{1}(3)^{\frac{n-4}{-4}}} = \frac{\sqrt{6}}{1}$$

$$\frac{(2)^{\frac{n-8}{4}}}{(3)^{\frac{-(n-8)}{4}}} = (6)^{\frac{1}{2}}$$

$$(6)^{\frac{n-8}{4}} = (6)^{\frac{1}{2}}$$

$$\frac{n-8}{4} = \frac{1}{2}$$

$$\Rightarrow 2n-16 = 4$$

$$n = 10$$

2.

Let three consecutive terms in the expansion of $(1+a)^{n}$ are $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ term

$$T_{r+1} = {}^{n}C_{r}(1)^{n-r}.(a)^{r}$$

$$T_{r+1} = {}^{n}C_{r}\ (a)^{r}$$

Coefficients are

${}^{n}C_{r-2},\ {}^{n}C_{r-1}$ and ${}^{n}C_{r}$ respectively

ATQ $$\frac{{}^{n}C_{r-2}}{{}^{n}C_{r-1}} = \frac{1}{7}$$

$$\Rightarrow n-8r+9 = 0 \text{.......}(i)$$

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{7}{42}$$

$$\Rightarrow n-7r+1 = 0 \text{......}(ii)$$

On solving eq. (i) and (ii) we get $n = 55$

3.

$T_2 = 240$

${}^{n}C_{1} x^{n-1} a = 240 \ldots\ldots (i)$

${}^{n}C_{2} x^{n-2} a^2 = 720 \ldots\ldots (ii)$

${}^{n}C_{3} x^{n-3} a^3 = 1080 \ldots\ldots (iii)$

Divide (ii) by (i) and (iii) by (ii)

We get

$\frac{a}{x} = \frac{6}{n-1}$ and $\frac{a}{x} = \frac{9}{2(n-2)}$

$\Rightarrow n = 5$

On solving we get

$x = 2$
$a = 3$

4.

Let $a^n = (a-b+b)^n$

$a^n = (b+a-b)^n$

$= {}^{n}C_{0} b^n + {}^{n}C_{1} b^{n-1}(a-b) + {}^{n}C_{2} b^{n-2}.(a-b)^2 + {}^{n}C_{3} b^{n-3}.(a-b)^3 + \ldots + {}^{n}C_{n}(a-b)^n$

$a^n = b^n + (a-b)\left[{}^{n}C_{0} b^n + {}^{n}C_{1} b^{n-1}(a-b) + {}^{n}C_{2} b^{n-2}.(a-b)^2 + {}^{n}C_{3} b^{n-3}.(a-b)^3 + \ldots + {}^{n}C_{n}(a-b)^n\right]$

$a^n - b^n = (a-b)k$

Where

${}^{n}C_{1} b^{n-1} + {}^{n}C_{2} b^{n-2}(a-b) + \ldots\ldots + (a-b)^{n-1} = k$

$H.P$

5. The coeff. Of the first three terms of $\left(x - \frac{3}{x^2}\right)^m$ are ${}^{m}C_{0}, (-3)\,{}^{m}C_{1}$ and $9\ {}^{m}C_{2}$.

Therefore, by the given condition

${}^{m}C_{0} - 3\,{}^{m}C_{1} + 9\,{}^{m}C_{2} = 559$

$1 - 3m + \frac{9m(m-1)}{2} = 559$

On solving we get $m = 12$

$T_{r+1} = {}^{12}C_r (x)^{12-r} \left(\frac{-3}{x^2}\right)^r$

$= {}^{12}C_r (x)^{12-r} .(-3)^r .(x)^{-2r}$

$= {}^{12}C_r (x)^{12-3r} .(-3)^r$

$12 - 3r = 3 \Rightarrow r = 3$, *req*. term is -5940 x^3