MATHEMATICS

Important Questions

Multiple Choice questions-

1. If cos (α + β) = 0, then sin (α – β) can be reduced to

(a) cos β

(b) cos 2β

(c) sin α

(d) sin 2α

2. If cos (40° + A) = sin 30°, the value of A is:?

(a) 60°

(b) 20°

(c) 40°

(d) 30°

3. If sin x + cosec x = 2, then $\sin^{19}x + \text{cosec}^{20}x =$

(a) 2^{19}

(b) 2^{20}

(c) 2

(d) 2^{39}

4. If cos 9a = sin a and 9a < 90°, then the value of tan 5a is

(a) $\frac{1}{\sqrt{3}}$

(b) $\sqrt{3}$

(c) 1

(d) 0

5. (1 + tanθ + secθ) (1 + cotθ – cosecθ) is equal to

(a) 0

(b) 1

(c) 2

(d) -1

6. Ratios of sides of a right triangle with respect to its acute angles are known as

(a) trigonometric identities

(b) trigonometry

(c) trigonometric ratios of the angles

(d) none of these

7. The value of cos θ cos(90° – θ) – sin θ sin (90° – θ) is:

(a) 1

(b) 0

(c) -1

(d) 2

8. If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ =

(a) ab

(b) $b^2 + a^2$

(c) a^2b^2

(d) a^4b^4

9. If x and y are complementary angles, then

(a) sin x = sin y

(b) tan x = tan y

(c) cos x = cos y

(d) sec x = cosec y

10. sin (45° + θ) – cos (45° – θ) is equal to

(a) $2 \cos \theta$

(b) 0

(c) $2 \sin \theta$

(d) 1

Very Short Questions:

1. Find maximum value of $\frac{1}{\sec\theta}$, $0° \leq \theta \leq 90°$.
2. Given that $\sin \theta = \frac{a}{b}$, find the value of $\tan \theta$.
3. If $\sin \theta = \cos \theta$, then find the value of $2 \tan \theta + \cos^2 \theta$.
4. If $\sin (x - 20)° = \cos (3x - 10)°$, then find the value of x.
5. If $\sin^2 A = \frac{1}{2}\tan^2 45°$, where A is an acute angle, then find the value of A.
6. If $x = a \cos \theta$, $y = b \sin \theta$, then find the value of $b^2x^2 + a^2y^2 - a^2b^2$.
7. If $\tan A = \cot B$, prove that $A + B = 90°$.
8. If $\sec A = 2x$ and $\tan A = 2x$, find the value of $2(x^2 - \frac{1}{x^2})$.
9. In a ΔABC, if $\angle C = 90°$, prove that $\sin^2 A + \sin^2 B = 1$.
10. If $\sec 4A = \text{cosec} (A - 20°)$ where 4 A is an acute angle, find the value of A.

Short Questions :

1. If $\sin A = \frac{3}{4}$, calculate cos A and tan A.
2. Given $15 \cot A = 8$, find sin A and sec A.
3. In Fig. 10.5, find $\tan P - \cot R$.

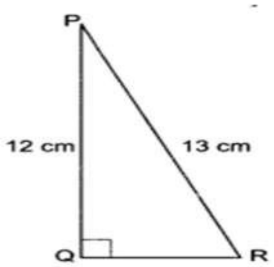


Fig. 10.5

4. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

5. Prove that $\frac{1-s}{1+sin}$ = $(\sec\theta - \tan\theta)^2$

6. $$\frac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° . \sec^2 52° - \sin^2 45°.$$

7. $$\frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45° . \tan 20° . \tan 40° . \tan 50° . \tan 70°}{5}$$

8. $$\frac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\frac{\sin(90° - \theta) . \sin\theta}{\tan\theta} + \frac{\cos(90° - \theta) . \cos\theta}{\cot\theta}\right]$$

9. Evaluate: sin 25° cos 65° + cos 25° sin 65°.

10. Without using tables, evaluate the following:

$3\cos 68°. \text{cosec } 22° - \frac{1}{2}\tan 43°. \tan 47°. \tan 12°. \tan 60°. \tan 78°$

Long Questions :

1. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

2. In triangle ABC right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ find the value of:

(i) sin A cos C + cos A sin C (ii) cos A cos C – sin A sin C.

3. If $\cot\theta = \frac{7}{8}$, evaluate:

(i)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

(ii) $\cot^2\theta$

4. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

5. Write all the other trigonometric ratios of ∠A in terms of sec A.

6. Prove that

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$$

7. Prove that:

$$\tan^2 A-\tan^2 B=\frac{\cos^2 B-\cos^2 A}{\cos^2 B\cos^2 A}=\frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}.$$

8. Prove that:

$$\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2+2\tan^2 A=2\sec^2 A.$$

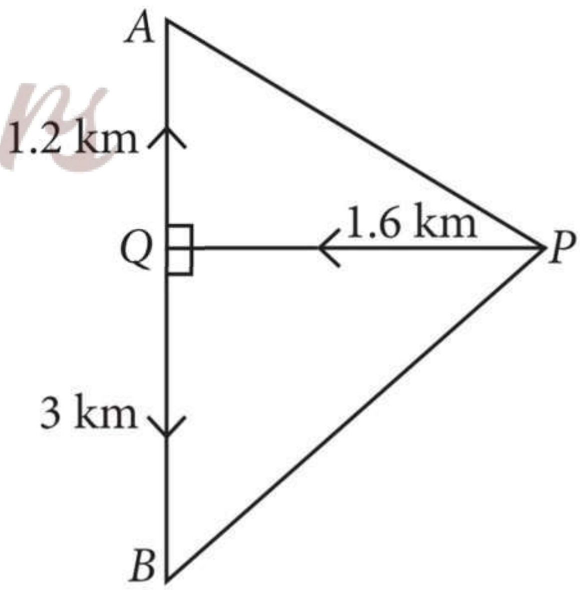
9. Prove that: $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2=(1+\sec\theta\operatorname{cosec}\theta)^2$.

10. Prove that:

$$\frac{1}{(\operatorname{cosec} x+\cot x)}-\frac{1}{\sin x}=\frac{1}{\sin x}-\frac{1}{(\operatorname{cosec} x-\cot x)}.$$

Assertion Reason Questions-

1. Two aeroplanes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aeroplanes is 400km/ hr and 500km/ hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.



i. Find $\tan\theta$ if $\angle APQ = \theta$.

a. $\frac{1}{2}$

b. $\frac{1}{\sqrt{2}}$

c. $\frac{\sqrt{3}}{2}$

d. $\frac{3}{4}$

ii. Find $\cot B$.

a. $\frac{3}{4}$

b. $\frac{15}{4}$

c. $\frac{3}{8}$

d. $\frac{15}{8}$

iii. Find $\tan A$.

a. 2

b. $\sqrt{2}$

c. $\frac{4}{3}$

d. $\frac{2}{\sqrt{3}}$

iv. Find $\sec A$.

a. 1

b. $\frac{2}{3}$

c. $\frac{4}{3}$

d. $\frac{5}{3}$

v. Find cosec B.

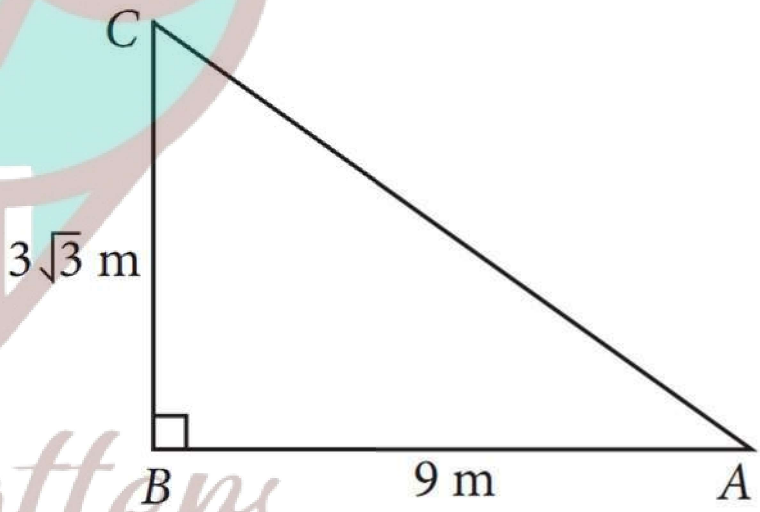
a. $\frac{17}{8}$

b. $\frac{12}{5}$

c. $\frac{5}{12}$

d. $\frac{8}{17}$

2. Three friends - Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to find both of them. If the positions of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that AB = 9 m, BC = $\sqrt{3}$ M and $\sqrt{3}$ m and $\angle B = 90°$, then answer the following questions.



i. The measure of $\angle A$ is:

a. 30º
b. 45º
c. 60º
d. None of these.

ii. The measure of $\angle C$ is:

a. 30º
b. 45º
c. 60º
d. None of these.

iii. The length of AC is:

a. $2\sqrt{3}$ m

b. $\sqrt{3}$ m

c. $4\sqrt{3}$ m

d. $6\sqrt{3}$ m

iv. $\cos 2A =$

a. 0

b. $\frac{1}{2}$

c. $\frac{1}{\sqrt{2}}$

d. $\frac{\sqrt{3}}{2}$

v. $\text{Sin}\left(\frac{C}{2}\right) =$

a. 0

b. $\frac{1}{2}$

c. $\frac{1}{\sqrt{2}}$

d. $\frac{\sqrt{3}}{2}$

Assertion Reason Questions-

1. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is false.

   d. A is false; R is true.

   Assertion: The value of each of the trigonometric ratios of an angle does not depend on the size of the triangle. It only depends on the angle.

   Reason: In right $\Delta ABC, \angle B = 90^\circ$ and $\angle A = \theta^\circ$ $\sin\theta = \frac{BC}{AC} < 1$ and $\cos\theta = \frac{AB}{AC} < 1$ as hypotenuse is the longest side.

2. **Directions:** Each of these questions contains two statements: Assertion [A] and Reason [R]. Each of these questions also has four alternative choices, any one of which is the correct answer. You have to select one of the codes [a], [b], [c] and [d] given below.

   a. A is true, R is true; R is a correct explanation for A.

   b. A is true, R is true; R is not a correct explanation for A.

   c. A is true; R is false.

   d. A is false; R is true.

   Assertion: Sin 60° = Cos 30°

   Reason: Sin 2θ = Sin θ where θ is an acute angle.

Answer Key-

Multiple Choice questions-

1. (b) cos 2β

2. (b) 20°
3. (c) 2
4. (c) 1
5. (c) 2
6. (c) trigonometric ratios of the angles
7. (b) 0
8. (c) a^2b^2
9. (d) sec x = cosec y
10. (b) 0

Very Short Answer :

1. $\frac{1}{sec\theta}$, (0° ≤ θ ≤ 90°) (Given)

∵ sec θ is in the denominator

∴ The min. value of sec θ will return max. value for $\frac{1}{sec\theta}$.

But the min. value of sec θ is sec 0° = 1.

Hence, the max. value of $\frac{1}{sec\theta^\circ} = \frac{1}{1} = 1$

2. $\sin\theta = \frac{a}{b}$

$$\Rightarrow \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{\frac{b^2-a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a/b}{\frac{\sqrt{b^2-a^2}}{b}} = \frac{a}{\sqrt{b^2-a^2}}$$

3. sin θ = cos θ (Given)

It means value of θ = 45°

Now, 2 tan θ + $\cos^2$ θ = 2 tan 45° + $\cos^2$ 45°

4. $\sin(x - 20)^\circ = \cos(3x - 10)^\circ$

$\Rightarrow \cos[90^\circ - (x - 20)^\circ] = \cos(3x - 10)^\circ$

By comparing the coefficient

$90^\circ - x^\circ + 20^\circ = 3x^\circ - 10^\circ = 110^\circ + 10^\circ = 3x^\circ + x^\circ$

$120^\circ = 4x^\circ$

$\Rightarrow \frac{120^\circ}{4} = 30^\circ$

5. $\sin^2 A = 12\tan^2 45^\circ$

$\Rightarrow \sin 2A = \frac{1}{2}(1)^2$ [$\because \tan 45^\circ = 1$]

$= \sin 2A = \frac{1}{2}$

$\Rightarrow \sin A = \frac{1}{\sqrt{2}}$

Hence, $\angle A = 45^\circ$

6. Given $x = a\cos\theta$, $y = b\sin\theta$

$b^2x^2 + a^2y^2 - a^2b^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2 - a^2b^2$

$= a^2b^2\cos^2\theta + a^2b^2\sin^2\theta - a^2b^2 = a^2b^2(\sin^2\theta + \cos^2\theta) - a^2b^2$

$= a^2b^2 - a^2b^2 = \theta$ ($\because \sin 2\theta + \cos^2\theta = 1$)

7. We have

$\tan A = \cot B$

$\Rightarrow \tan A = \tan(90^\circ - B)$

$A = 90^\circ - B$

[$\because$ Both A and B are acute angles]

$\Rightarrow A + B = 90^\circ$

8.

$$2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\frac{\sec^2 A}{4} - \frac{\tan^2 A}{4}\right) = \frac{2}{4}(\sec^2 A - \tan^2 A) = \frac{1}{2} \times 1 = \frac{1}{2}$$

9. Since $\angle C = 90°$

$\therefore \angle A + \angle B = 180° - \angle C = 90°$

Now, $\sin^2 A + \sin^2 B = \sin^2 A + \sin^2 (90° - A) = \sin^2 A + \cos^2 A = 1$

10. We have

$\sec 4A = \text{cosec}(A - 20°)$

$\Rightarrow \text{cosec}(90° - 4A) = \text{cosec}(A - 20°)$

$\therefore 90° - 4A = A - 20°$

$\Rightarrow 90° + 20° = A + 4A$

$\Rightarrow 110° = 5A$

$\therefore A = \frac{110}{5} = 22°$

Short Answer :

1. Let us first draw a right ΔABC in which $\angle C = 90°$.

Now, we know that

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{3}{4}$$

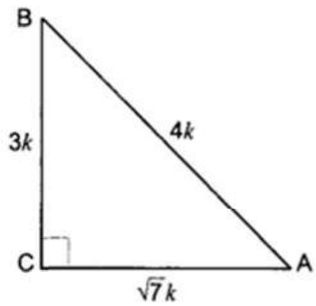


Fig. 10.3

Let $BC = 3k$ and $AB = 4k$, where k is a positive number.

Then, by Pythagoras Theorem, we have

$AB^2 = BC^2 + AC^2 \quad \Rightarrow \quad (4k)^2 = (3k)^2 + AC^2$

$\Rightarrow \quad 16k^2 - 9k^2 = AC^2 \quad \Rightarrow \quad 7k^2 = AC^2$

$\therefore \quad AC = \sqrt{7}k$

$\therefore \quad \cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$ and $\tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

2. Let us first draw a right ΔABC in which $\angle B = 90°$.

Now, we have, $15 \cot A = 8$

$$\therefore \quad \cot A = \frac{8}{15} = \frac{AB}{BC} = \frac{\text{Base}}{\text{Perpendicular}}$$

Let $AB = 8k$ and $BC = 15k$

Then, $AC = \sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras Theorem)

$$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$$

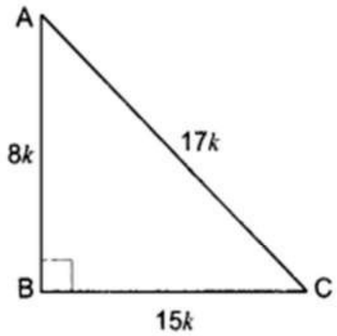


Fig. 10.4

$$\therefore \quad \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\text{and,} \quad \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$$

3. Using Pythagoras Theorem, we have

$PR^2 = PO^2 + QR^2$

$\Rightarrow (13)2 = (12)^2 + QR^2$

$\Rightarrow 169 = 144 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow QR = 5$ cm

Now, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$ and $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

4. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\cos\theta = 2$ ($\because \sin^2\theta + \cos^2\theta = 1$)

$\Rightarrow \sin\theta.\cos\theta = 1 = \sin^2\theta + \cos^2\theta$

$$\Rightarrow \quad 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$\Rightarrow 1 = \tan\theta + \cot\theta = 1$

Therefore $\tan\theta + \cot\theta = 1$

5.

$$\text{LHS} = \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta} \quad \text{[Rationalising the denominator]}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\sec\theta-\tan\theta)^2 = \text{RHS}$$

6.

We have, $$\frac{\sec^2 54° - \cot^2 36°}{\operatorname{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° . \sec^2 52° - \sin^2 45°$$

$$= \frac{\sec^2(90°-36°)-\cot^2 36°}{\operatorname{cosec}^2(90°-33°)-\tan^2 33°} + 2\sin^2 38° . \sec^2(90°-38°) - \sin^2 45°$$

$$= \frac{\operatorname{cosec}^2 36° - \cot^2 36°}{\sec^2 33° - \tan^2 33°} + 2\sin^2 38° . \operatorname{cosec}^2 38° - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{1} + 2.1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}$$

7.

We have, $$\frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45° . \tan 20° . \tan 40° . \tan 50° . \tan 70°}{5}$$

$$= \frac{2\sin(90°-22°)}{\cos 22°} - \frac{2\cot 15°}{5\tan(90°-15°)} - \frac{3\tan 45° . \tan 20° . \tan 40° . \tan(90°-40°) . \tan(90°-20°)}{5}$$

$$= \frac{2\cos 22°}{\cos 22°} - \frac{2\cot 15°}{5\cot 15°} - \frac{3\tan 45° . \tan 20° . \tan 40° . \cot 40° . \cot 20°}{5}$$

$$= 2 - \frac{2}{5} - \frac{3\tan 45° . (\tan 20° . \cot 20°) . (\tan 40° . \cot 40°)}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} . 1 . 1 . 1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1$$

8.

We have $\dfrac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\dfrac{\sin(90° - \theta).\sin\theta}{\tan\theta} + \dfrac{\cos(90° - \theta).\cos\theta}{\cot\theta}\right]$

$$= \frac{\sin^2 20° + \sin^2(90° - 20°)}{\cos^2 20° + \cos^2(90° - 20°)} + \left[\frac{\cos\theta.\sin\theta}{\tan\theta} + \frac{\cos\theta.\sin\theta}{\cot\theta}\right]$$

$$= \frac{\sin^2 20° + \cos^2 20°}{\cos^2 20° + \sin^2 20°} + \left[\frac{\cos\theta.\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta.\sin\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$= \frac{1}{1} + [\cos^2\theta + \sin^2\theta] = 1 + 1 = 2.$$

9. sin 25°. cos 65° + cos 25° . sin 65°

= sin (90° – 65°). cos 65° + cos (90° – 65°). sin 65°

= cos 65° . cos 65° + sin 65°. sin 65°

= cos2 65° + sin2 65° = 1.

10. We have,

3 cos 68°. cosec 22° – $\frac{1}{2}$ tan 43°. tan 47°. tan 12°. tan 60°. tan 78°.

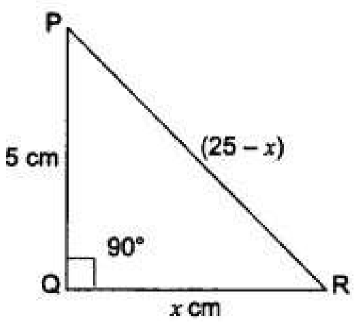
= 3 cos (90° – 22°). cosec 22° – $\frac{1}{2}$. {tan 43° . tan (90° – 43°)}. {tan 12°. tan (90° – 12°). tan 60°}

= 3 sin 22°. cosec 22° – $\frac{1}{2}$ (tan 43° . cot 43°). (tan 12°. cot 12°). tan 60°

= 3 × 1 – × 1 × 1 × √3 = $3 - \frac{3}{\sqrt{2}} = \frac{6-\sqrt{3}}{\sqrt{2}}$.

Long Answer :

1.



We have a right-angled ΔPQR in which ∠Q = 90°.

Let QR = x cm

Therefore, PR = (25 – x) cm

By Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$(25 - x)^2 = 52 + x^2$

$= (25 - x)^2 - x^2 = 25$

(25 – x – x) (25 – x + x) = 25

(25 – 2x) 25 = 25

25 – 2x = 1

25 – 1 = 2x

= 24 = 2x

∴ x = 12 cm

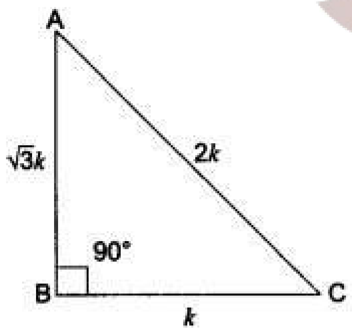
Hence, QR = 12 cm

PR = (25 – x) cm = 25 – 12 = 13 cm

PQ = 5 cm

$$\therefore \quad \sin P = \frac{QR}{PR} = \frac{12}{13}; \quad \cos P = \frac{PQ}{PR} = \frac{5}{13}; \quad \tan P = \frac{QR}{PQ} = \frac{12}{5} \text{ cm}$$

2.



We have a right-angled ΔABC in which ∠B = 90°.

and, $\tan A = \frac{1}{\sqrt{3}}$

Now, tan A = $\frac{1}{\sqrt{3}}$ = BCAB

Let BC = k and AB = √3k

∴ By Pythagoras Theorem, we have

⇒ $AC^2 = AB^2 + BC^2$

⇒ $AC^2 = (\sqrt{3}k)^2 + (k)^2 = 3k^2 + k^2$

⇒ $AC^2 = 4k^2$

Now, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$; $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$; $\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cdot \cos C + \cos A \cdot \sin C = \frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{1}{4}+\frac{3}{4} = \frac{4}{4} = 1$.

(ii) $\cos A \cdot \cos C - \sin A \cdot \sin C = \frac{\sqrt{3}}{2}\times\frac{1}{2}-\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} = 0$.

3. Let us draw a right triangle ABC in which ∠B = 90° and ∠C = θ.

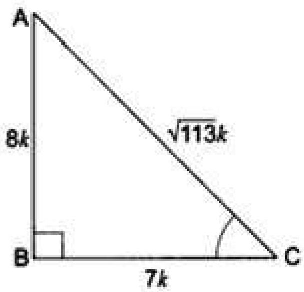


Fig. 10.9

We have, $\cot\theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB}$ (given)

Let $BC = 7k$ and $AB = 8k$

Therefore, by Pythagoras Theorem

$$AC^2 = AB^2 + BC^2 = (8k)^2 + (7k)^2 = 64k^2 + 49k^2$$

$$AC^2 = 113k^2 \quad \therefore \quad AC = \sqrt{113}k$$

$$\therefore \quad \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\text{and} \quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$$

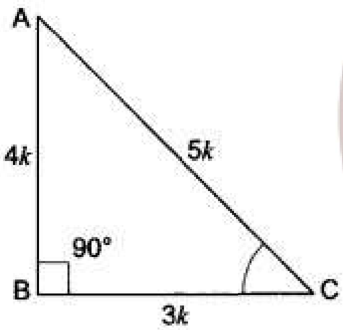
$$= \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}} = \frac{49}{64}$$

Alternate method:

(*i*) $$\frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

(*ii*) $$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

4.



Let us consider a right triangle ABC in which ∠B = 90°

$$\text{Now,} \quad \cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$$

Let AB = 4k and BC = 3k

∴ By Pythagoras Theorem

$AC^2 = AB^2 + BC^2$

$AC = (4k)^2 + (3k)^2 = 16k^2 + 9k^2$

$AC^2 = 25k^2$

∴ AC = 5k

$$\text{Therefore, } \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$$

$$\text{and, } \quad \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

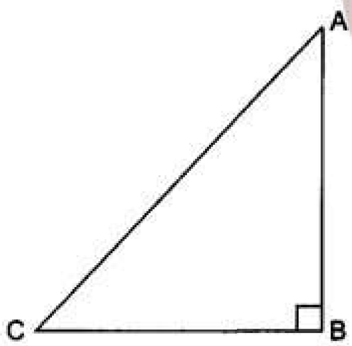
Now, $\text{LHS} = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Hence, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

5.



Let us consider a right-angled ΔABC in which ∠B = 90°.

For ∠A we have

$$\therefore \quad \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\Rightarrow \quad \frac{\sec A}{1} = \frac{AC}{AB} \quad \Rightarrow \quad AC = AB \sec A$$

Let $AB = k$ and $AC = k \sec A$

∴ By Pythagoras Theorem, we have

$$AC^2 = AB^2 + BC^2 \quad \Rightarrow \quad k^2 \sec^2 A = k^2 + BC^2$$

$$\therefore BC^2 = k^2 \sec^2 A - k^2 \quad \Rightarrow \quad BC = k\sqrt{\sec^2 A - 1}$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{k\sqrt{\sec^2 A-1}}{k \sec A} = \frac{\sqrt{\sec^2 A-1}}{\sec A}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k\sec A} = \frac{1}{\sec A}$$

$$\tan A = \frac{BC}{AB} = \frac{k\sqrt{\sec^2 A-1}}{k} = \sqrt{\sec^2 A-1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$$

$$\operatorname{cosec} A = \frac{AC}{BC} = \frac{k\sec A}{k\sqrt{\sec^2 A-1}} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$$

6.

$$\text{LHS} = \left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$\text{RHS} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A-1}\times\tan A\right)^2 = (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

7.

$$\text{LHS} = \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A\cos^2 B - \cos^2 A\sin^2 B}{\cos^2 A\cos^2 B} = \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A\cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A\cos^2 B - \cos^2 A + \cos^2 A\cos^2 B}{\cos^2 A\cos^2 B} = \frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B}$$

Also $$\frac{\cos^2 B - \cos^2 A}{\cos^2 A\cos^2 B} = \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A\cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \text{RHS.}$$

8.

$$\text{LHS} = \frac{\text{cosec } A}{(\text{cosec } A - 1)} + \frac{\text{cosec } A}{(\text{cosec } A + 1)}$$

$$= \frac{\text{cosec } A(\text{cosec } A + 1) + \text{cosec } A(\text{cosec } A - 1)}{(\text{cosec } A - 1)(\text{cosec } A + 1)}$$

$$= \frac{\text{cosec}^2 A + \text{cosec} A + \text{cosec}^2 A - \text{cosec } A}{(\text{cosec}^2 A - 1)} = \frac{2\,\text{cosec}^2 A}{1 + \cot^2 A - 1} = \frac{2\,\text{cosec}^2 A}{\cot^2 A}$$

$= 2\,\text{cosec}^2 A \tan^2 A = 2(1 + \cot^2 A).\ \tan^2 A$

$= 2\tan^2 A + 2\tan^2 A.\ \cot^2 A$ ($\because \tan A \cot A = 1$)

$= 2 + 2\tan^2 A = 2(1 + \tan^2 A) = 2\sec^2 A$ = RHS.

9.

LHS = $(\sin\theta + \sec\theta)^2 + (\cos\theta + \text{cosec}\,\theta)^2$

$$= \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2 = \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta}\right)^2 + \left(\frac{\cos\theta\sin\theta + 1}{\sin\theta}\right)^2$$

$$= \frac{(\sin\theta\cos\theta + 1)^2}{\cos^2\theta} + \frac{(\cos\theta\sin\theta + 1)^2}{\sin^2\theta} = (\sin\theta\cos\theta + 1)^2\left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\right)$$

$$= (\sin\theta\cos\theta + 1)^2\left(\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}\right) = (\sin\theta\cos\theta + 1)^2.\left(\frac{1}{\cos^2\theta\sin^2\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta + 1}{\cos\theta\sin\theta}\right)^2 = \left(1 + \frac{1}{\cos\theta\sin\theta}\right)^2$$

$$= (1 + \sec\theta\,\text{cosec}\,\theta)^2 = \text{RHS.}$$

$= (1 + \sec\theta\ \text{cosec}\,\theta)^2$ = RHS.

10. In order to show that,

It is sufficient to show

$$\frac{1}{\text{cosec } x + \cot x} + \frac{1}{\text{cosec } x - \cot x} = \frac{1}{\sin x} + \frac{1}{\sin x}$$

$$\Rightarrow \quad \frac{1}{(\text{cosec } x + \cot x)} + \frac{1}{(\text{cosec } x - \cot x)} = \frac{2}{\sin x} \quad \ldots(i)$$

Now, LHS of above is

$$\frac{1}{(\text{cosec } x + \cot x)} + \frac{1}{(\text{cosec } x - \cot x)}$$

$$= \frac{(\text{cosec } x - \cot x) + (\text{cosec } x + \cot x)}{(\text{cosec } x - \cot x)(\text{cosec } x + \cot x)}$$

$$= \frac{2 \text{ cosec } x}{\text{cosec}^2 x - \cot^2 x} \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{2 \text{ cosec } x}{1} = \frac{2}{\sin x} = \text{RHS of } (i)$$

Hence, $\frac{1}{(\text{cosec } x + \cot x)} + \frac{1}{(\text{cosec } x - \cot x)} = \frac{1}{\sin x} + \frac{1}{\sin x}$

or $\frac{1}{(\text{cosec } x + \cot x)} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{(\text{cosec } x - \cot x)}$.

Case Study Answers:

1. **Answer :**

i. (d) $\frac{3}{4}$

Solution:

In $\triangle \text{APQ}, \tan\theta = \frac{\text{AQ}}{\text{PQ}} = \frac{1.2}{1.6} = \frac{3}{4}$

ii. (d) $\frac{15}{8}$

Solution:

In $\triangle \text{PBQ}, \cot \text{B} = \frac{\text{QB}}{\text{PQ}} = \frac{3}{1.6} = \frac{15}{8}$

iii. (c) $\frac{4}{3}$

Solution:

In $\triangle \text{APQ}, \tan \text{A} = \frac{\text{PQ}}{\text{AQ}} = \frac{1.6}{1.2} = \frac{4}{3}$

iv. (d) $\frac{5}{3}$

Solution:

We have, $\tan^2 A + 1 = \sec^2 A$

$\Rightarrow \sqrt{\left(\frac{4}{3}\right)^2 + 1}$

$= \sqrt{\frac{16}{9} + 1} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

v. (a) $\frac{17}{8}$

Solution:

Since, $\operatorname{cosec} B = \sqrt{\cot^2 B + 1}$

$= \sqrt{\left(\frac{15}{8}\right)^2 + 1}$

$= \frac{17}{8}$

2. Answer :

i. (a) 30º

Solution:

We have, AB = 9 m, BC $= \sqrt{3}\,\mathrm{m}$ in $\triangle ABC$, we have

$\tan A = \frac{BC}{AB} = \frac{3\sqrt{3}}{9} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan A = \tan 30^\circ \Rightarrow \angle A = 30^\circ$

ii. (c) 60º

Solution:

Similarly, $\tan C = \frac{AB}{BC} = \frac{9}{3\sqrt{3}} = \sqrt{3}$

$\Rightarrow \tan C = \tan 60^\circ \Rightarrow \angle C = 60^\circ$

iii. (d) $6\sqrt{3}\ \mathrm{m}$

Solution:

Since, $\sin A = \frac{BC}{AC} \Rightarrow \sin 30^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{3\sqrt{3}}{AC} \Rightarrow AC = 6\sqrt{3}\ \mathrm{m}$

iv. (b) $\frac{1}{2}$

Solution:

$\because \angle A = 30^\circ$

$\therefore \cos 2A = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}$

v. (b) $\frac{1}{2}$

Solution:

$\because \angle C = 60^\circ$

$\therefore \sin\left(\frac{C}{2}\right) = \sin\left(\frac{60^\circ}{2}\right) = \sin 30^\circ = \frac{1}{2}$

Assertion Reason Answer-

1. (b) A is true, R is true; R is not a correct explanation for A.
2. (c) A is true; R is false.