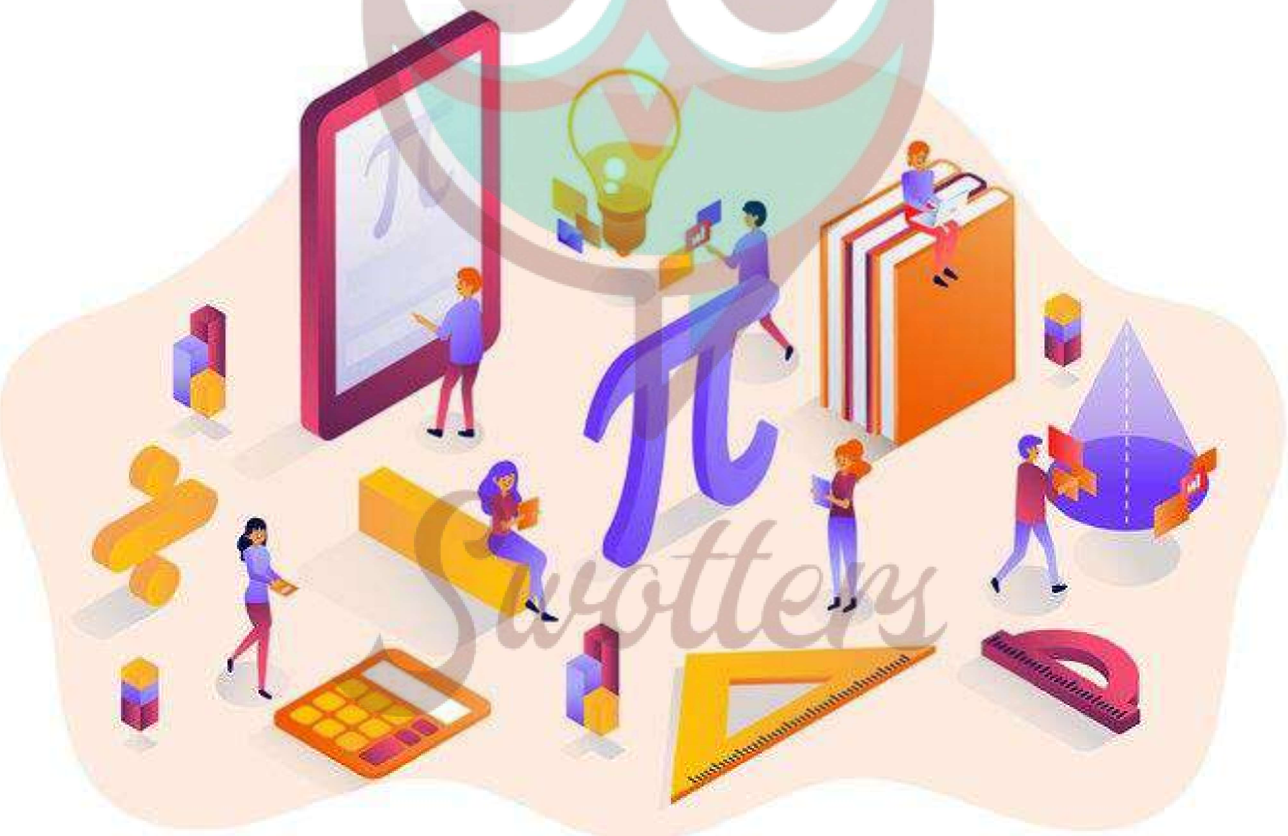
MATHEMATICS



Important Questions

Multiple Choice questions-

1. The degree of the differential equation:

$(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + \sin(\frac{dy}{dx}) + 1 = 0$ is

(a) 3

(b) 2

(c) 1

(d) not defined.

2. The order of the differential equation:

$2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

(a) 2

(b) 1

(c) 0

(d) not defined.

3. The number of arbitrary constants in the general solution of a differential equation of fourth order is:

(a) 0

(b) 2

(c) 3

(d) 4.

4. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(a) 3

(b) 2

(c) 1

(d) 0.

5. Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?

(a) $\frac{d^2y}{dx^2} + y = 0$

(b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + 1 = 0$

(d) $\frac{d^2y}{dx^2} - 1 = 0$

6. Which of the following differential equations has y = x as one of its particular solutions?

(a) $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$

(b) $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = x$

(c) $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$

(d) $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = 0$

7. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} = c$

(b) $e^x + e^y = c$

(c) $e^{-x} + e^y = c$

(d) $e^{-x} + e^{-y} = c$.

8. Which of the following differential equations cannot be solved, using variable separable method?

(a) $\frac{dy}{dx} + e^{x+y} + e^{-x+y}$

(b) $(y^2 - 2xy)\, dx = (x^2 - 2xy)\, dy$

(c) $xy \frac{dy}{dx} = 1 + x + y + xy$

(d) $\frac{dy}{dx}$ + y = 2.

9. A homogeneous differential equation of the form $\frac{dy}{dx} = h(\frac{x}{y})$ can be solved by making the substitution.

(a) y = vx

(b) v = yx

(c) x = vy

(d) x = v

10. Which of the following is a homogeneous differential equation?

(a) (4x + 6y + 5)dy – (3y + 2x + 4)dx = 0

(b) xy dx – ($x^3 + y^2$)dy = Q

(c) ($x^3 + 2y^2$) dx + 2xy dy = 0

(d) y^2 dx + ($x^2 - xy - y^2$)dy = 0.

Very Short Questions:

1. Find the order and the degree of the differential equation: $x^2 \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^4$ (Delhi 2019)

2. Determine the order and the degree of the differential equation: $\left(\frac{dy}{dx}\right)^3 + 2y\frac{d^2y}{dx^2} = 0$ (C.B.S.E. 2019 C)

3. Form the differential equation representing the family of curves: y = b (x + a), where « and b are arbitrary constants. (C.B.S.E. 2019 C)

4. Write the general solution of differential equation:

   $\frac{dy}{dx} = e^{x+y}$ (C.B.S.E. Sample Paper 2019-20)

5. Find the integrating factor of the differential equation:

   $y\frac{dy}{dx} - 2x = y^3e^{-y}$

6. Form the differential equation representing the family of curves y = a sin (3x – b), where a

and b are arbitrary constants. (C.B.S.E. 2019C)

Short Questions:

1. Determine the order and the degree of the differential equation:

2. Form the differential equation representing the family of curves: $y = e^{2x}(a + bx)$, where 'a' and 'h' are arbitrary constants. (Delhi 2019)

3. Solve the following differentia equation:

   $\frac{dy}{dx} + y = \cos x - \sin x$ (Outside Delhi 2019)

4. Solve the following differential equation:

   $\frac{dx}{dy} + x = (\tan y + \sec 2y)$. (Outside Delhi 2019 C)

Long Questions:

1. Find the area enclosed by the circle:

   $x^2 + y^2 = a^2$. (N.C.E.R.T.)

2. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. (C.B.S.E. 2018)

3. Find the area bounded by the curves $y = \sqrt{x}$, $2y + 3 = Y$ and Y-axis. (C.B.S.E. Sample Paper 2018-19)

4. Find the area of region:

   $\{(x,y): x^2 + y^2 < 8, x^2 < 2y\}$. (C.B.S.E. Sample Paper 2018-19)

Case Study Questions:

1. If the equation is of the form $\frac{dy}{dx} = py = Q,$ where P, Q are functions of x, then the solution of the differential equation is given by ye $ye^{\int pdx} = \int Q\, e^{\int pdx} dx + c,$ where $e^{\int pdx}$ is called the integrating factor (I.F.).

Based on the above information, answer the following questions.

i. The integrating factor of the differential equation

$\sin x \frac{dy}{dx} + 2y \cos x = 1$ is $(\sin x)^{\lambda}$, where $\lambda =$

a. 0
b. 1
c. 2
d. 3

ii. Integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$ is:

a. $-x$

b. $\frac{x}{1+x^2}$

c. $\sqrt{1 - x^2}$

d. $\frac{1}{2}\log(1 - x^2)$

iii. The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$, is:

a. $y = e^x(x - 1)$

b. $y = xe^{-x}$

c. $y = xe^{-x} + 1$

d. $y = (x + 1)e^{-x}$

iv. General solution of $\frac{dy}{dx} + y \tan x = \sec x$ is:

a. $y \sec y = \tan x + c$

b. $y \tan x = \sec x + c$

c. $\tan x = y \tan x + c$

d. $x \sec x = \tan y + c$

v. The integrating factor of differential equation $\frac{dy}{dx} - 3y = \sin 2x$ is:

a. e^{3x}

b. e^{-2x}

c. e^{-3x}

d. xe^{-3x}

2. If the equation is of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ or $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, where f(x, y), g(x, y) are homogeneous functions of the same degree in x and y, then put y = vx And $\frac{dy}{dx} = v + x\left(\frac{dv}{dx}\right)$, so that the dependent variable y is changed to another variable v and then apply variable separable method.

Based on the above information, answer the following questions.

i. The general solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is:

a. $\tan^{-1}\frac{x}{y} = \log|x| + c$

b. $\tan^{-1}\frac{y}{x} = \log|x| + c$

c. $y = x\log|x| + c$

d. $x = y\log|y| + c$

ii. Solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is:

a. $x^3 + y^2 = cx^2$

b. $\frac{x^2}{2} + \frac{y^3}{3} = y^2 + c$

c. $x^2 + y^3 = cx^2$

d. $x^2 + y^2 = cx^3$

iii. General solution of the differential equation $(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$ is:

a. $\frac{x+y}{y} - \log x = c$

b. $\frac{x+y}{y} + \log x = c$

c. $\frac{x}{x+y} - \log x = c$

d. $\frac{x}{x+y} + \log x = c$

iv. General solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$ is:

a. $\log(xy) = c$

b. $\log y = cx$

c. $\log \frac{y}{x} = cx$

d. $\log x = cy$

v. Solution of the differential equation $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x^2 \cos x$ is:

a. $e^{\frac{y}{x}} - \sin x = c$

b. $e^{\frac{y}{x}} + \sin x = c$

c. $e^{\frac{-y}{x}} - \sin x = c$

d. $e^{\frac{-y}{x}} + \sin x = c$

Answer Key-

Multiple Choice questions-

1. Answer: (a) 3
2. Answer: (a) 2
3. Answer: (d) 4.

4. Answer: (d) 0.
5. Answer: (b) $\frac{d^2y}{dx^2} - y = 0$
6. Answer: (c) $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$
7. Answer: (a) $e^x + e^{-y} = c$
8. Answer: (b) $(y^2 - 2xy)\, dx = (x^2 - 2xy)\, dy$
9. Answer: (c) $x = vy$
10. Answer: (d) $y^2\, dx + (x^2 - xy - y^2)dy = 0$.

Very Short Answer:

1. Solution: Here, order = 2 and degree = 1.
2. Solution: Order = 2 and Degree = 1.
3. Solution:

   We have: $y= b(x + a)$ ...(1)

   Diff. w.r.t. x, b.

   Again diff. w.r.t. x, $\frac{d^2y}{dx^2} = 0$,

   which is the reqd. differential equation.
4. Solution:

   We have: $\frac{dy}{dx} = e^{x+y}$

   $\Rightarrow e^{-y}\, dy = e^x\, dx$ [Variables Separable

   Integrating, $\int e^{-y} dy + c = \int e^x dx$

   $\Rightarrow - e^{-y} + c = e^x$

   $\Rightarrow e^x + e^{-y} = c$.
5. Solution:

   The given equation can be written as.

$$\frac{dy}{dx} - \frac{2x}{y} = y^2e^{-y}.$$

$$\therefore \quad \text{I.F.} = e^{-\int \frac{2}{y}dy}$$

$$= e^{-2\log|y|} = e^{\log\frac{1}{y^2}} = \frac{1}{y^2}$$

6. Solution:

We have: y – a sin (3x – b) ...(1)

Diff. W.r.t y $\frac{dy}{dx}$ = a cos (3x – b) .3

= 3a cos (3x – b)

$\frac{d^2y}{dx^2}$ = -3a sin (3x – b) 3

= -9a sin (3x – b)

= -9y [Using (1)]

$\frac{d^2y}{dx^2}$ + 9y = 0,m

which in the reqd. differential equation.

Short Answer:

1. Solution: Order = 2 and Degree = 1.
2. Solution:

We have: y = e^{2x} (a + bx) ...(1)

Diff. w.r.t. x, $\frac{dy}{dx}$ = e^{2x} (b) + 2e2x (a + bx)

⇒ $\frac{dy}{dx}$ = be^{2x} + 2y .... (2)

Again diff. w.r.t. x,

$\frac{d^2y}{dx^2}$ = $2be^{2x}$ + 2^{2x}

$\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - 2y\right) + \frac{dy}{dx}$

[Using (2)]

Hence, $\frac{d^2y}{dx^2}$ -4 $\frac{dy}{dx}$ + 4y = 0, which is the reqd. differential equation.

3. Solution:

   The given differential equation is:

   $\frac{dy}{dx}$ + y = cos x – sin x dx Linear Equation

   ∴ I.F. = $e^{\int 1dx}$ = ex

   The solution is :

   y.e^x = ∫ (cos x — sin x) e^x dx + C

   ⇒ y.e^x = e^x cos x + C

   or y = cos x + C e^{-x}

4. Solution:

   The given differential equation is:

   $\frac{dx}{dy}$ + x = (tany + $\sec^2$y).

   Linear Equation

   ∵ I.F. = Jldy = ey

   ∴ The solution is:

   x. ey = ∫ ey (tan y + $\sec^2$ y)dy + c

   ⇒ x. ey = ey tan y + c

   = x = tan y + c e^{-y}, which is the reqd. solution.

Long Answer:

1. Solution:

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{\frac{y^2}{x^2} - 1}{\frac{2y}{x}}$$

Put $\frac{y}{x} = v$

$\Rightarrow$ $y = vx$ and so $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ $v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\Rightarrow$ $\frac{x\,dv}{dx} = -\frac{(1 + v^2)}{2v}$

$\Rightarrow$ $\int \frac{dx}{x} = -\int \frac{2v\,dv}{1 + v^2}$

$\log x = -\log(1 + v^2) + \log C$

$x(1 + v^2) = C$

$x\left(1 + \frac{y^2}{x^2}\right) = C$

$x^2 + y^2 = C.$

2. Solution:

$$\frac{dy}{dx} + \frac{2x}{1 + x^2}y = \frac{1}{(1 + x^2)^2}$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}} + e^{\log(1+x^2)} = (1 + x^2).$$

Solution is $y(1 + x^2) = \int \frac{1}{1+x^2}dx$

$= \tan^{-1} x + C$

When $y = 0$, $x = 1$,

then $0 = \frac{\pi}{4} + C$

$C = \frac{\pi}{4}$

$\therefore$ y (1 + x^2) = $\tan^{-1} x - \frac{\pi}{4}$

i.e, y = $\frac{\tan^{-1} x}{1+x^2} - \frac{\pi}{4(1+x^2)}$

3. Solution:

We have: y = ae^{bx+5} + 5 ...(1)

Diff. w.r.t. x, $\frac{dy}{dx}$ = ae^{bx+5}. (b)

$\frac{dy}{dx}$ = dy ……(2) [Using (1)]]

Again diff. w.r.t x.,

$\frac{d^2y}{dx^2}$ = b $\frac{dy}{dx}$ ………(3)

Dividing (3) by (2),

$$\frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{\frac{dy}{dx}}{y}$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0,$$

which is the required differential equation.

4. Solution:

The given differential equation is:

$x\,dx - ye^y\sqrt{1+x^2}\,dy = 0$

$\Rightarrow \dfrac{x\,dx}{\sqrt{1+x^2}} - ye^y\,dy = 0$ [*Variables Separable*]

Integrating, $\int \dfrac{x\,dx}{\sqrt{1+x^2}} - \int ye^y\,dy = c$ ...(1)

Now, $\int \dfrac{x}{\sqrt{1+x^2}}\,dx = \dfrac{1}{2}\int (1+x^2)^{-1/2}(2x)\,dx$

$= \dfrac{1}{2}\dfrac{(1+x^2)^{1/2}}{1/2} = \sqrt{1+x^2}$.

And, $\int y\,e^y\,dy = y\cdot e^y - \int (1)\,e^y\,dy$

[*Integrating by parts*

$= ye^y - e^y$.

$\therefore$ From (1), $\sqrt{1+x^2} - (ye^y - e^y) = c$

$\Rightarrow \sqrt{1+x^2} = c + e^y(y-1)$ ...(2)

When x = 0, y = 1, ∴ 1 = c + c(0) ⇒ c = 1.

Putting in (2), $\sqrt{1 + x^2} = 1 + e^y(y - 1)$,

which is the reqd. particular solution.

Case Study Answers:

1. Answer :

i. (c) 2

Solution:

The given differential equation can be written as $\dfrac{dy}{dx} + 2y\cot x = \operatorname{cosec} x$

$\therefore$ I.F $= e^{\int 2\cot x dx} = e^{2\log|\sin x|} = (\sin x)^2$

$\therefore \lambda = 2$

ii. (c) $\sqrt{1-x^2}$

Solution:

We have, $(1-x^2)\frac{dy}{dx} - xy = 1$

$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2} \cdot y = \frac{1}{1-x^2}$

$\therefore$ I.F. $= e^{-\int \frac{x}{1-x^2} dx} = e^{\frac{1}{2}\int \frac{-2x}{1-x^2} dx}$

$= e^{\frac{1}{2}\log(1-x^2)} = e^{\log(1-x^2)^{\frac{1}{2}}} = \sqrt{1-x^2}$

iii. (b) $y = xe^{-x}$

Solution:

We have, $\frac{dy}{dx} + y = e^{-x}$

It is a linear differential equation with I.F. $= e^{\int dx} = e^x$

Now, solution is $y \cdot e^x = \int e^{-x}dx + c$

$\Rightarrow ye^x = \int dx + c$

$\Rightarrow ye^x = x + c$

$\Rightarrow y = xe^{-x} + ce^{-x}$

$\because y(0) = 0 \Rightarrow c = 0$

$\therefore y = xe^{-x}$

iv. (a) $y \sec y = \tan x + c$

Solution:

We have, $\frac{dy}{dx} + y \tan x = \sec x$

It is a linear differential equation with,

$$I.F. = e^{\int \tan x dx} = e^{\log|\sec x|} = \sec x$$

Now, solution is $y \sec x = \int \sec^2 x \, dx + c$

$$\Rightarrow y \sec x = \tan x + c$$

v. (c) e^{-3x}

Solution:

We have, $\frac{dy}{dx} 3y = \sin 2x$

It is a linear differential equation with,

$$I.F. = e^{\int -3dx} = e^{-3x}$$

2. Answer :

i. (b) $\tan^{-1} \frac{y}{x} = \log|x| + c$

Solution:

We have, $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

Put y = vx and $\frac{dy}{dx} = v+x\frac{dv}{dx}$

$$\therefore\ v+x\frac{dv}{dx} = \frac{x^2+x\times vx+v^2x^2}{x^2} = 1 + v + v^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v^2 \Rightarrow \int \frac{dv}{1+v^2} = \int \frac{dx}{x} + c$$

$$\Rightarrow \tan^{-1} v = \log|x| + c$$

$$\Rightarrow \tan^{-1} \frac{y}{x} = \log|x| + c$$

ii. (d) $x^2 + y^2 = cx^3$

Solution:

We have,

$$2xy\frac{dy}{dx} = x^2 + 3y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

Put y = vx and $\frac{dy}{dx} = v+x\frac{dv}{dx}$

$$\therefore\ v+x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2vx^2}$$

$$\Rightarrow\ x\frac{dv}{dx} = \frac{1+3v^2}{2v} - v$$

$$\Rightarrow\ x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow \int \frac{2v}{1+v^2}dv = \int \frac{dx}{x} + \log c$$

$$\Rightarrow \log|1+v^2| = \log|x| + \log|c|$$

$$\Rightarrow \log|v^2+1| = \log|xc|$$

$$\Rightarrow v^2 + 1 = xc \Rightarrow \frac{y^2}{x^2} + 1 = xc$$

$$\Rightarrow x^2 + y^2 = x^3c$$

iii. (d) $\frac{x}{x+y} + \log x = c$

Solution:

We have,

$(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$

$\Rightarrow \frac{x^2+3xy+y^2}{x^2} = \frac{dy}{dx}$

Put y= vx and $\frac{dy}{dx} = v+x\frac{dv}{dx}$

$\therefore \frac{x^2+3x^2v+x^2v^2}{x^2} = \left(v+x\frac{dv}{dx}\right)$

$\Rightarrow 1 + 3v + v^2 = v+x\frac{dv}{dx}$

$\Rightarrow 1 + 2v + v^2 = x\frac{dv}{dx}$

$\Rightarrow \int \frac{dx}{x} - \int (v+1)^{-2} = dv = c$

$\log x + \frac{1}{v+1} = c$

$\Rightarrow \log x + \frac{x}{x+y} = c$

iv. (c) $\log \frac{y}{x} = cx$

Solution:

We have, $\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore\ v + x\frac{dv}{dx} = v\{\log(v + 1\}$

$\Rightarrow x\frac{dv}{dx} = v\log v$

$\Rightarrow \int \frac{dv}{v\log v} = \int \frac{dx}{x} \Rightarrow \log|\log v| = \log|x| + \log|c|$

$\Rightarrow \log\left(\frac{y}{x}\right) = cx$

v. (a) $e^{\frac{y}{x}} - \sin x = c$

Solution:

We have, $\left(x\frac{dy}{dx} - y\right)e^{\frac{y}{x}} = x^2\ \cos x$

$\Rightarrow \left(\frac{dy}{dx} - \frac{y}{x}\right)e^{\frac{y}{x}} = x\ \cos x$

Put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \left(v + x\frac{dv}{dx} - v\right)e^{v} = x\cos x$

$\Rightarrow xe^{v}\frac{dv}{dx} = x\cos x$

$\Rightarrow \int e^{v}dv = \int \cos x\, dx$

$\Rightarrow e^{v} = \sin x + c$

$\Rightarrow\ e^{\frac{y}{x}} - \sin x = c$